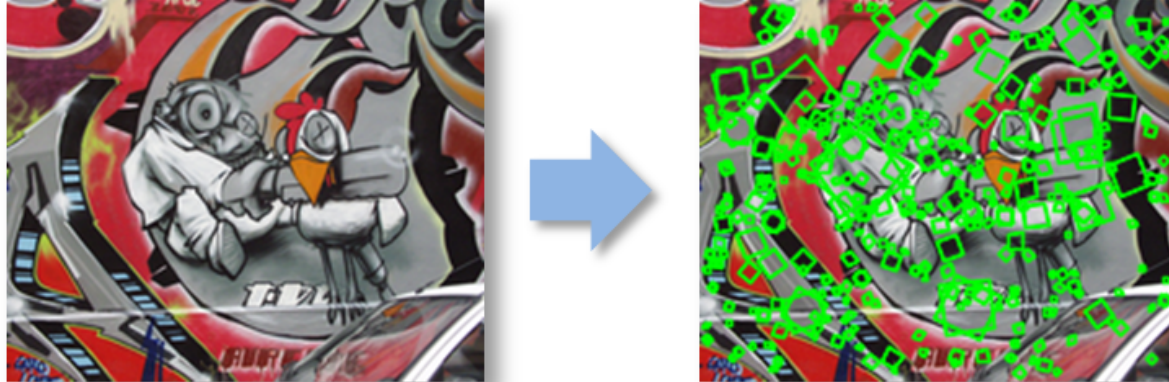


CS4670: Computer Vision

Kavita Bala

Lecture 6: Feature Detection



Quiz

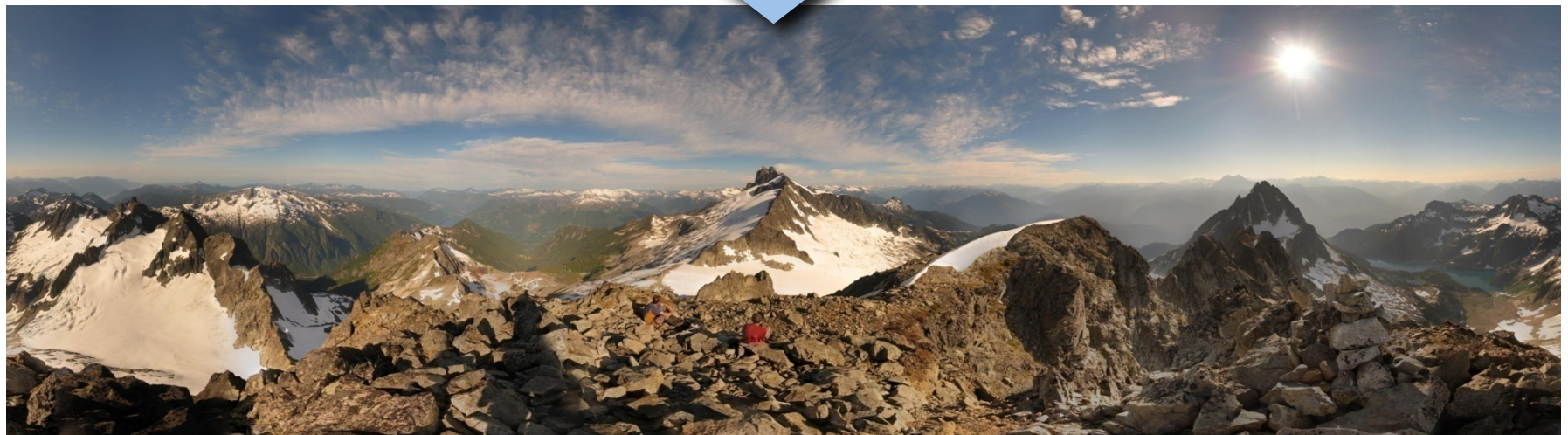
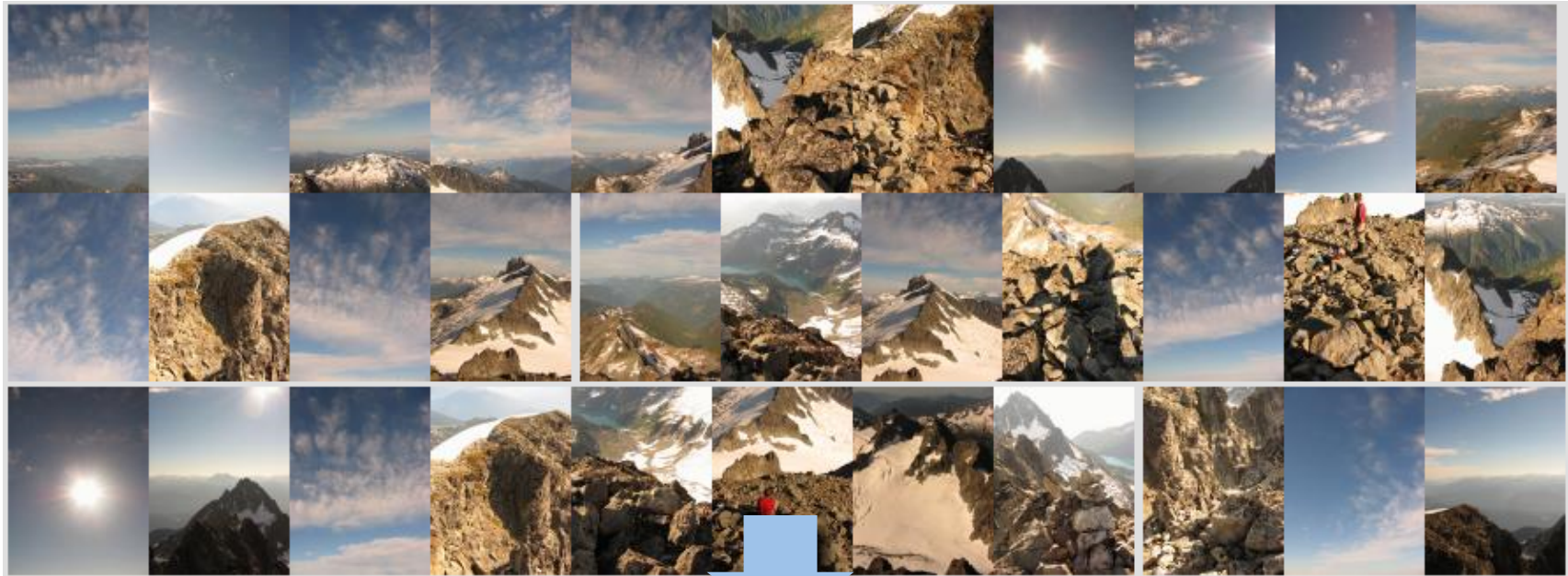
- Write your name and netid
- ... now

