CS4670/5670: Computer Vision

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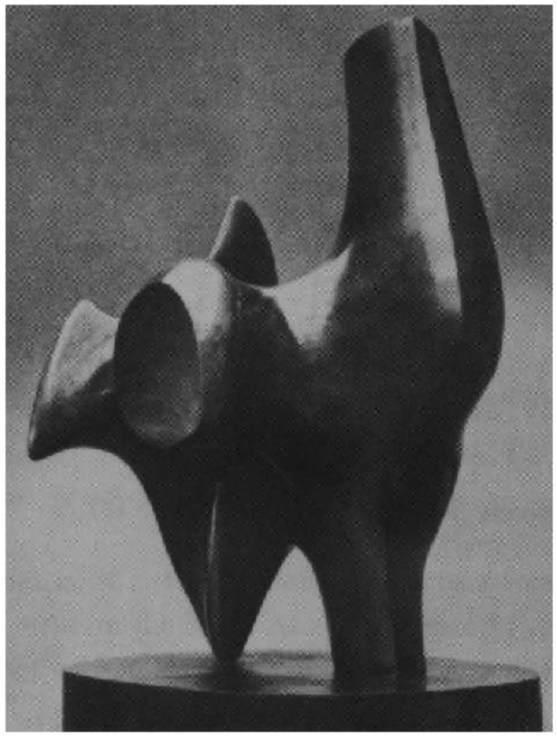
Lecture 3: Edge detection

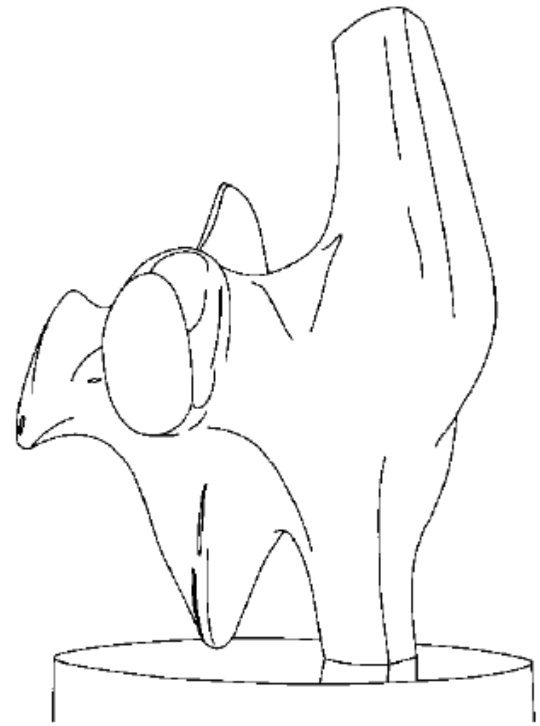
From Sandlot Science

Announcements

- Find partners on piazza
- PA 1 will be out on Monday
- Quiz on Monday or Wednesday, beginning of class

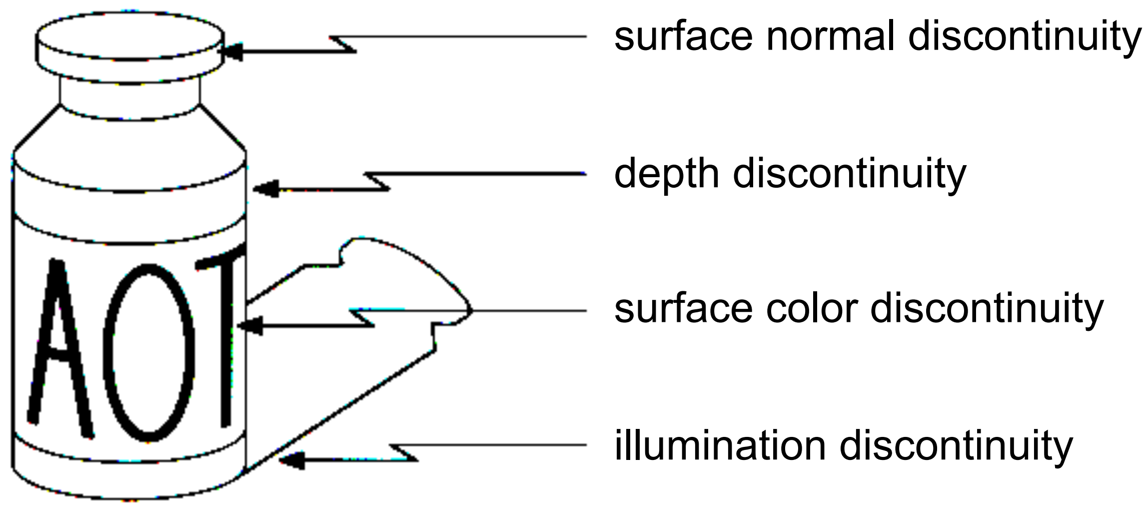
Why edges?





