# CS4670: Computer Vision

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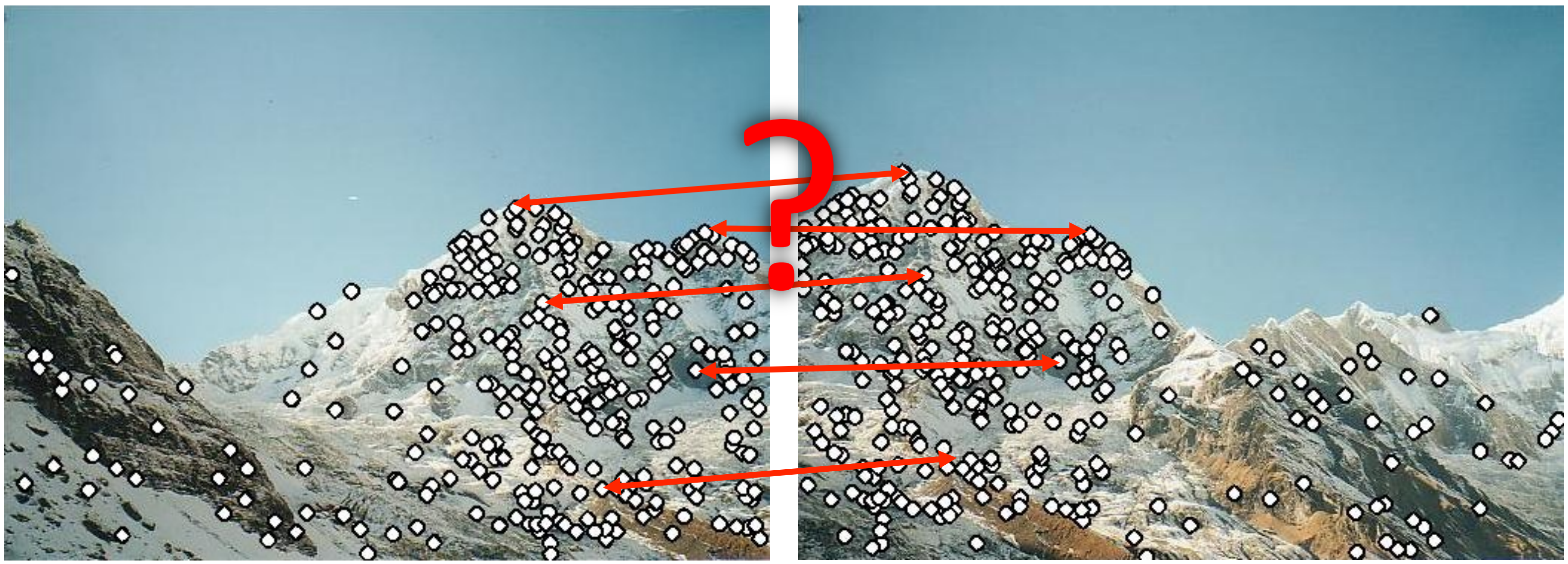
## Lecture 10: Feature Matching and Transforms

# Announcements

- HW 1
  - Some clarifications will be posted (check FAQ on piazza)
- HW 1 will be due a bit later (prob Fri)
- In class review: Fri Feb 27
- Prelim: Mar 5 2015

# Feature descriptors

We know how to detect good points
Next question: **How to match them?**

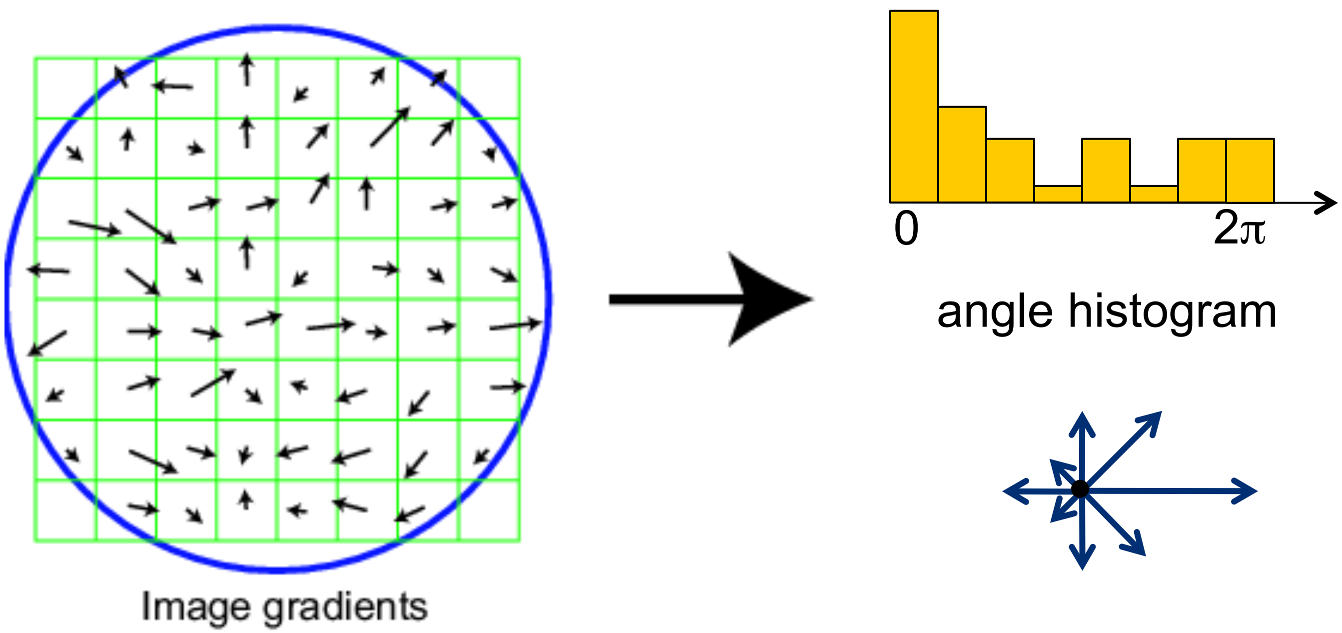



**Answer:** Come up with a *descriptor* for each point,
find similar descriptors between the two images

# Scale Invariant Feature Transform

Basic idea:

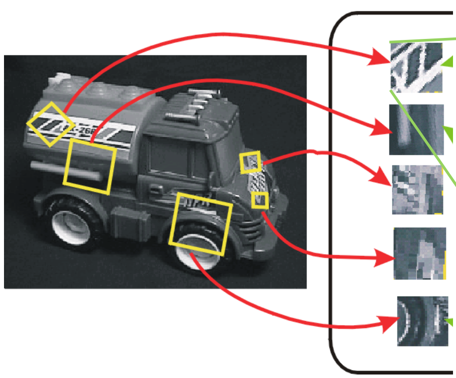
- Take 16x16 square window around detected feature
- Compute gradient orientation for each pixel
- Throw out weak edges (threshold gradient magnitude)
- Create histogram of surviving edge orientations

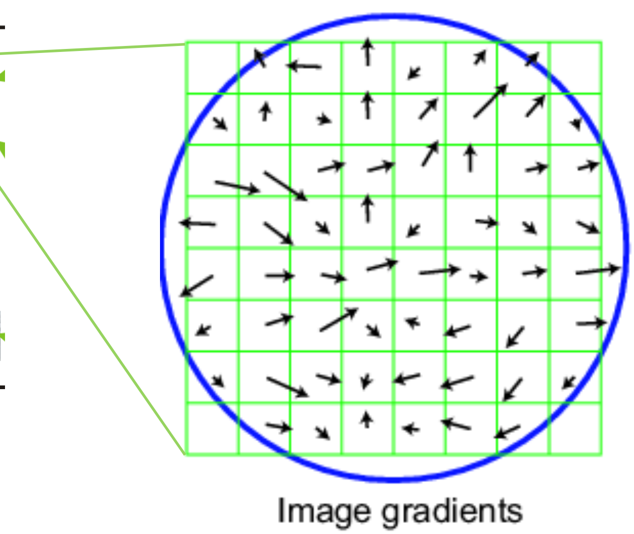



Adapted from slide by David Lowe

# SIFT vector formation

- Computed on rotated and scaled version of window according to computed orientation & scale
  – resample the window
- Based on gradients weighted by a Gaussian of variance 1.5 times the window (for smooth falloff)





# SIFT vector formation

- 4x4 array of gradient orientation histogram weighted by magnitude
- 8 orientations x 4x4 array = 128 dimensions
- Motivation:  some sensitivity to spatial layout, but not too much