Name (please print):

NETID (i.e., email address @cornell.edu):

This test is closed-book / closed-note. You have 10 minutes.

Motivation: Automatic panoramas



Credit: Matt Brown

Microsoft puts some pizzazz into panoramic photos

The company's ICE software now can stitch video frames into panoramic images and fill in inevitable gaps. It shows the field of computational photography is still in its early days.

by **Stephen Shankland** [@stshank](#) / February 5, 2015 9:30 AM PST

[1](#) / [f](#) 88 / [t](#) 139 / [in](#) / [g+](#) / [more +](#)



Microsoft ICE stitches still photos or video frames into a single panoramic image. Version 2.0 can fill in gaps so you don't have to crop as much.

Screenshot by Stephen Shankland/CNET

<http://gigapixelartzoom.com>

Approach

Feature detection: find it

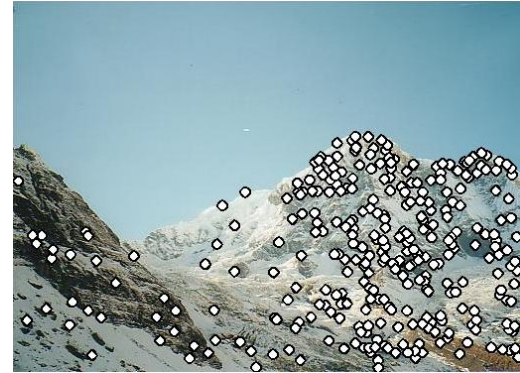
Feature descriptor: represent it

Feature matching: match it

Feature tracking: track it, when motion

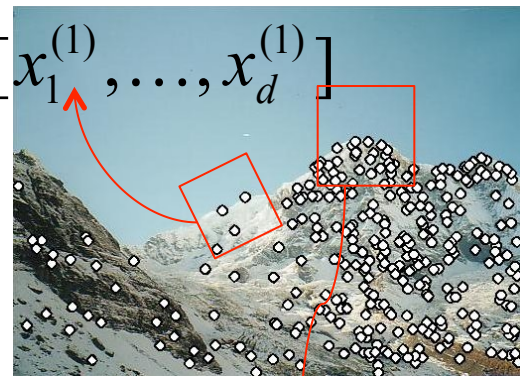
Local features: main components

1) Detection: Identify the interest points



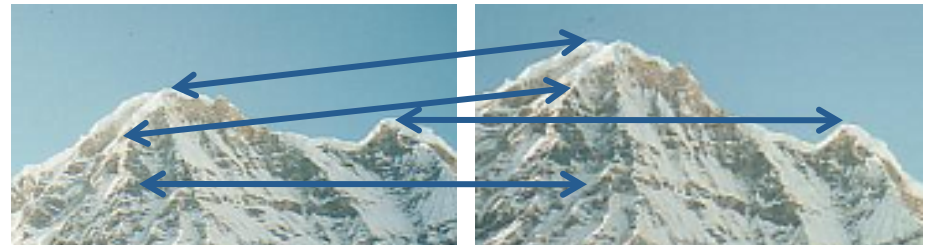
2) Description: Extract vector feature descriptor surrounding each interest point.