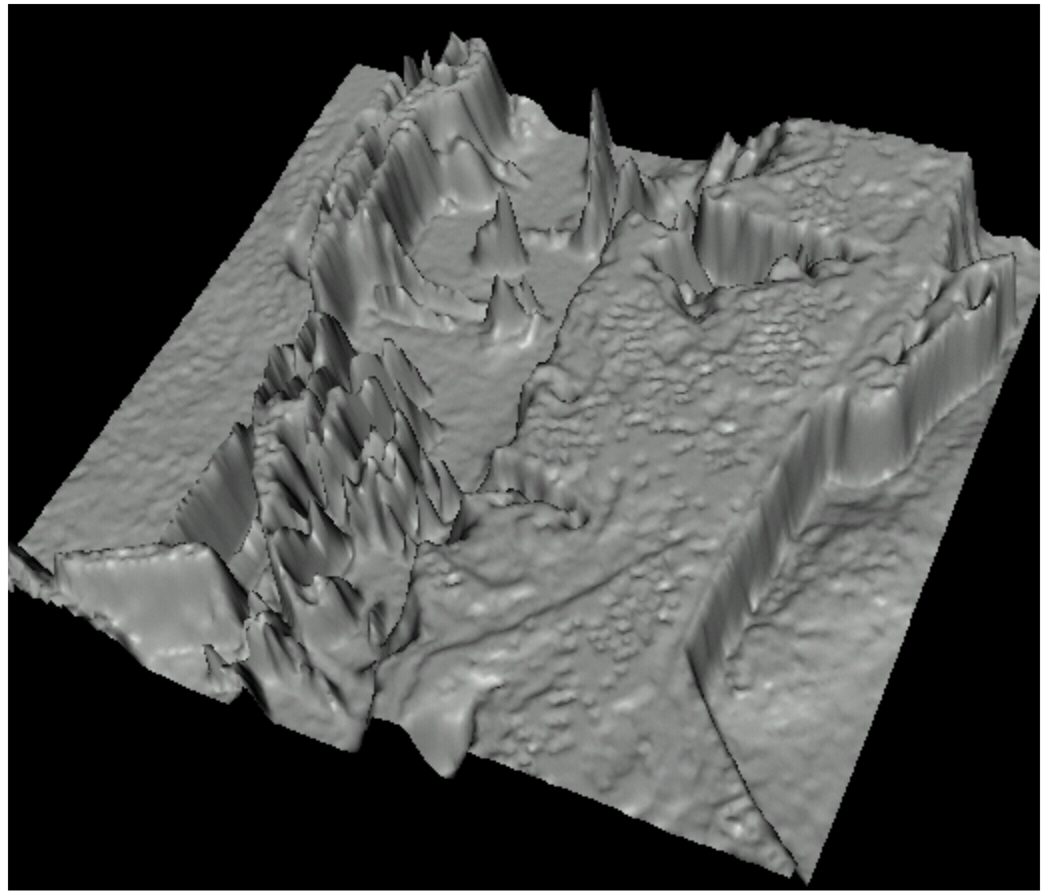
- Humans are sensitive to edges
- Convert a 2D image into a set of curves
  - Extracts salient features of the scene, more compact