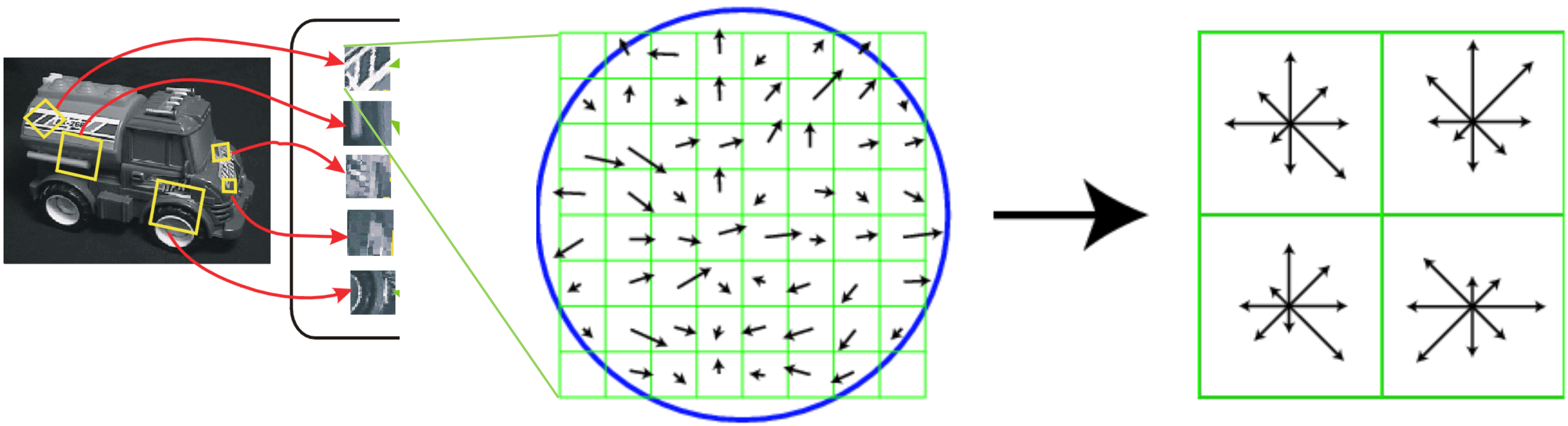


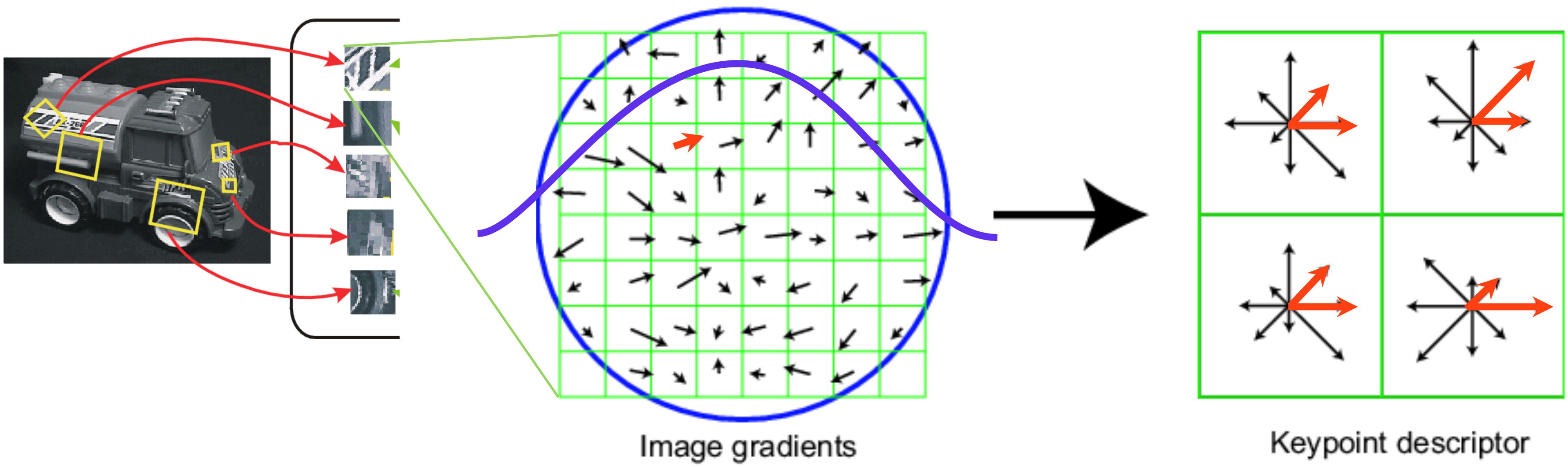
Image gradients

Keypoint descriptor

showing only 2x2 here but is 4x4

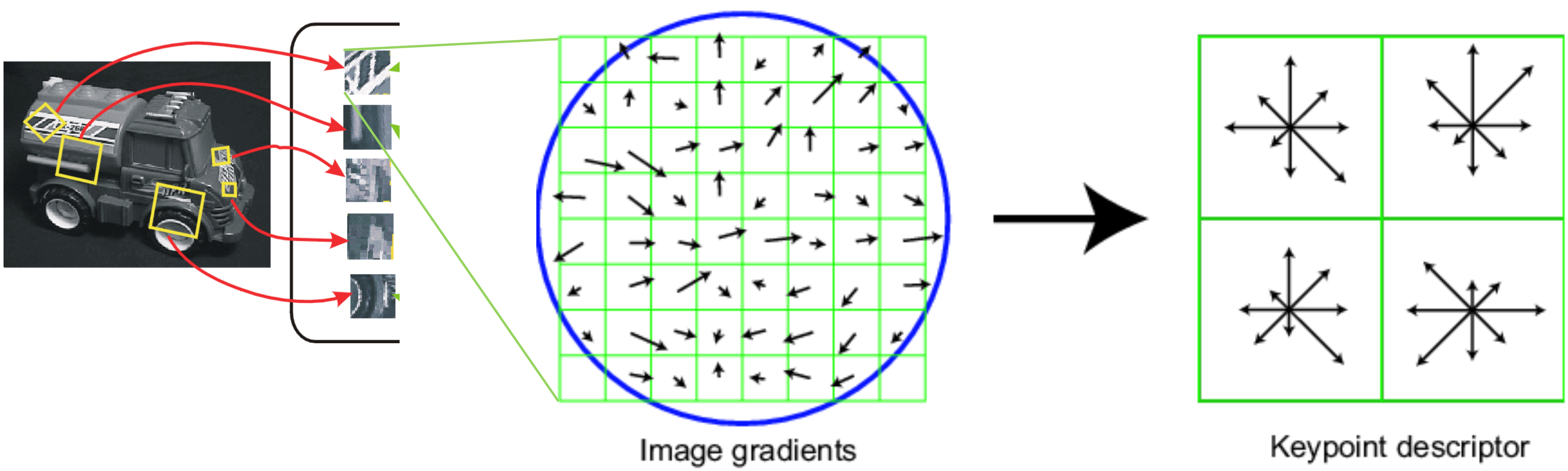
# Ensure smoothness

- Gaussian weight
- Trilinear interpolation
  - a given gradient contributes to 8 bins: 4 in space times 2 in orientation



# Reduce effect of illumination

- 128-dim vector normalized to 1
- Threshold gradient magnitudes to avoid excessive influence of high gradients
  - after normalization, clamp gradients >0.2
  - renormalize



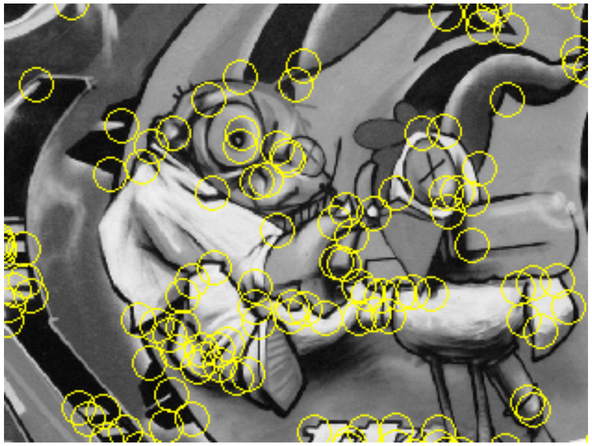
# Properties of SIFT

Extraordinarily robust matching technique

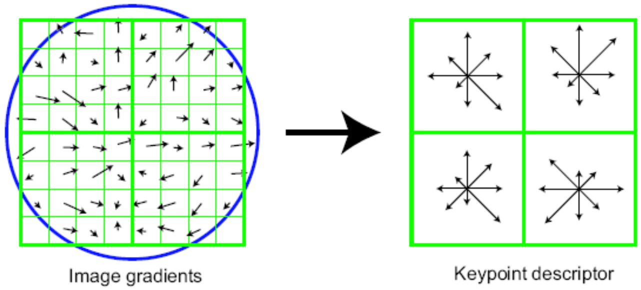
- Can handle changes in viewpoint
  - Up to about 60 degree out of plane rotation
- Can handle significant changes in illumination
  - Sometimes even day vs. night (below)
- Fast and efficient—can run in real time
- Lots of code available: http://people.csail.mit.edu/albert/ladypack/wiki/index.php/Known_implementations_of_SIFT

# Summary

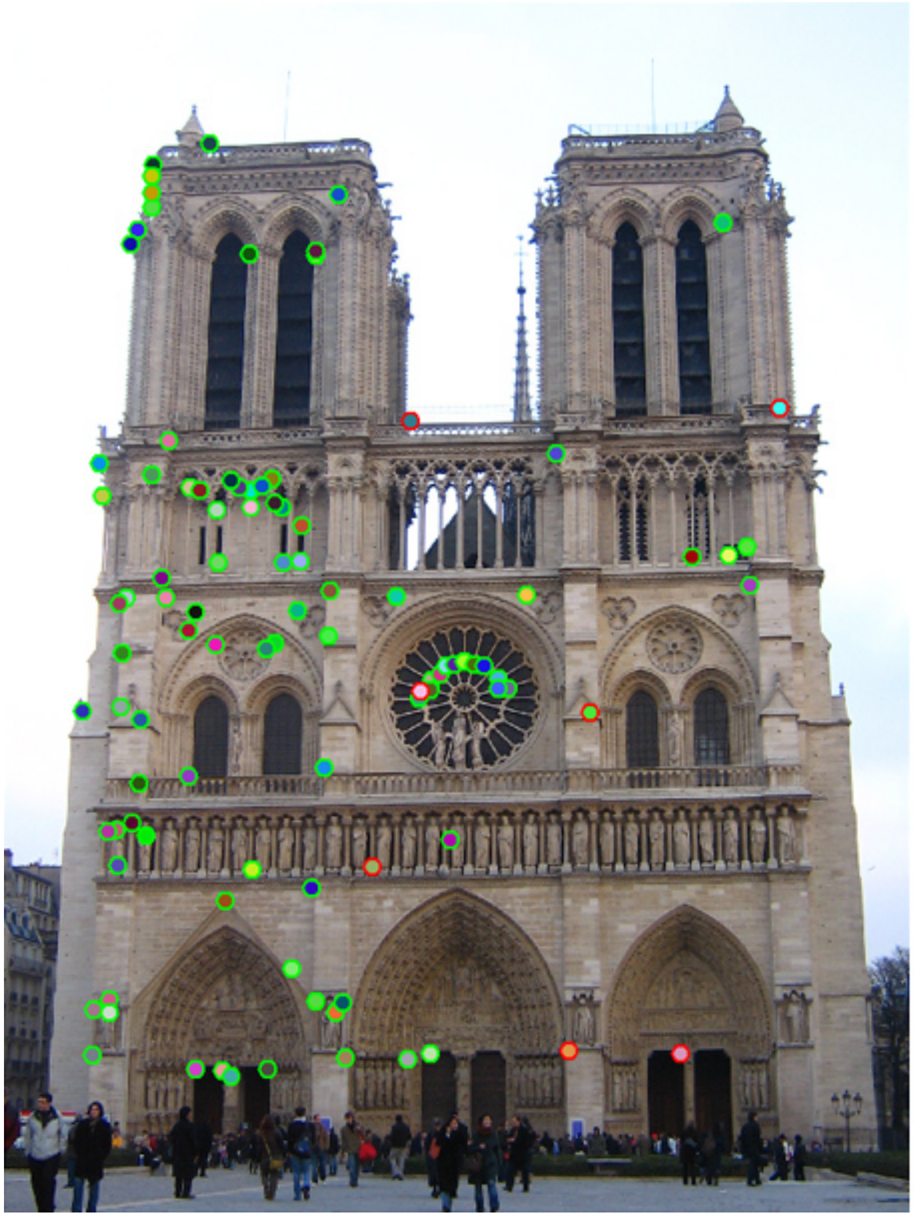
- Keypoint detection: repeatable and distinctive
  - Corners, blobs, stable regions
  - Harris, DoG