$$\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$$

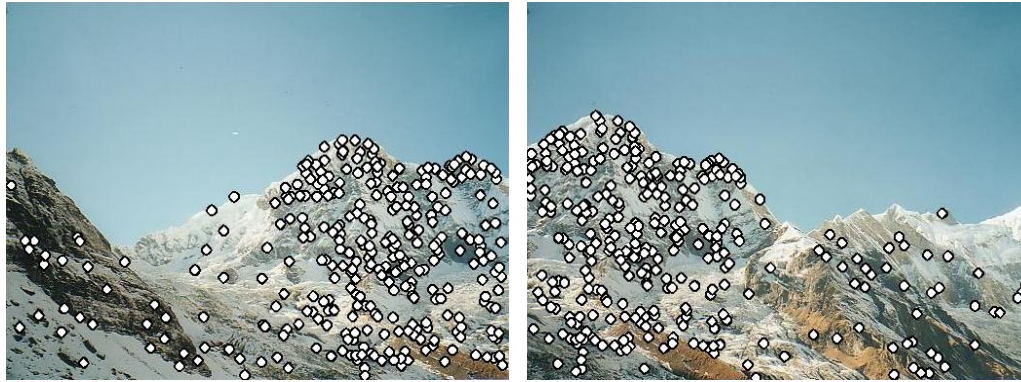


$$\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$$

3) Matching: Determine correspondence between descriptors in two views



Characteristics of good features



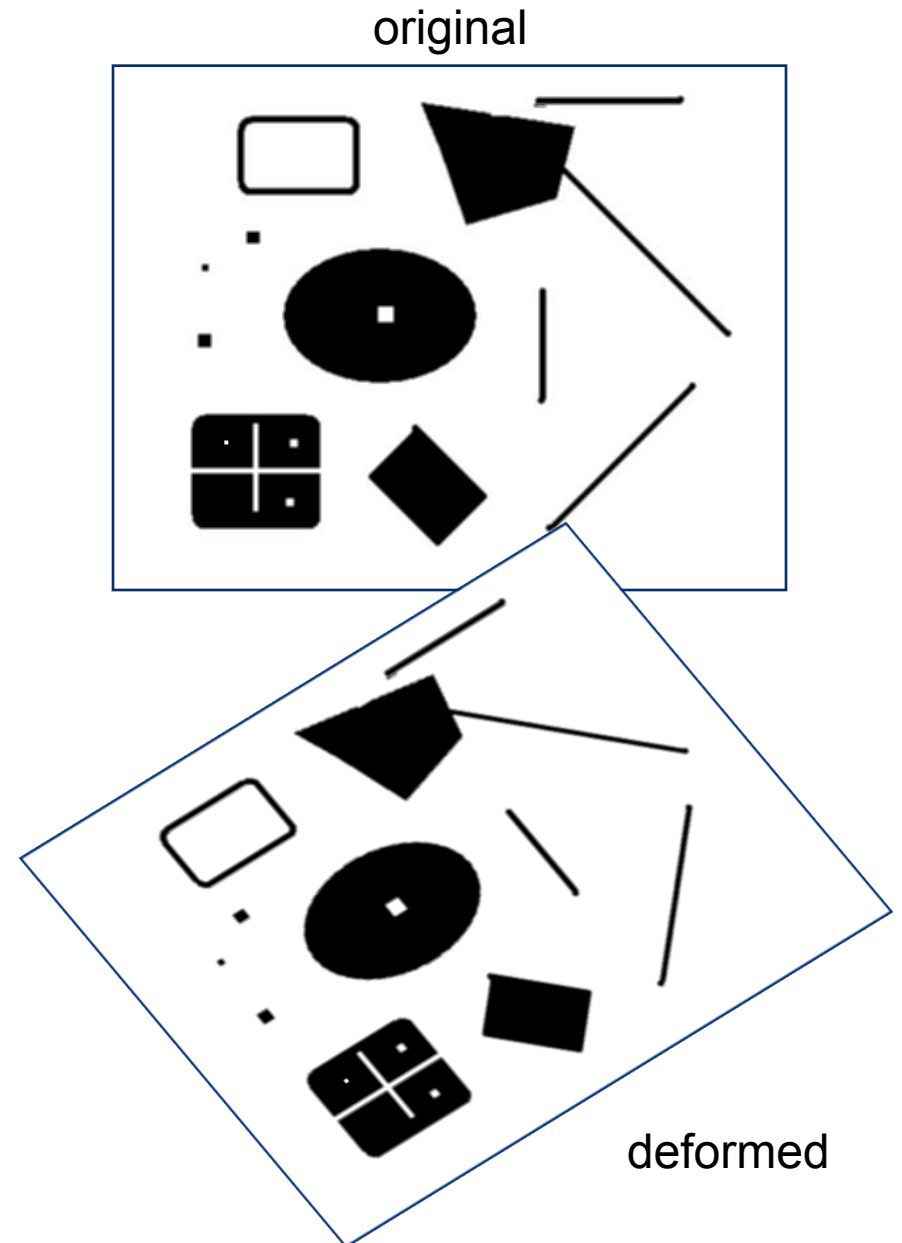
- **Repeatability**
 - The same feature can be found in several images despite geometric and photometric transformations
- **Saliency**
 - Each feature is distinctive
- **Compactness and efficiency**
 - Many fewer features than image pixels
- **Locality**
 - A feature occupies a relatively small area of the image; robust to clutter and occlusion

What makes a good feature?



What is a good feature?

- Suppose you have to click on some point, go away and come back after I deform the image, and click on the same points again.
 - Which points would you choose?