Origin of Edges



- Edges are caused by a variety of factors

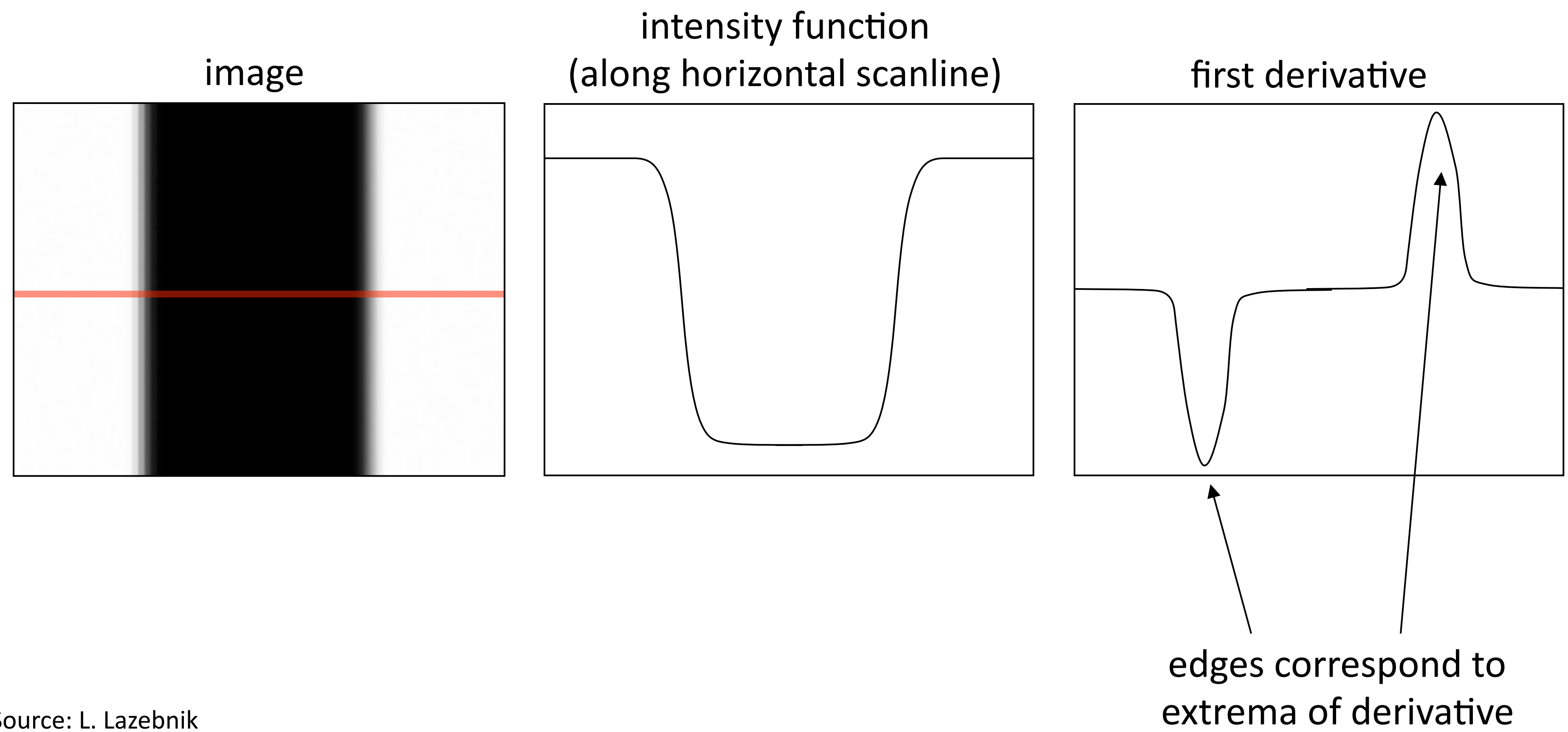
Images as functions…



- Edges look like steep cliffs

Characterizing edges

- An edge is a place of *rapid change* in the image intensity function



Source: L. Lazebnik

Image derivatives

- How can we differentiate a *digital* image F[x,y]?
  - Option 1: reconstruct a continuous image, *f,* then compute the derivative
  - Option 2: take discrete derivative (finite difference)

$$\frac{\partial f}{\partial x}[x, y] \approx F[x+1, y] - F[x, y]$$

How would you implement this as a linear filter?

$\frac{\partial f}{\partial x}$:

H_x

$\frac{\partial f}{\partial y}$:

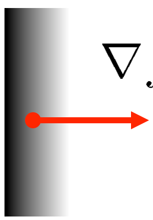
H_y

Source: S. Seitz

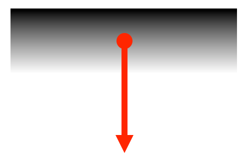
Image gradient

- The *gradient* of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

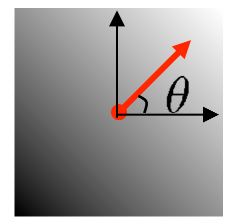
The gradient points in the direction of most rapid increase in intensity



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The *edge strength* is given by the gradient magnitude:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