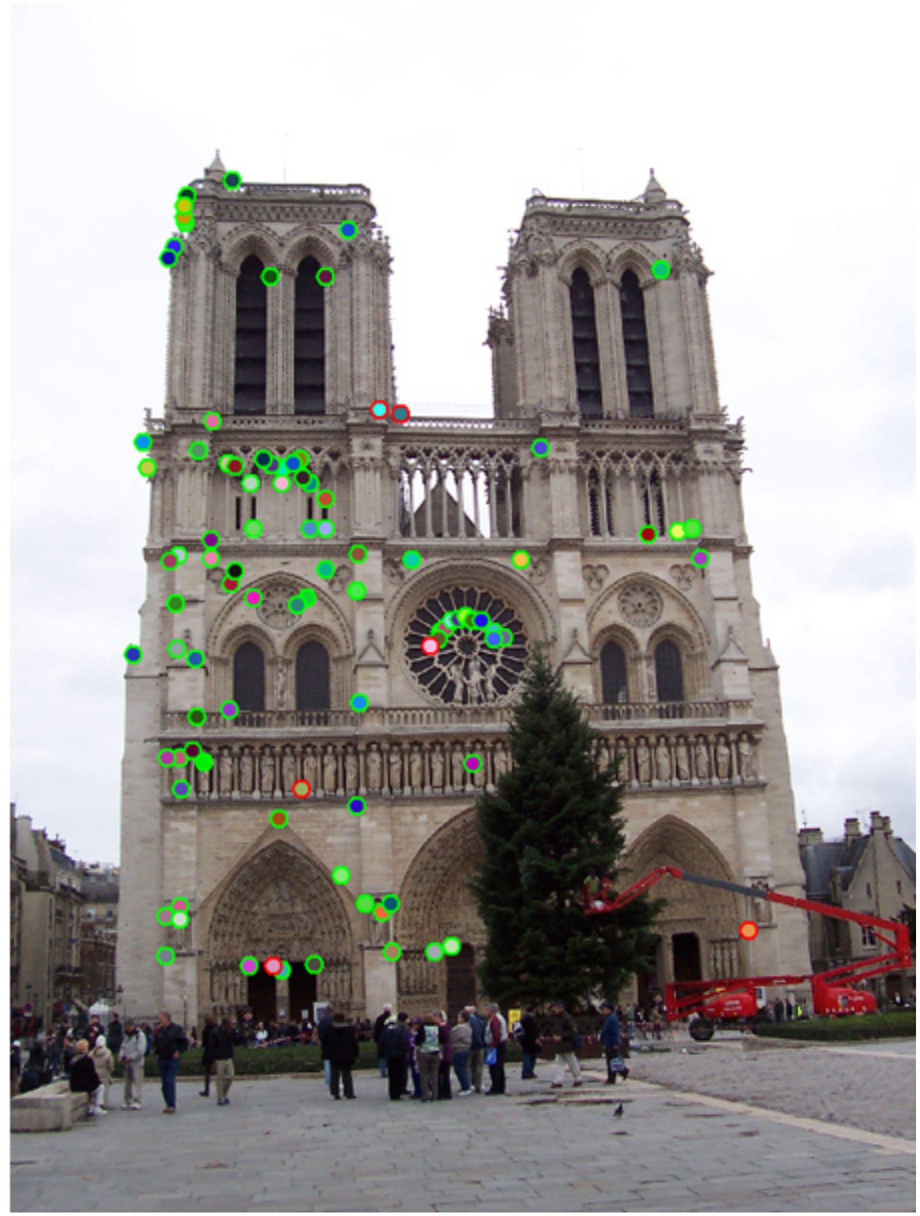


- Descriptors: robust and selective
  - spatial histograms of orientation
  - SIFT and variants are typically good for stitching and recognition
  - But, need not stick to one



# Which features match?





# Feature matching

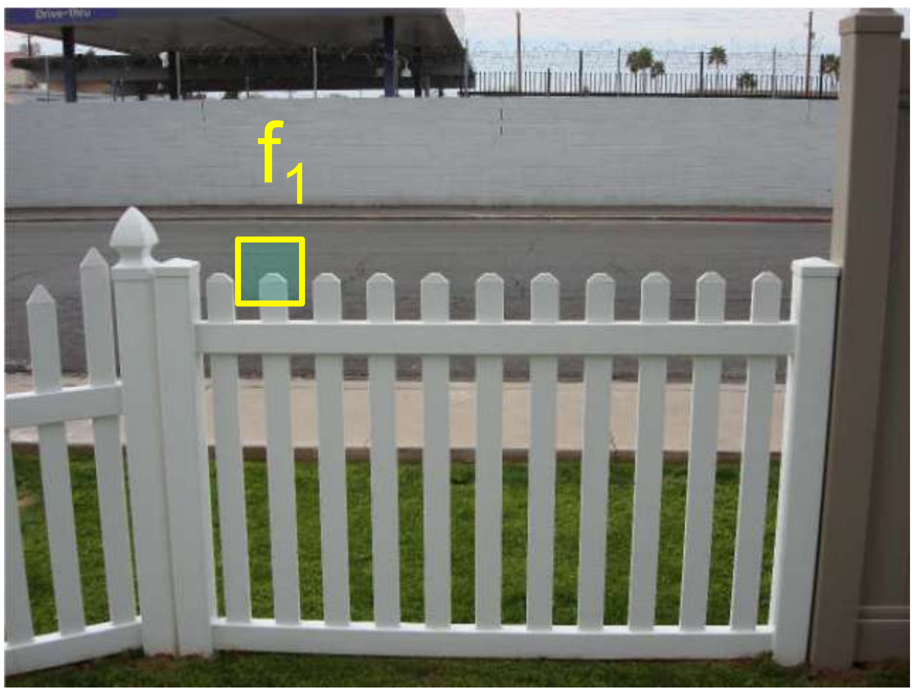
Given a feature in $I_1$, how to find the best match in $I_2$?

1. Define distance function that compares two descriptors
2. Test all the features in $I_2$, find the one with min distance

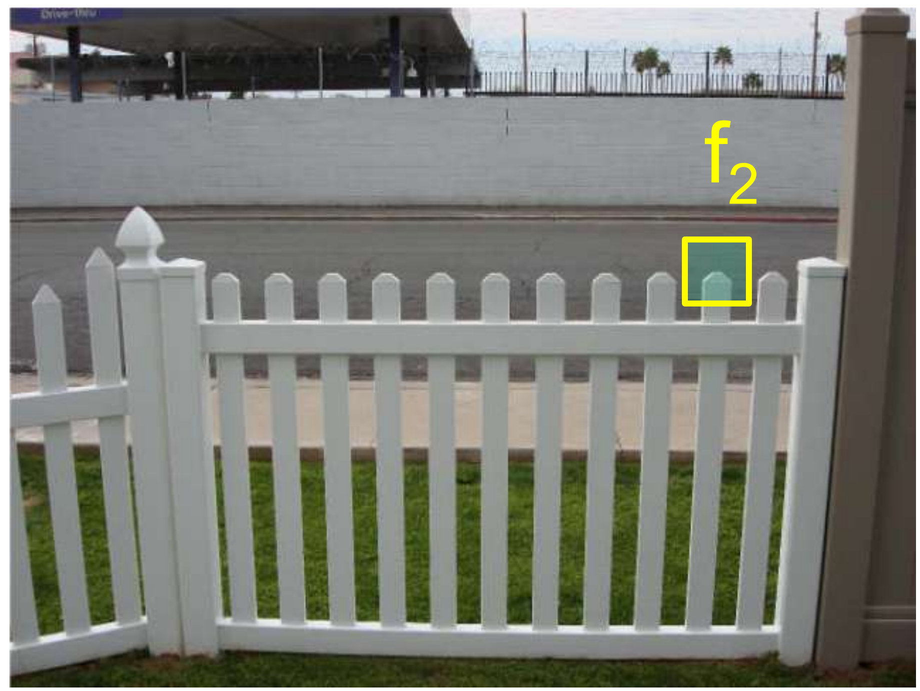
# Feature distance

How to define the difference between two features $f_1$, $f_2$?