Want uniqueness

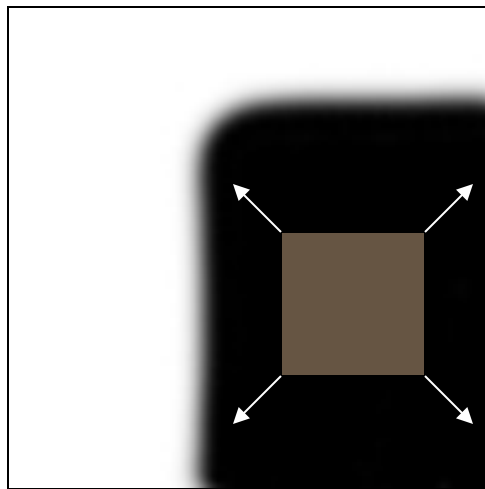
Look for image regions that are unusual

- Lead to unambiguous matches in other images

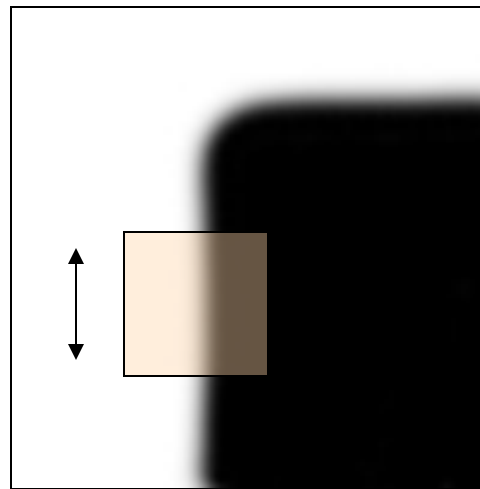
How to define “unusual”?

Corner Detection: Basic Idea

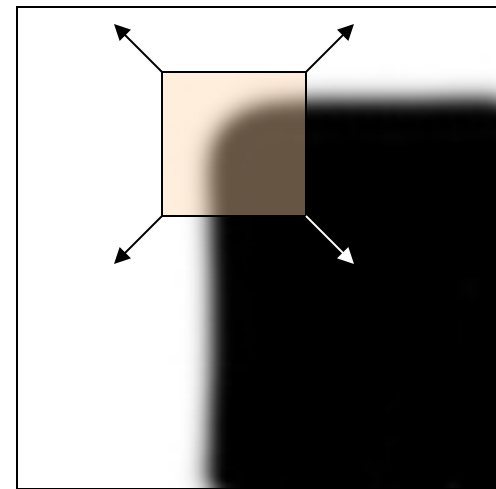
- We should easily recognize the point by looking through a small window



“flat” region



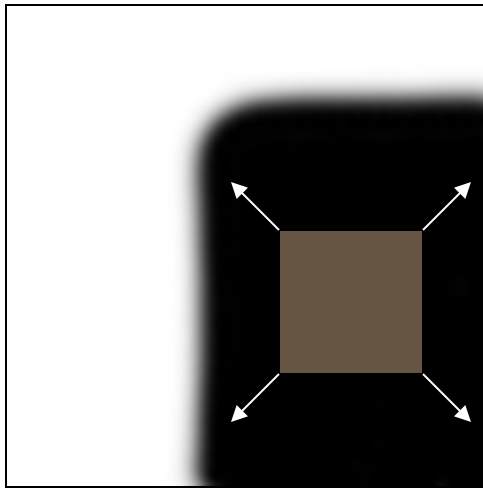
“edge”



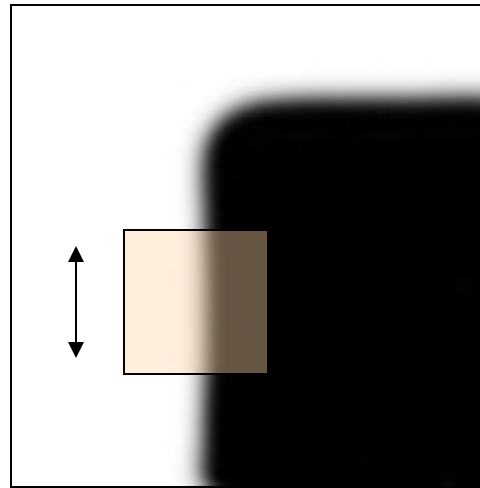
“corner”

Corner Detection: Basic Idea

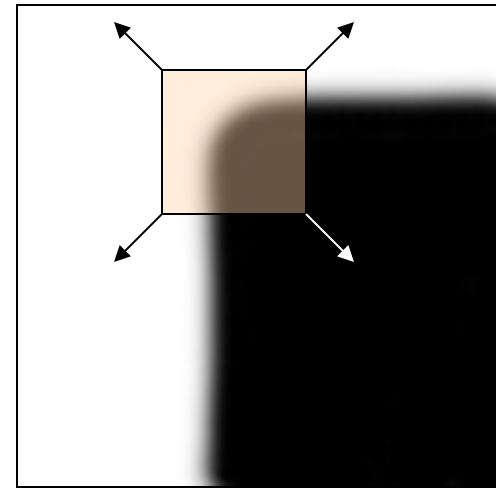
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give a *large change* in intensity



“flat” region:
no change in
all directions



“edge”:
no change along
the edge
direction



“corner”:
significant
change in all
directions

Many Existing Detectors Available

Hessian & Harris	[Beaudet '78], [Harris '88]
Laplacian, DoG	[Lindeberg '98], [Lowe 1999]
Harris-/Hessian-Laplace	[Mikolajczyk & Schmid '01]
Harris-/Hessian-Affine	[Mikolajczyk & Schmid '04]
EBR and IBR	[Tuytelaars & Van Gool '04]
MSER	[Matas '02]
Salient Regions	[Kadir & Brady '01]
Others...	