- Simple approach: $L_2$ distance, $||f_1 - f_2||$
- can give good scores to ambiguous (incorrect) matches




$I_1$

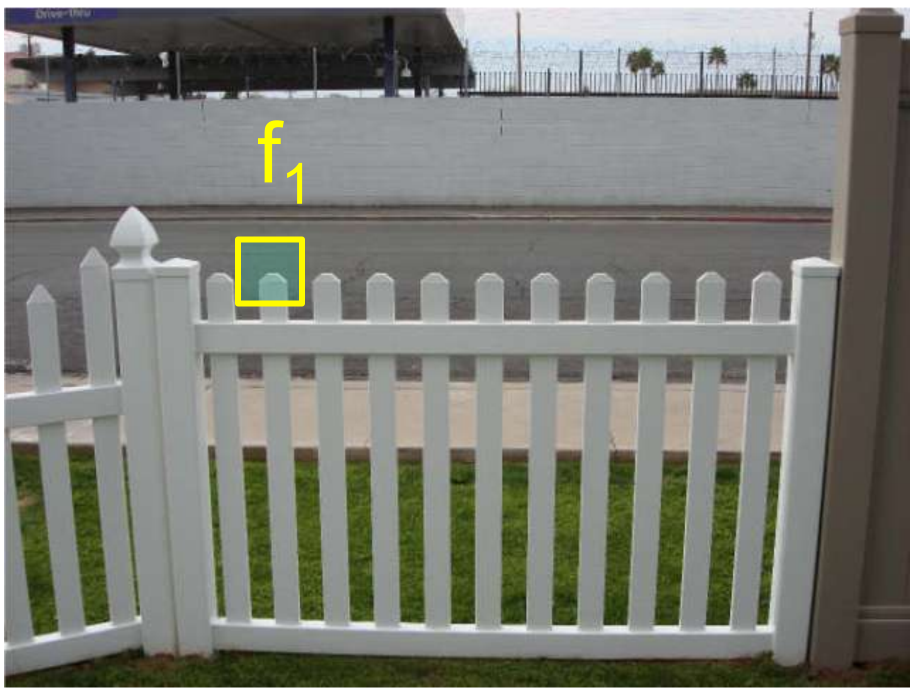



$I_2$

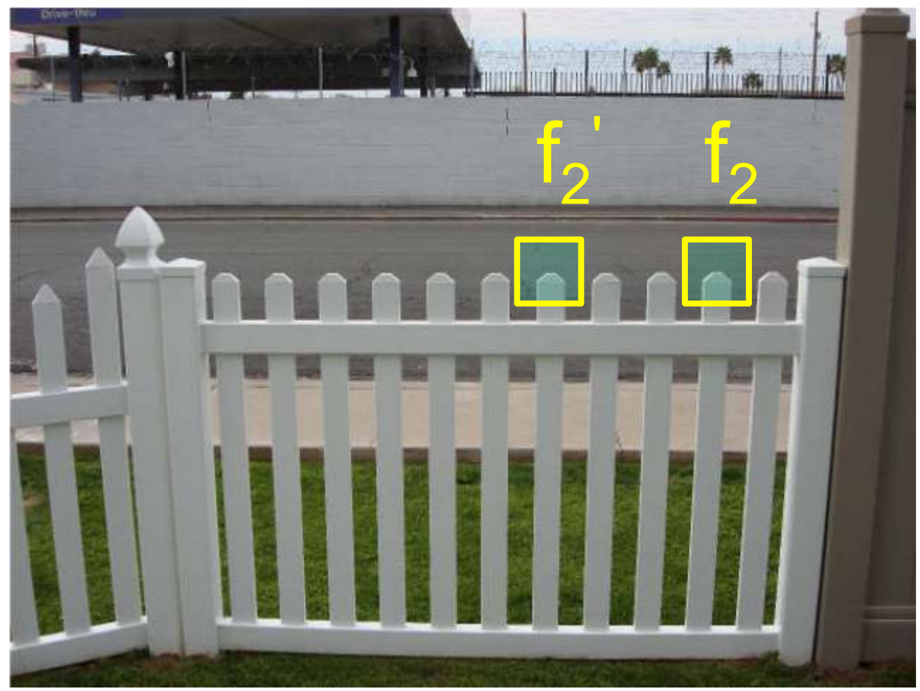
# Feature distance

How to define the difference between two features $f_1$, $f_2$?

- Better approach: ratio distance = $||f_1 - f_2|| / || f_1 - f_2' ||$
  - $f_2$ is best SSD match to $f_1$ in $I_2$
  - $f_2'$ is $2^{nd}$ best SSD match to $f_1$ in $I_2$
  - gives bad scores for ambiguous matches



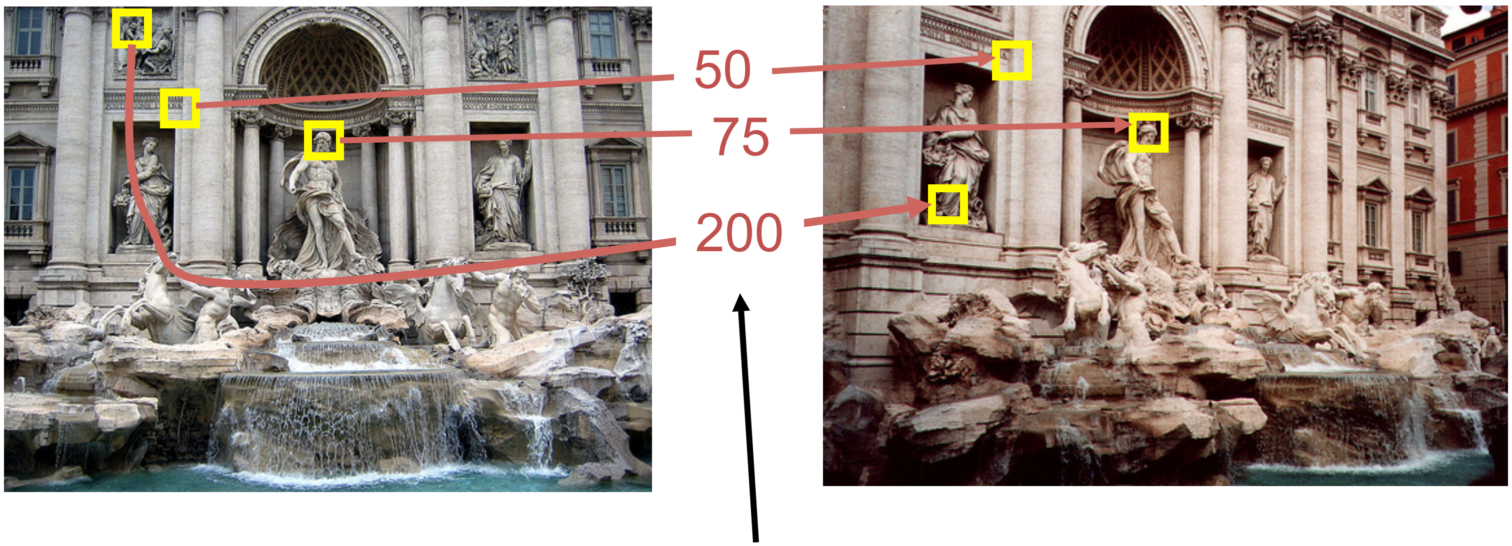

$I_1$




$I_2$

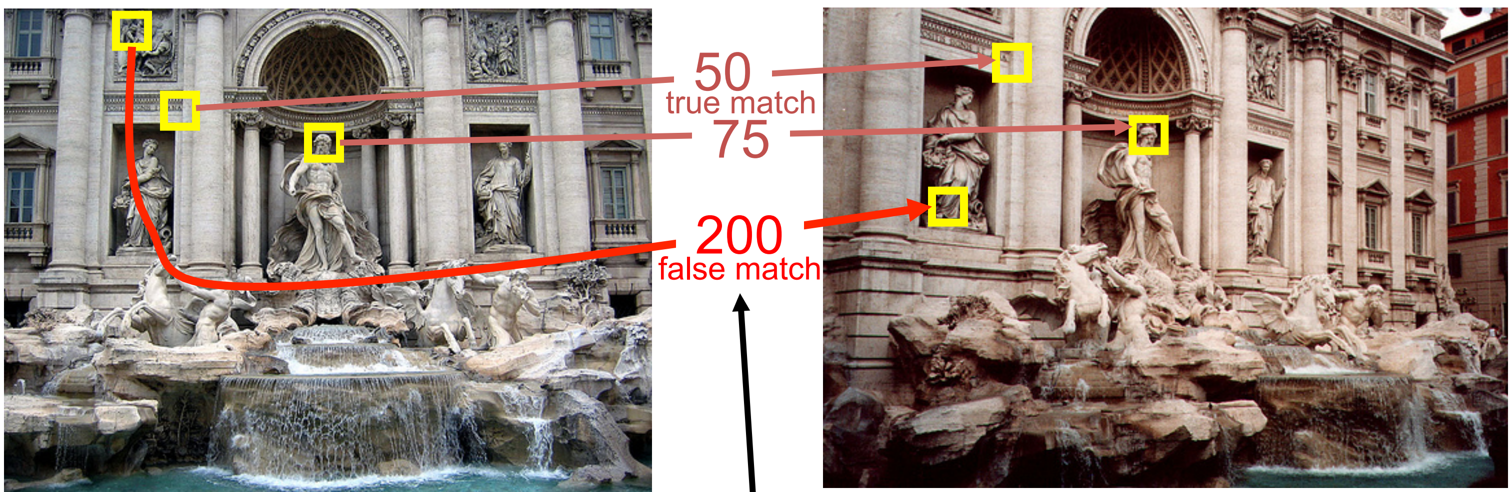
# Evaluating the results

How can we measure the performance of a feature matcher?



# True/false positives

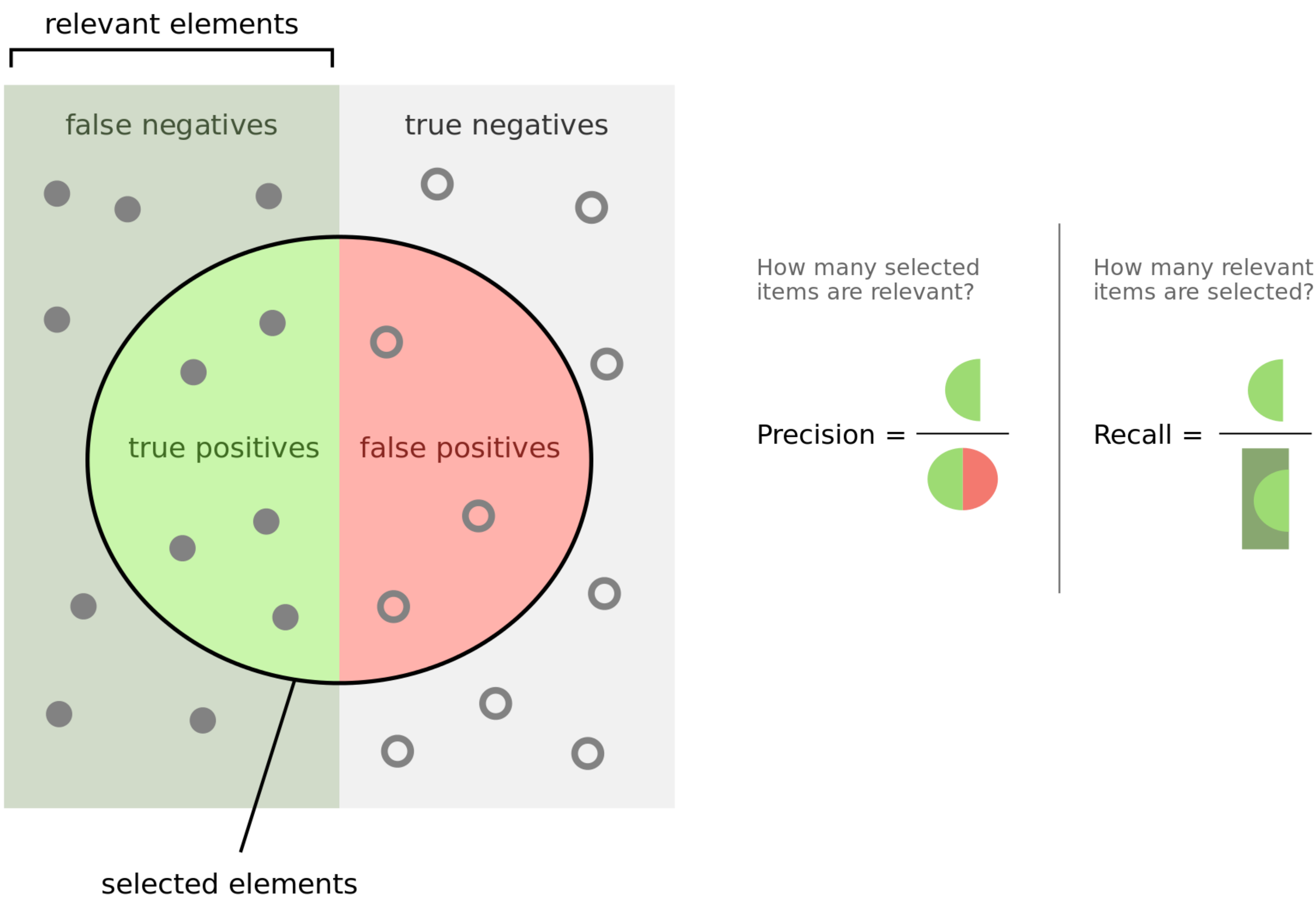
How can we measure the performance of a feature matcher?



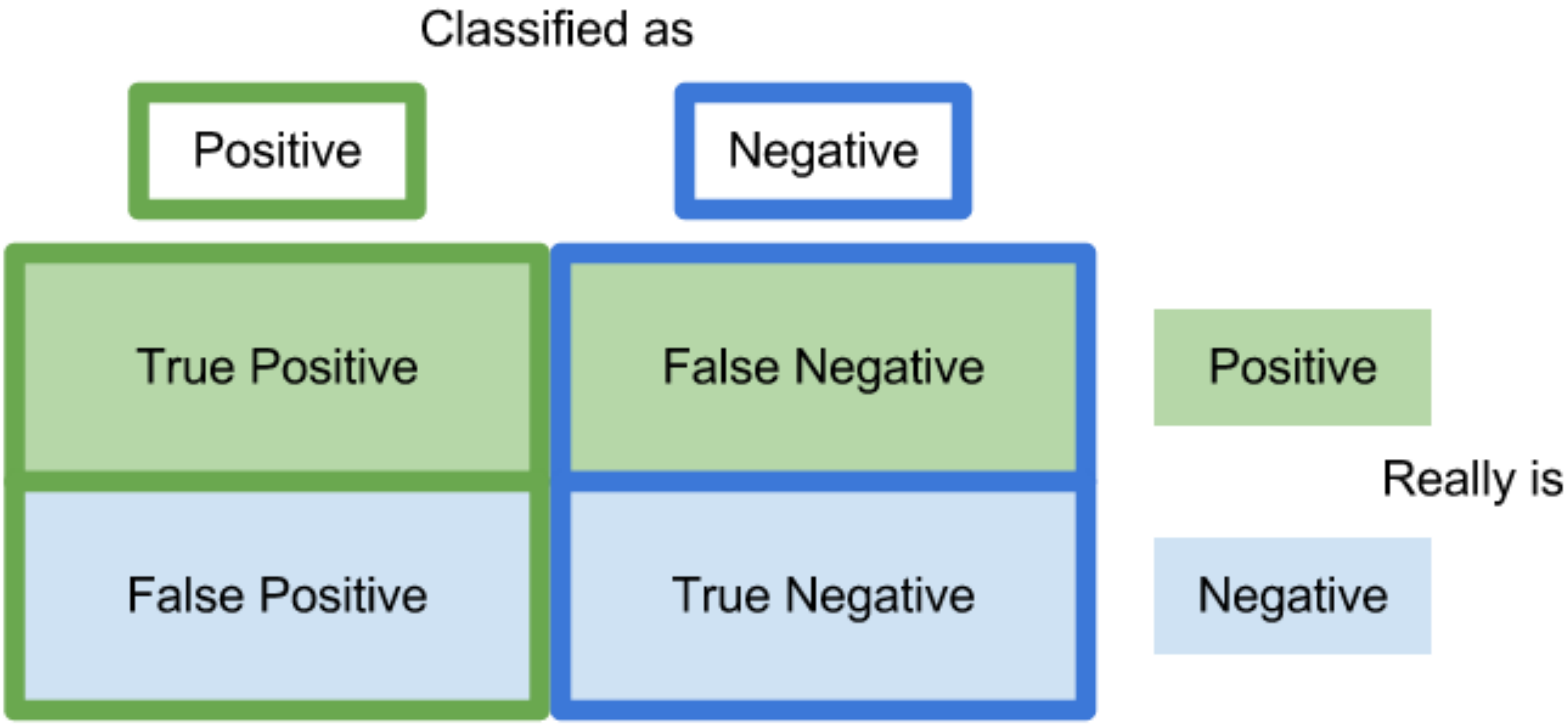

feature distance

The distance threshold affects performance

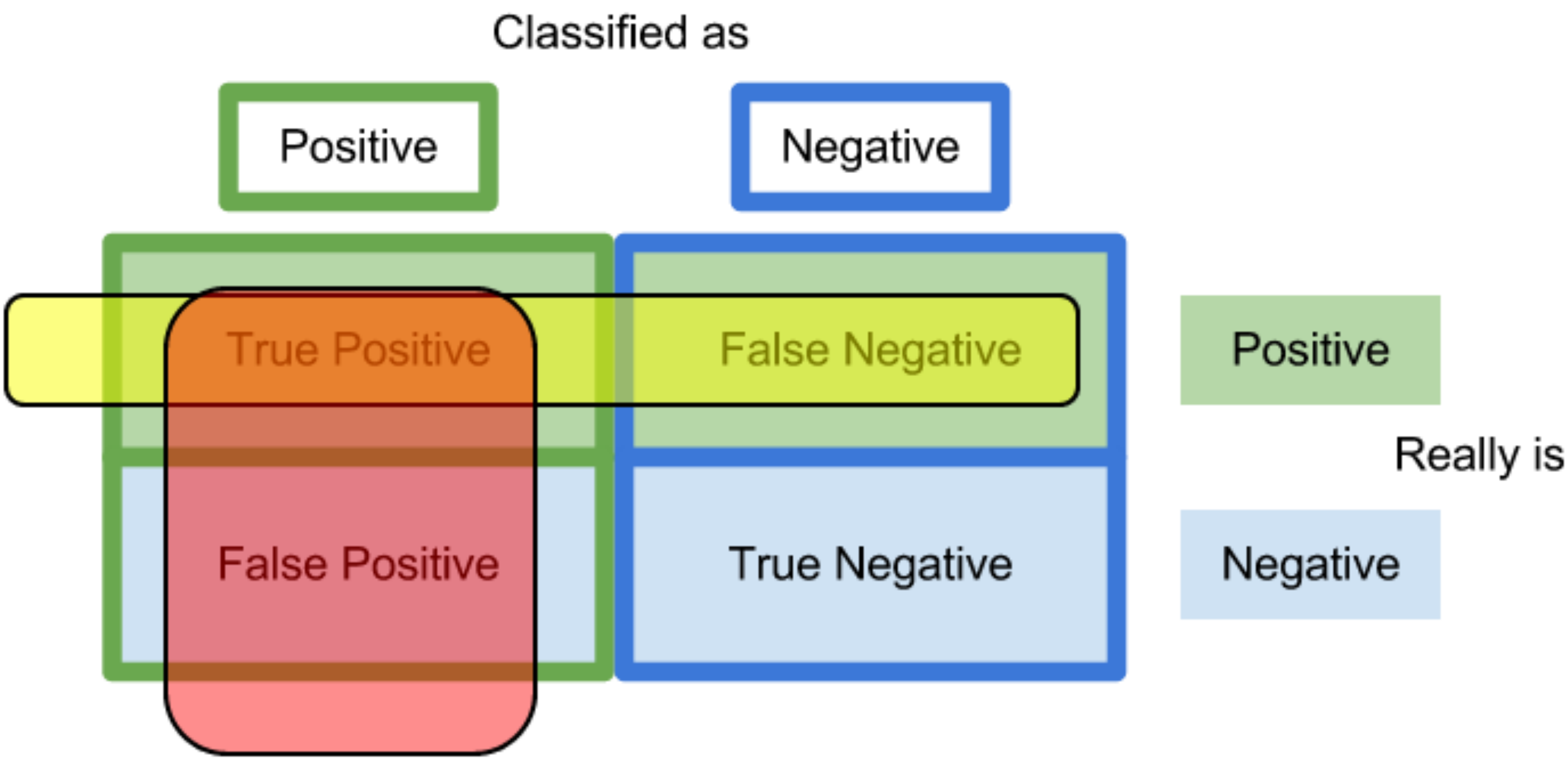
- True positives = # of detected matches that are correct
  - Suppose we want to maximize these—how to choose threshold?
- False positives = # of detected matches that are incorrect
  - Suppose we want to minimize these—how to choose threshold?