Finding Corners

- Corners are repeatable and distinctive
- Key property: in the region around a corner, image gradient has two or more dominant directions

C.Harris and M.Stephens. ["A Combined Corner and Edge Detector."](#)
Proceedings of the 4th Alvey Vision Conference: pages 147--151.

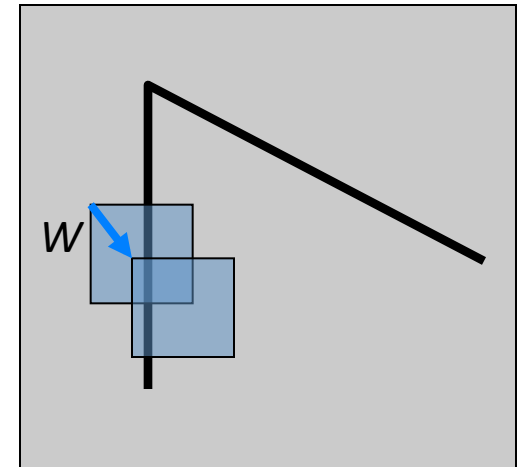
Feature extraction: Corners



Harris corner detection: the math

Consider shifting the window W by (u, v)

- how do the pixels in W change?
- compare each pixel before and after:
compute sum of squared differences (SSD)
- this defines an SSD “error” $E(u, v)$:



$$E(u, v) = \sum_{(x, y) \in W} [I(x + u, y + v) - I(x, y)]^2$$

Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u, v) = \sum_{x,y} [I(x+u, y+v) - I(x, y)]^2$$

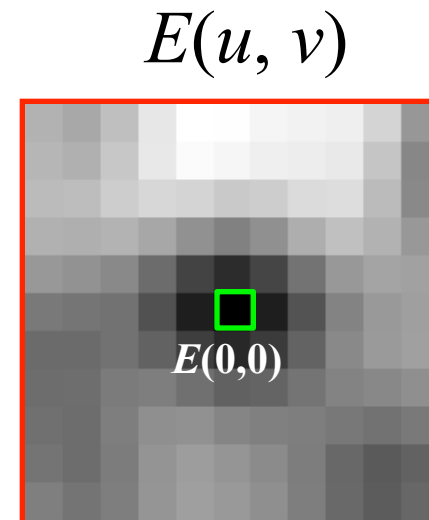
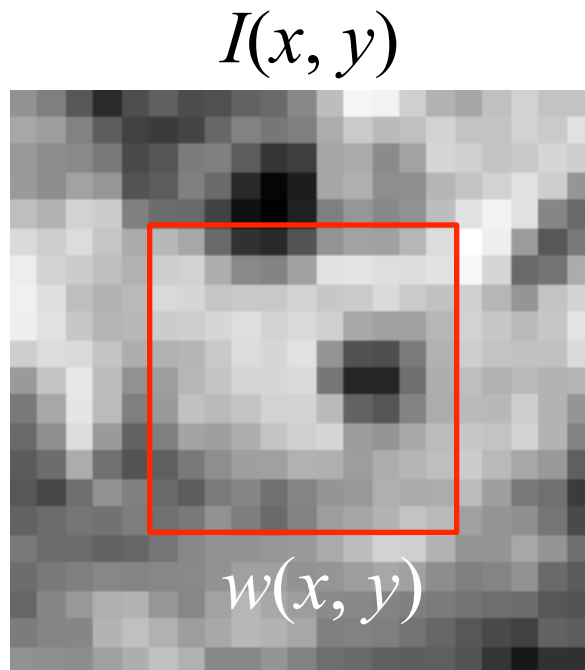
Shifted
intensity

Intensity

Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

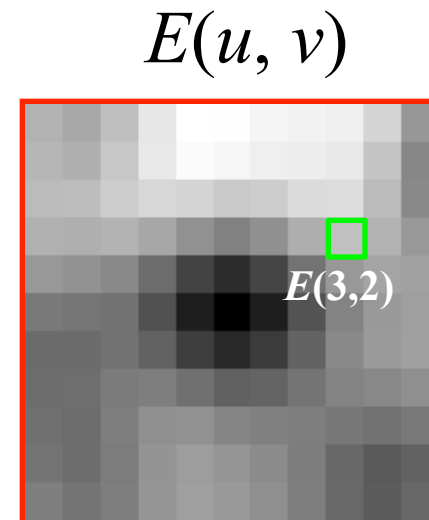
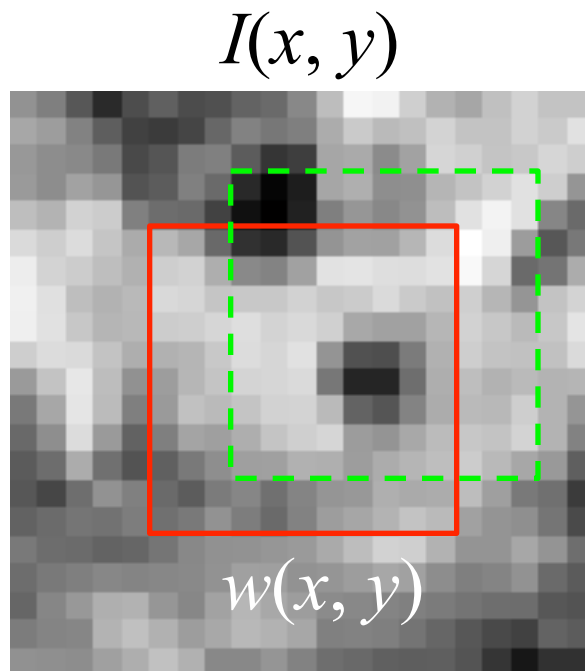
$$E(u, v) = \sum_{x,y} [I(x+u, y+v) - I(x, y)]^2$$



Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u, v) = \sum_{x,y} [I(x+u, y+v) - I(x, y)]^2$$



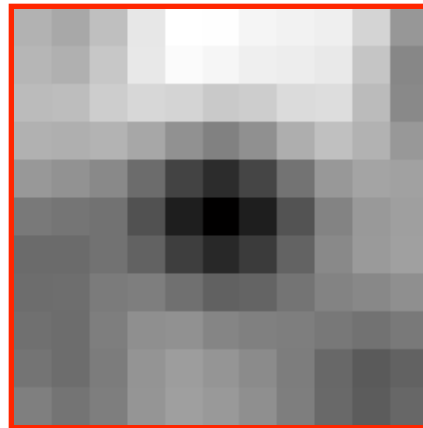
Corner Detection: Mathematics

Change in appearance of window W
for the shift $[u, v]$:

$$E(u, v) = \sum_{x, y} [I(x + u, y + v) - I(x, y)]^2$$

We want to find out how this function behaves for small shifts

$E(u, v)$



Small motion assumption

Taylor Series expansion of I :

$$I(x+u, y+v) = I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

If the motion (u,v) is small, then first order approximation is good

$$\begin{aligned} I(x+u, y+v) &\approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \\ &\approx I(x, y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} \end{aligned}$$

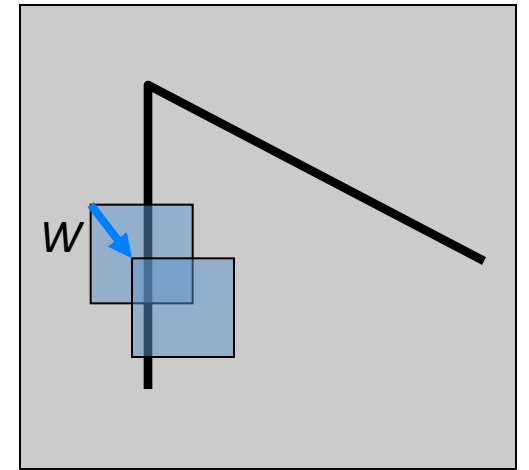
shorthand: $I_x = \frac{\partial I}{\partial x}$

Plugging this into the formula on the previous slide...

Feature detection: the math

Consider shifting the window W by (u, v)

- define an SSD “error” $E(u, v)$:



$$\begin{aligned} E(u, v) &= \sum_{(x, y) \in W} [I(x + u, y + v) - I(x, y)]^2 \\ &\approx \sum_{(x, y) \in W} [I(x, y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} - I(x, y)]^2 \\ &\approx \sum_{(x, y) \in W} \left[[I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} \right]^2 \end{aligned}$$

Corner detection: the math

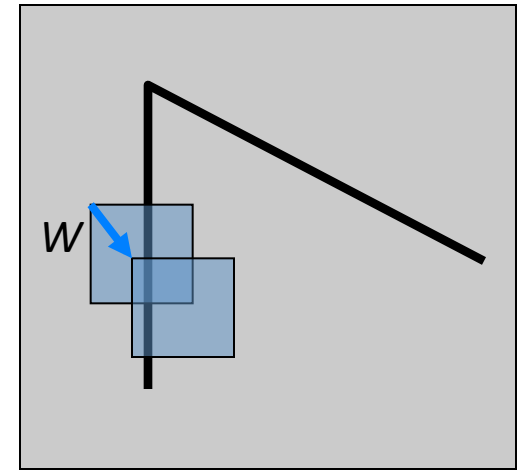
Consider shifting the window W by (u, v)

- define an SSD “error” $E(u, v)$:

$$E(u, v) \approx \sum_{(x, y) \in W} [I_x u + I_y v]^2$$

$$\approx Au^2 + 2Buv + Cv^2$$

$$A = \sum_{(x, y) \in W} I_x^2 \quad B = \sum_{(x, y) \in W} I_x I_y \quad C = \sum_{(x, y) \in W} I_y^2$$



- Thus, $E(u, v)$ is locally approximated as a quadratic error function

Corner Detection: Mathematics

The quadratic approximation simplifies to

$$E(u, v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a *second moment matrix* computed from image derivatives (aka structure tensor):

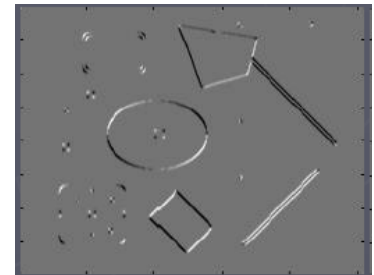
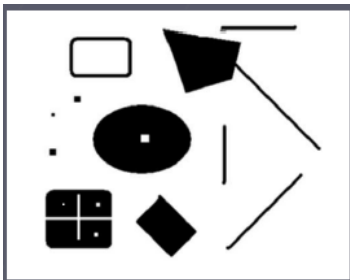
$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y] = \sum \nabla I (\nabla I)^T$$