relevant elements
false negatives
true negatives
true positives
false positives
selected elements
How many selected items are relevant?
Precision =
How many relevant items are selected?
Recall =

# Confusion Matrix




https://uberpython.wordpress.com/2012/01/01/precision-recall-sensitivity-and-specificity/




— Precision in red, recall in yellow

https://uberpython.wordpress.com/2012/01/01/precision-recall-sensitivity-and-specificity/

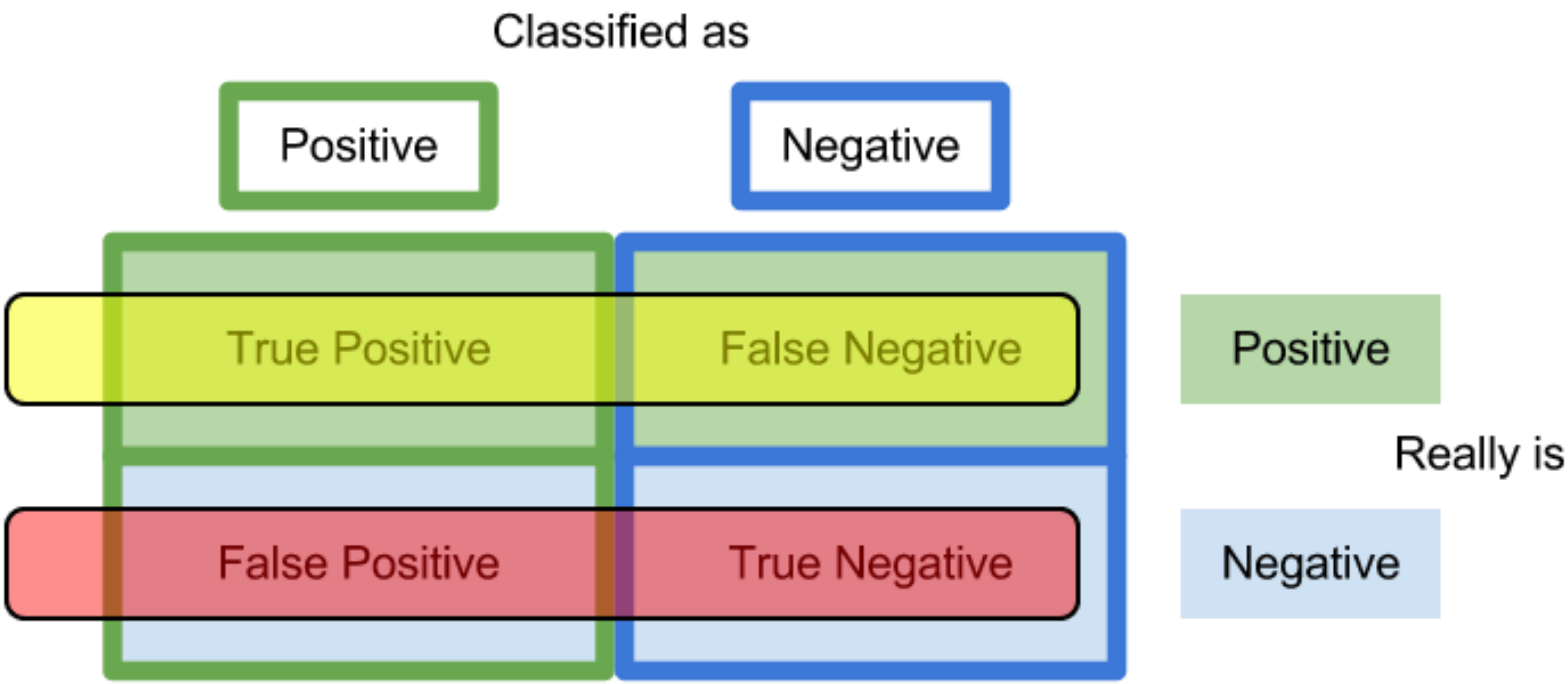
# Precision vs. Recall

Examples

- 1000 animals, 100 dogs
- Algorithm finds 50 (of which 40 are dogs, 10 are cats)
  - Precision =
  - Recall =

- Algorithm finds 10 (of which 10 are dogs)
  - Precision =
  - Recall =

- Algorithm returns 1000 (of which 100 are dogs)
  - Precision =
  - Recall =




— Sensitivity in yellow, specificity in red

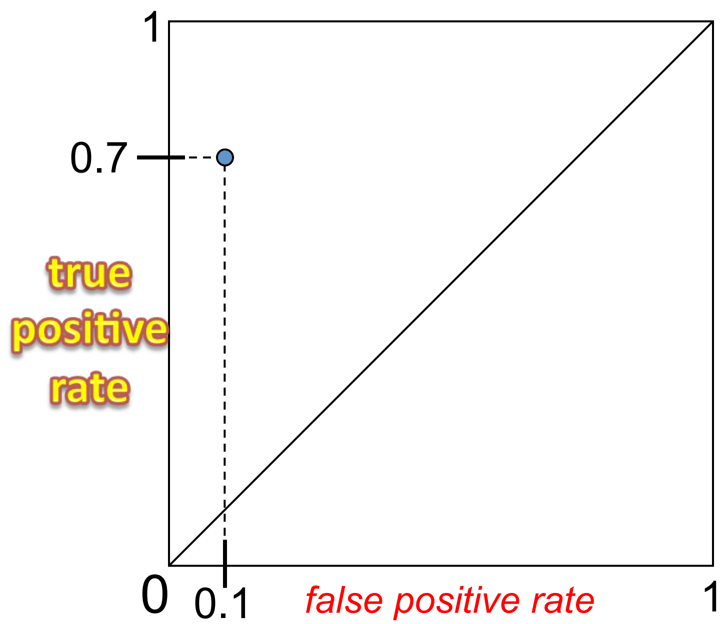
https://uberpython.wordpress.com/2012/01/01/precision-recall-sensitivity-and-specificity/

# Evaluating the results

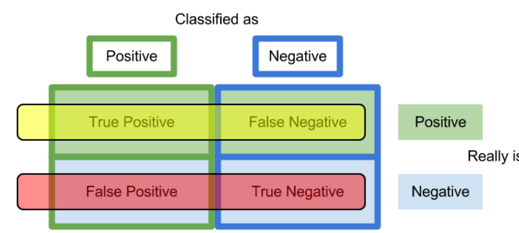
How can we measure the performance of a feature matcher?

$$\text{true positive rate} = \frac{\text{\# true positives}}{\text{\# matching features (positives)}}$$

"recall"

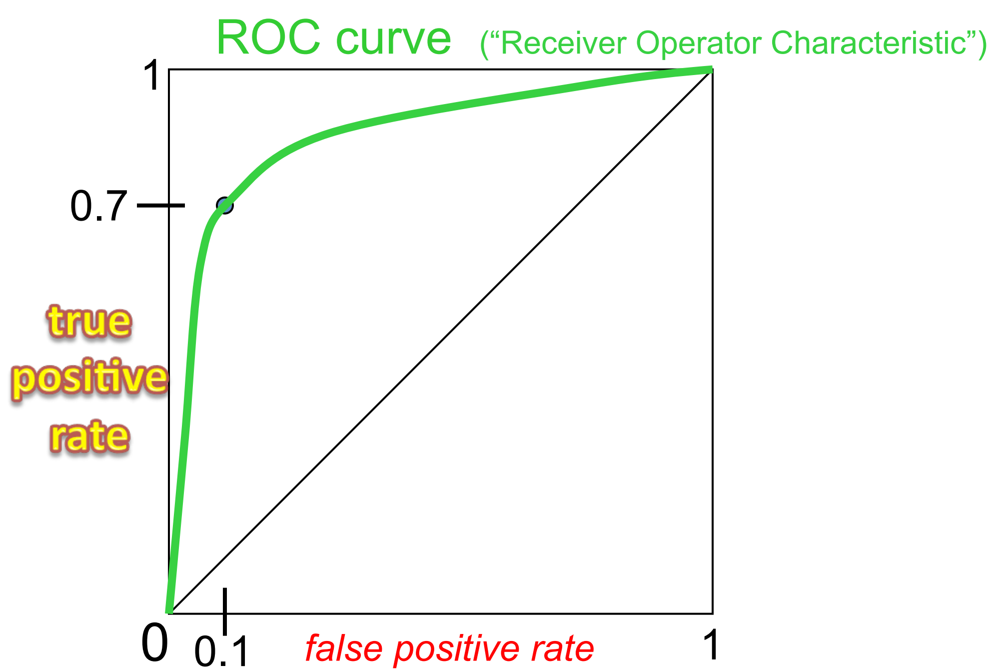



$$\text{false positive rate} = \frac{\text{\# false positives}}{\text{\# unmatched features (negatives)}}$$



# Evaluating the results

How can we measure the performance of a feature matcher?




$$\frac{\text{\# true positives}}{\text{\# matching features (positives)}}$$

"recall"

$$\frac{\text{\# false positives}}{\text{\# unmatched features (negatives)}}$$

AUC (area under curve)

# More on feature detection/description

Address http://www.robots.ox.ac.uk/~vgg/research/affine/

Google mikolajczyk Search Web

## Affine Covariant Regions

### Publications

**Region detectors**

- *Harris-Affine & Hessian Affine*: K. Mikolajczyk and C. Schmid, Scale and Affine invariant interest point detectors. In IJCV 1(60):63-86, 2004. PDF
- *MSER*: J.Matas, O. Chum, M. Urban, and T. Pajdla, Robust wide baseline stereo from maximally stable extremal regions. In BMVC p. 384-393, 2002. PDF
- *IBR & EBR*: T.Tuytelaars and L. Van Gool, Matching widely separated views based onaffine invariant regions. In IJCV 1 (59):61-85, 2004. PDF
- *Salient regions*: T. Kadir, A. Zisserman, and M. Brady, An affine invariant salient region detector. In ECCV p. 404-416, 2004. PDF