Corners as distinctive interest points

$$M = \sum \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

2 x 2 matrix of image derivatives (averaged in neighborhood of a point)



Notation:

$$I_x \Leftrightarrow \frac{\partial I}{\partial x}$$

$$I_y \Leftrightarrow \frac{\partial I}{\partial y}$$

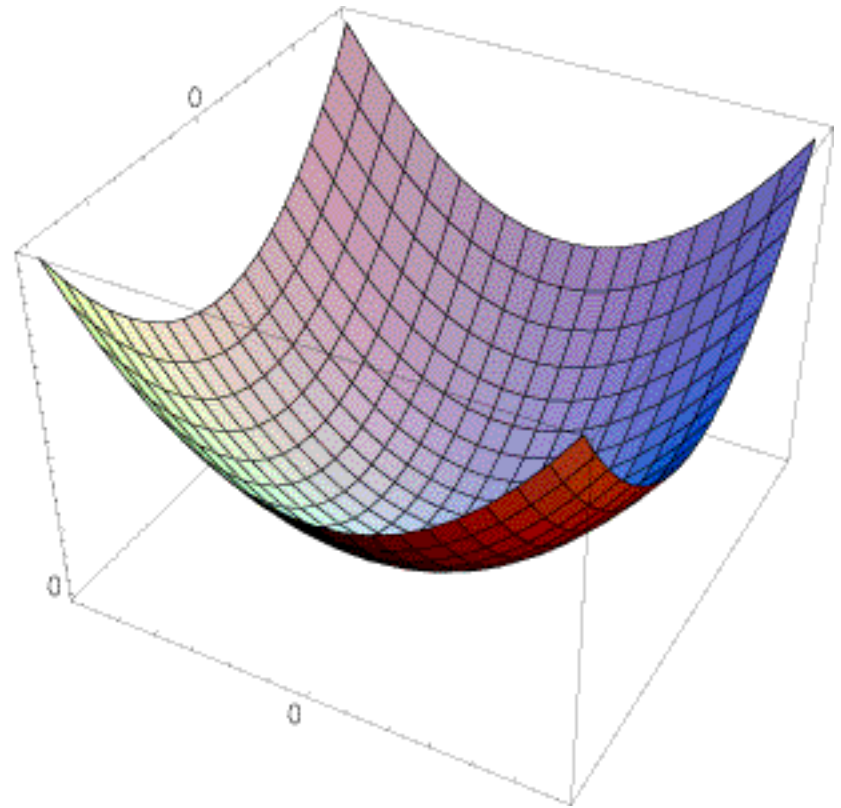
$$I_x I_y \Leftrightarrow \frac{\partial I}{\partial x} \frac{\partial I}{\partial y}$$

Interpreting the second moment matrix

The surface $E(u,v)$ is locally approximated by a quadratic form. Let's try to understand its shape.

$$E(u, v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

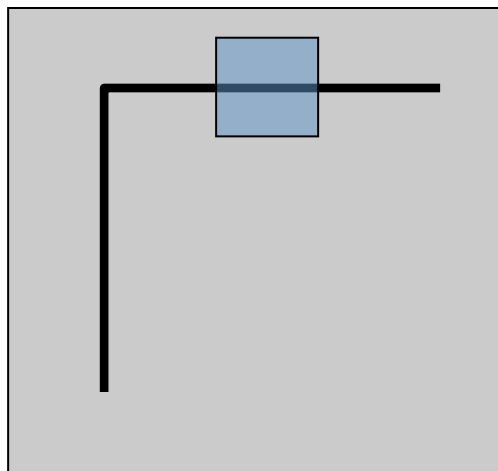


$$E(u, v) \approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y) \in W} I_x^2$$

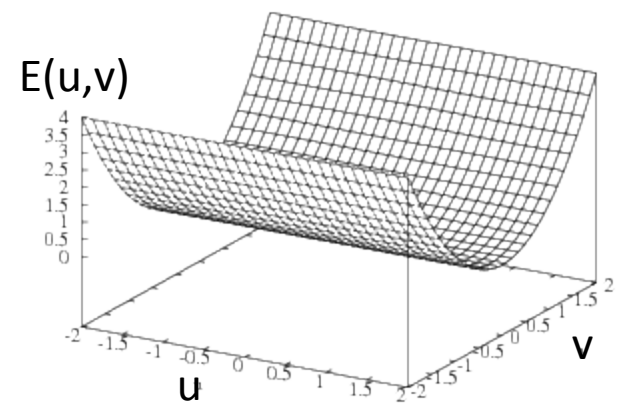
$$B = \sum_{(x,y) \in W} I_x I_y$$

$$C = \sum_{(x,y) \in W} I_y^2$$



Horizontal edge: $I_x = 0$

$$M = \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix}$$

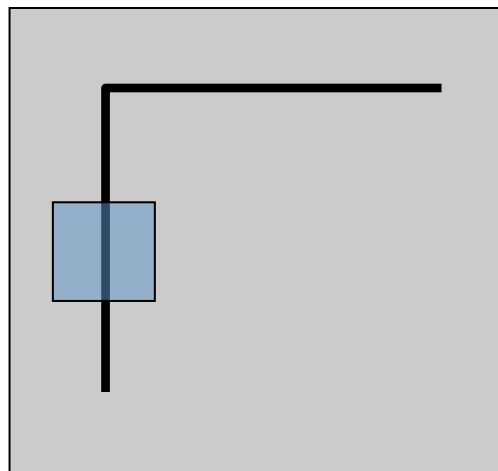


$$E(u, v) \approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y) \in W} I_x^2$$