**Region descriptors**

- *SIFT*: D. Lowe, Distinctive image features from scale invariant keypoints. In IJCV 2(60):91-110, 2004. PDF

**Performance evaluation**

- K. Mikolajczyk, T. Tuytelaars, C. Schmid, A. Zisserman, J. Matas, F. Schaffalitzky, T. Kadir and L. Van Gool, A comparison of affine region detectors. Technical Report, accepted to IJCV. PDF
- K. Mikolajczyk, C. Schmid, A performance evaluation of local descriptors. Technical Report, accepted to PAMI. PDF

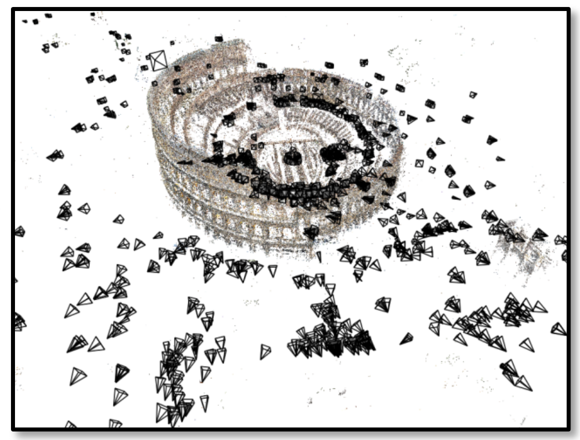
# Lots of applications

Features are used for:

- Image alignment (e.g., mosaics)
- 3D reconstruction
- Motion tracking
- Object recognition
- Indexing and database retrieval
- Robot navigation
- … other

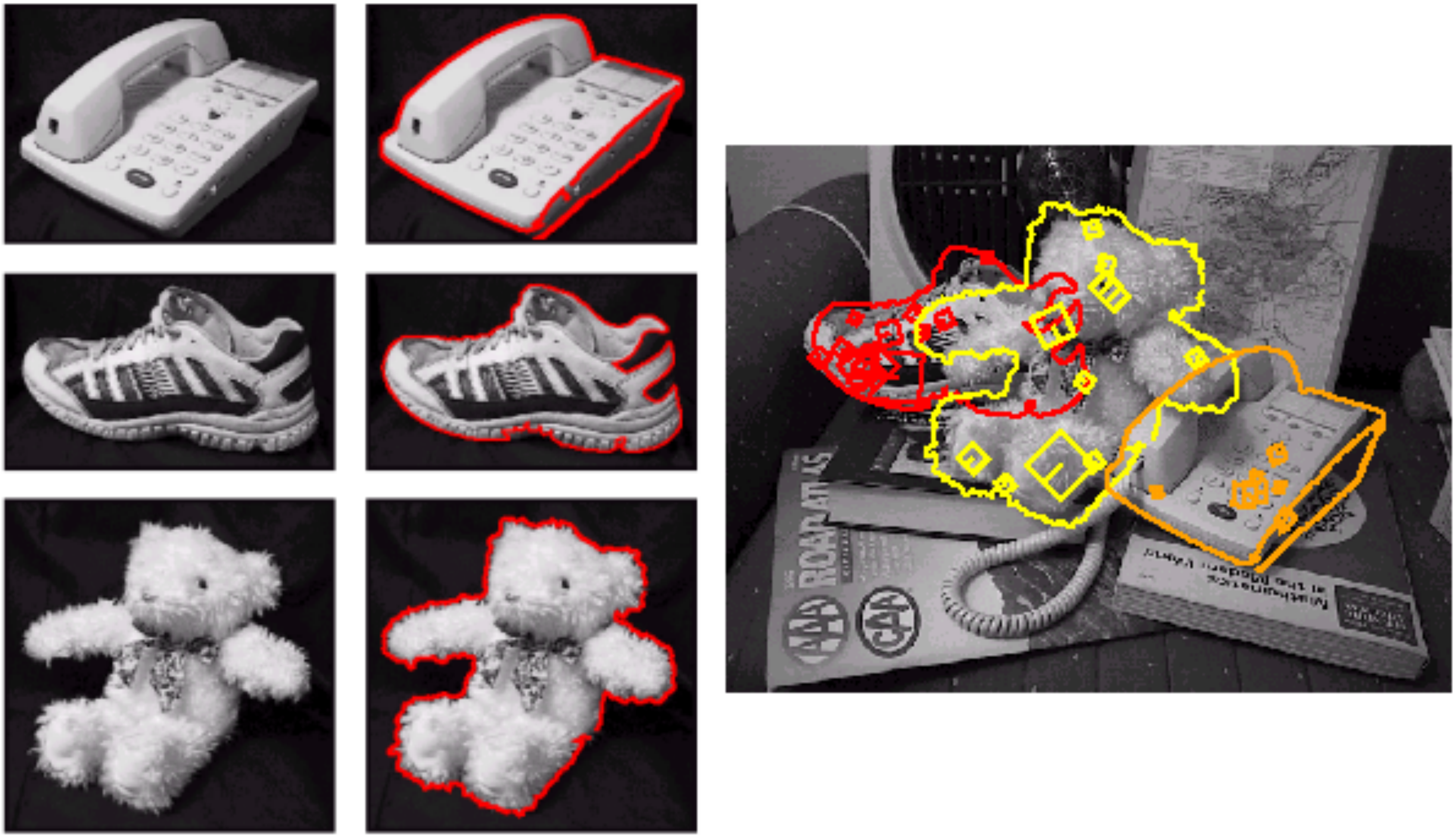
# 3D Reconstruction

Internet Photos ("Colosseum")



Reconstructed 3D cameras and points

# Object recognition (David Lowe)



# Sony Aibo

SIFT usage:

- Recognize charging station
- Communicate with visual cards
- Teach object recognition

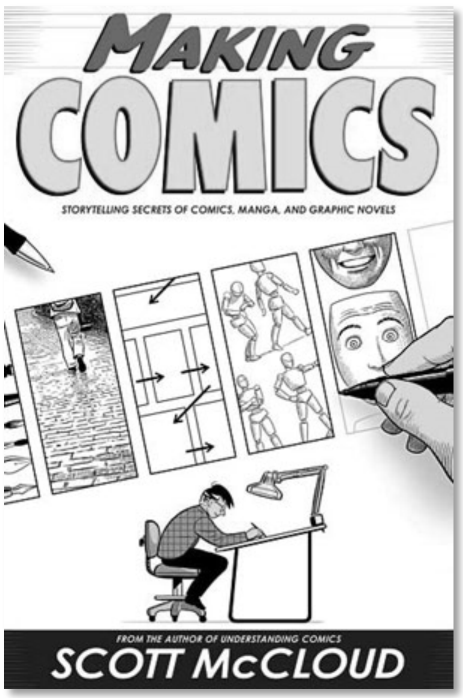
# Available at a web site near you…

- For most local feature detectors, executables are available online:
  - http://www.robots.ox.ac.uk/~vgg/research/affine
  - http://www.cs.ubc.ca/~lowe/keypoints/
  - http://www.vision.ee.ethz.ch/~surf

# Questions?

# Image alignment

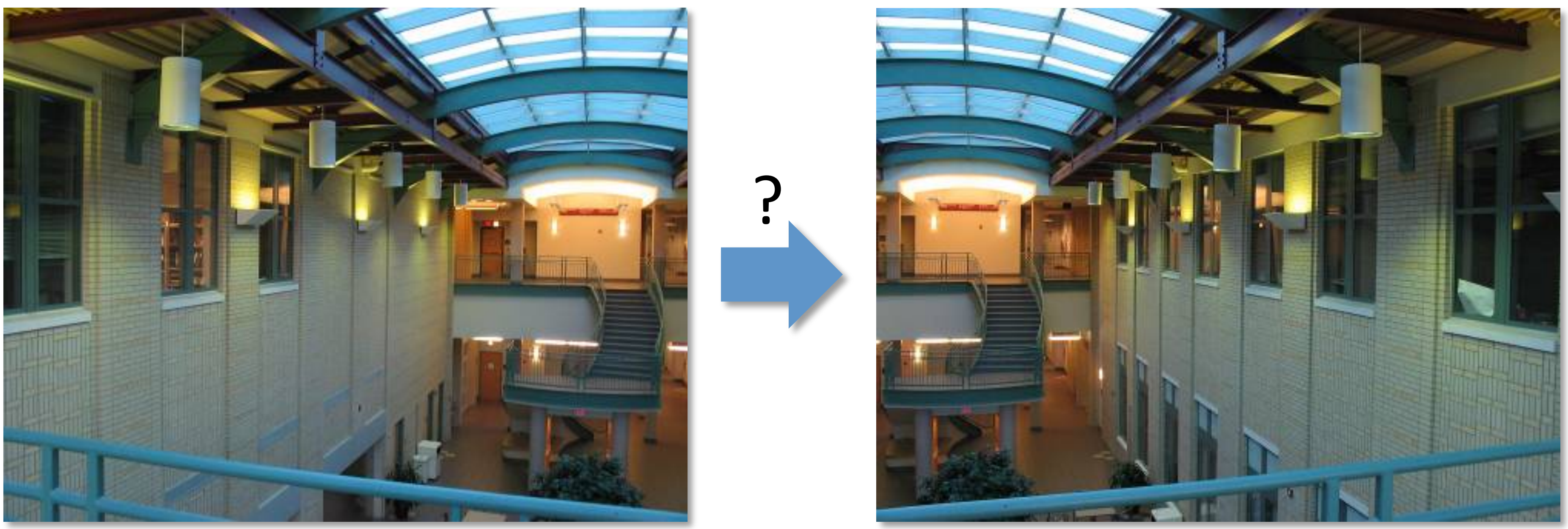
# What is the geometric relationship between these two images?




?