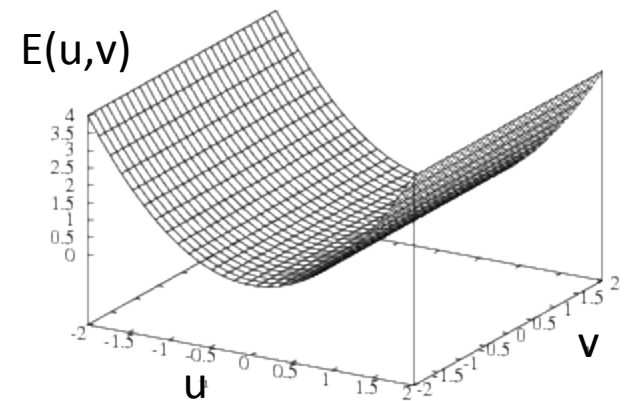
$$B = \sum_{(x,y) \in W} I_x I_y$$

$$C = \sum_{(x,y) \in W} I_y^2$$



Vertical edge: $I_y = 0$

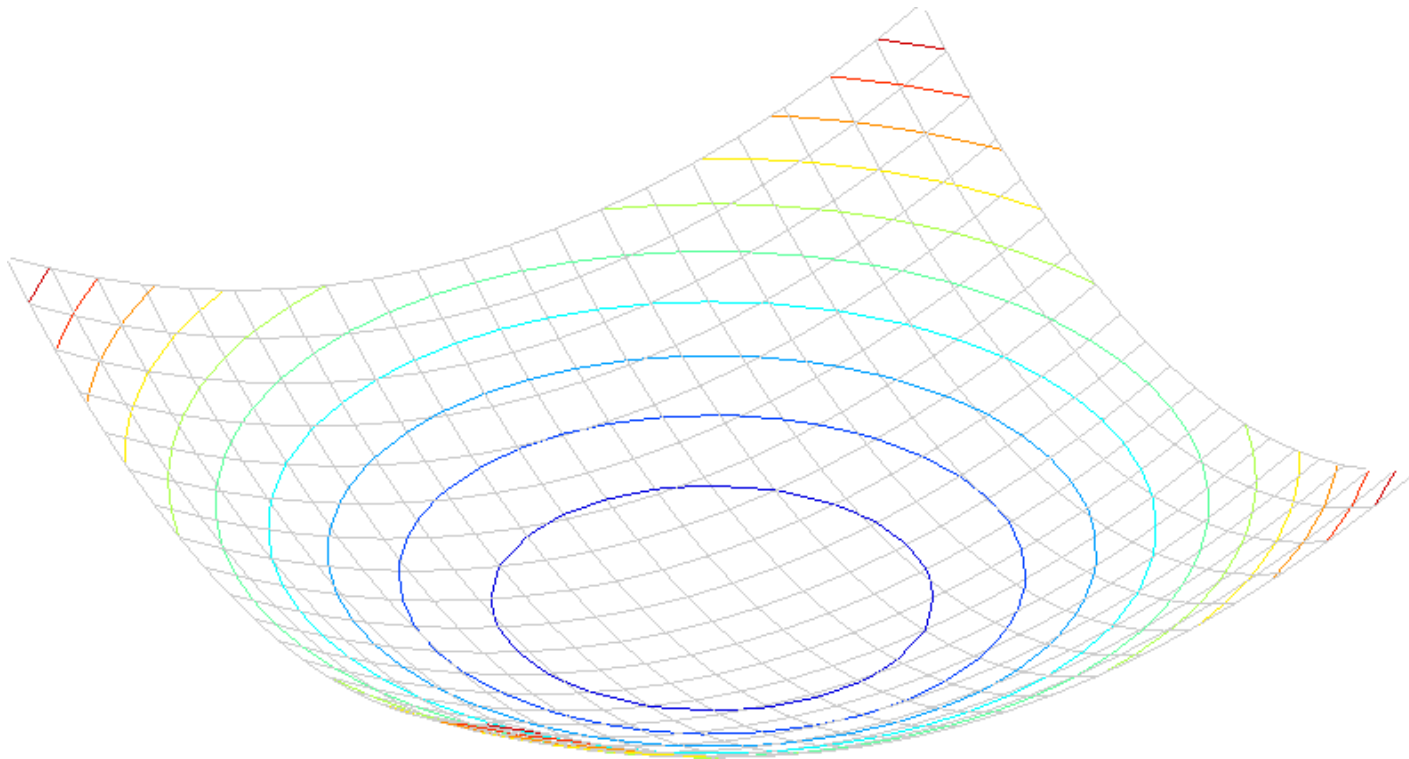
$$M = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$$



Interpreting the second moment matrix

Consider a horizontal “slice” of $E(u, v)$: $[u \ v] M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

This is the equation of an ellipse.



Questions?