**Answer: Similarity transformation** (translation, rotation, uniform scale)

# What is the geometric relationship between these two images?

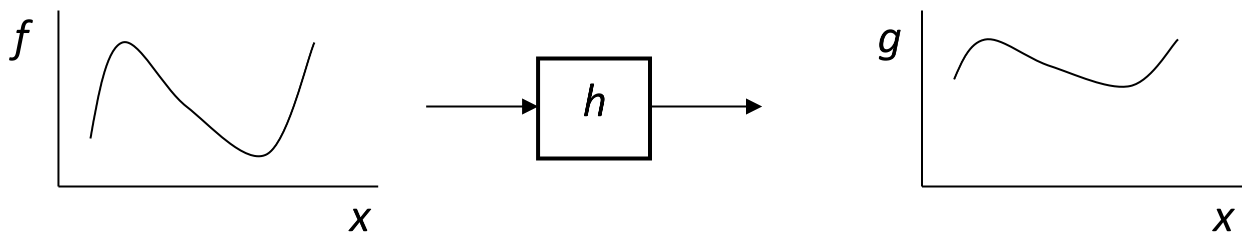


# What is the geometric relationship between these two images?

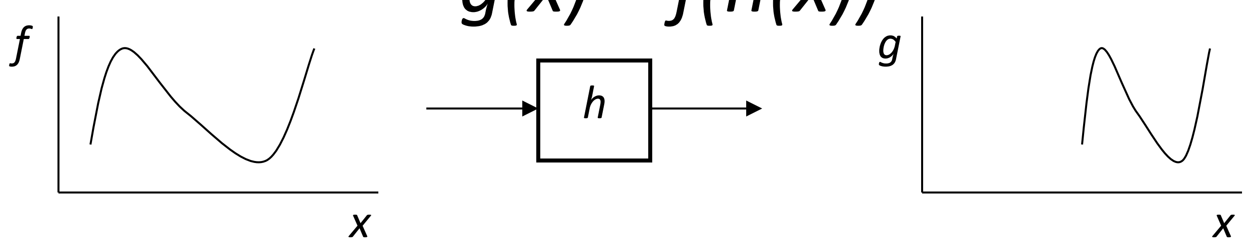
**Very important for creating mosaics!**

# Image Warping

- image filtering: change *range* of image
  - $g(x) = h(f(x))$




- image warping: change *domain* of image
  - $g(x) = f(h(x))$



# Image Warping

- image filtering: change *range* of image
  - $g(x) = h(f(x))$

$f$

$h$

$g$

- image warping: change *domain* of image
  - $g(x) = f(h(x))$

$f$

$h$

$g$

# Parametric (global) warping

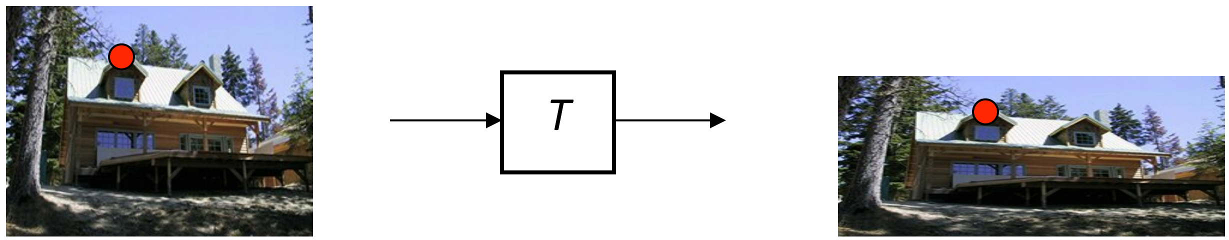
- Examples of parametric warps:

translation

rotation

aspect

# Parametric (global) warping




**p** = (x,y) **p'** = (x',y')

- Transformation T is a coordinate-changing machine:

  $$p' = T(p)$$

- What does it mean that *T* is global?
  - Is the same for any point p
  - can be described by just a few numbers (parameters)
- Let's consider *linear* xforms (can be represented by a 2D matrix):

$$\mathbf{p}' = \mathbf{T}\mathbf{p} \qquad \begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

# Common linear transformations

- Uniform scaling by *s*:

(0,0)

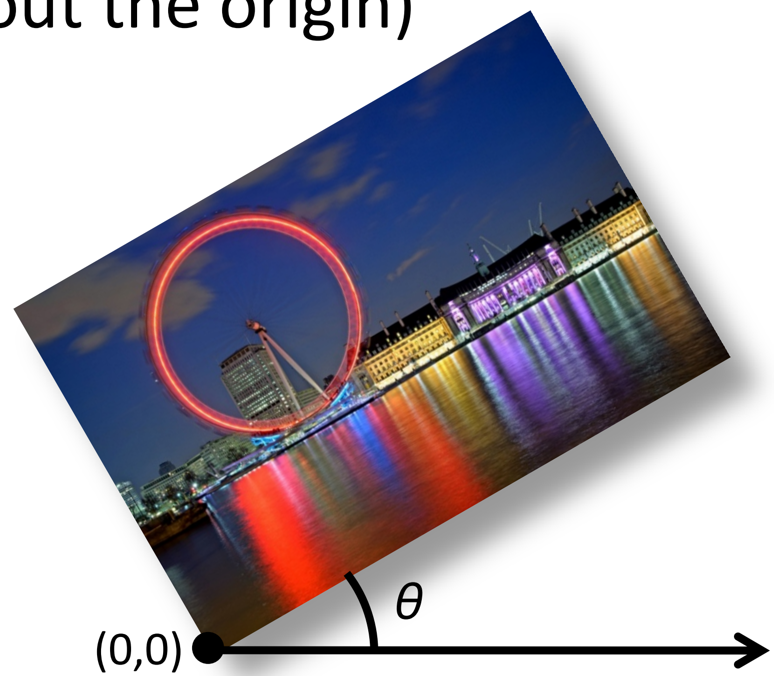
(0,0)

$$\mathbf{S} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

What is the inverse?

# Common linear transformations

- Rotation by angle $\theta$ (about the origin)


$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

What is the inverse?

For rotations:

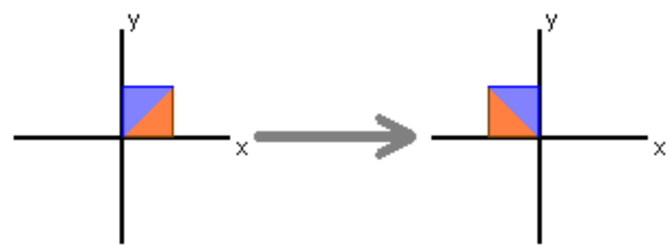
$$\mathbf{R}^{-1} = \mathbf{R}^{T}$$

# 2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

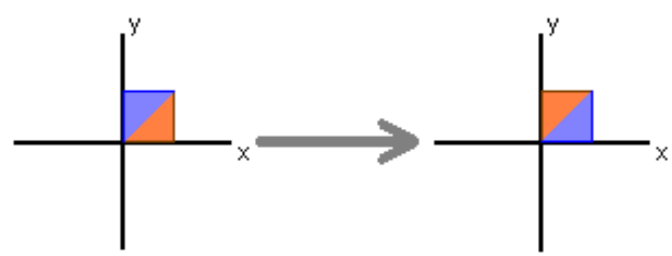
2D mirror about Y axis?

$$x' = -x \qquad T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$y' = y$$




2D mirror across line y = x?

$$x' = y \qquad \mathbf{T} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$y' = x$$



# 2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$
\begin{aligned}
x' &= x + t_x \\
y' &= y + t_y
\end{aligned}
$$

NO!

Translation is not a linear operation on 2D coordinates

# All 2D Linear Transformations

- Linear transformations are combinations of …
  - Scale,
  - Rotation,
  - Shear, and
  - Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Properties of linear transformations:
  - Origin maps to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition

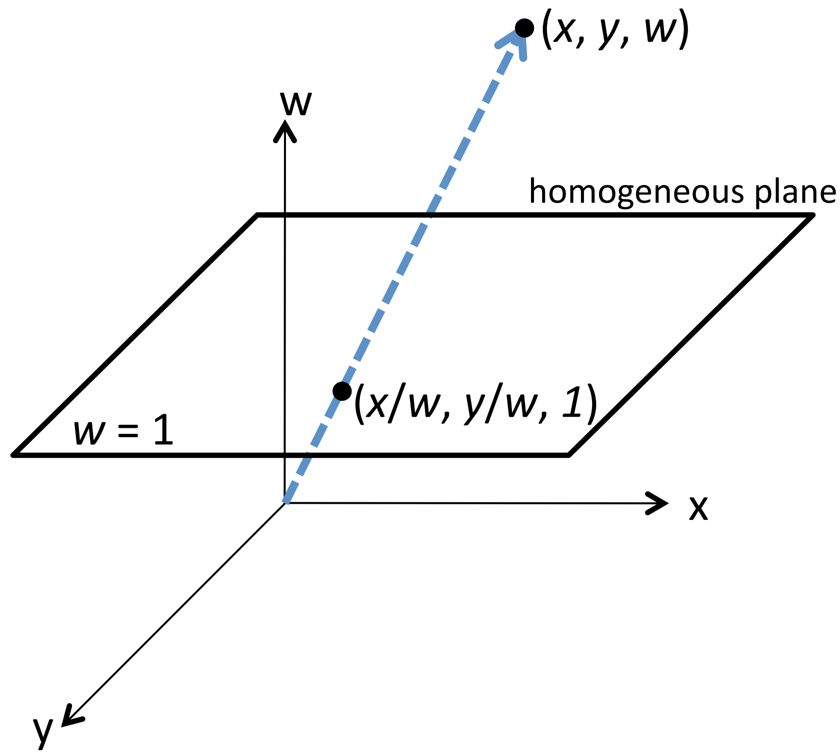
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

# Homogeneous coordinates

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates




Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

# Translation

- Solution: homogeneous coordinates to the rescue

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

# Affine transformations

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

any transformation with last row [ 0 0 1 ] we call an *affine* transformation

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

# Basic affine transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}_{\boldsymbol{x}} & 0 & 0 \\ 0 & \boldsymbol{s}_{\boldsymbol{y}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2D *in-plane* rotation

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{sh}_x & 0 \\ \boldsymbol{sh}_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Shear

# Affine Transformations

- Affine transformations are combinations of …
  - Linear transformations, and
  - Translations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Properties of affine transformations:
  - Origin does not necessarily map to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition

# Is this an affine transformation?