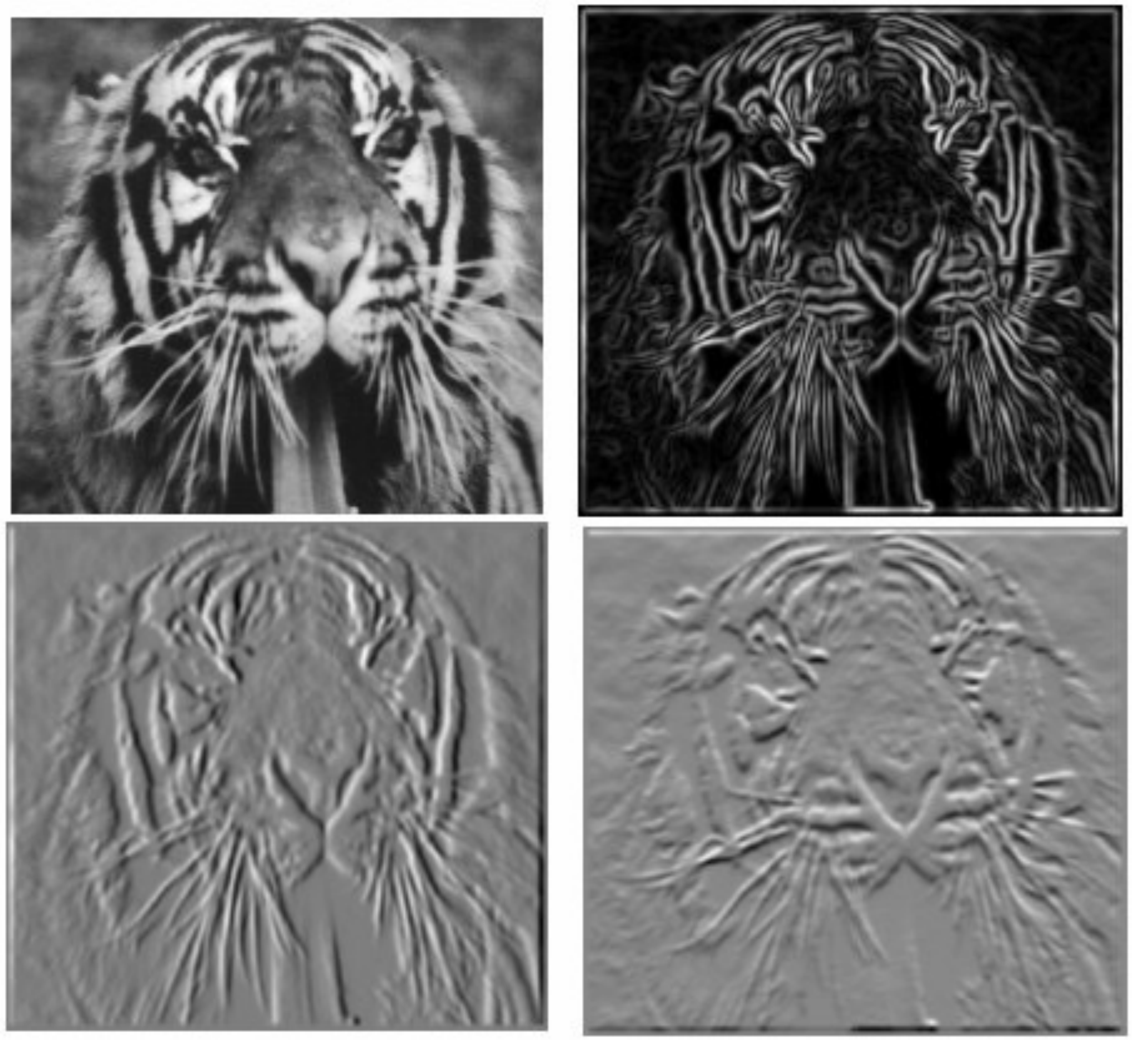
The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge?

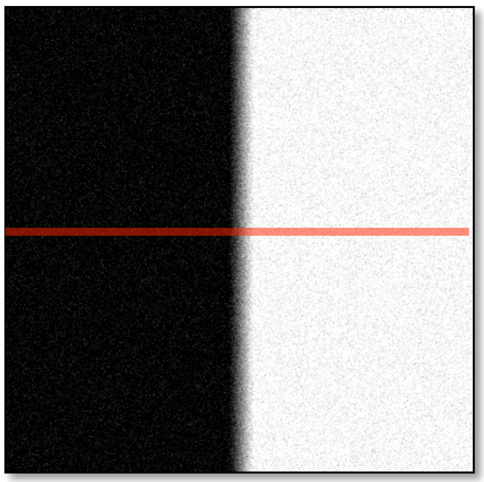
Source: Steve Seitz

Image gradient

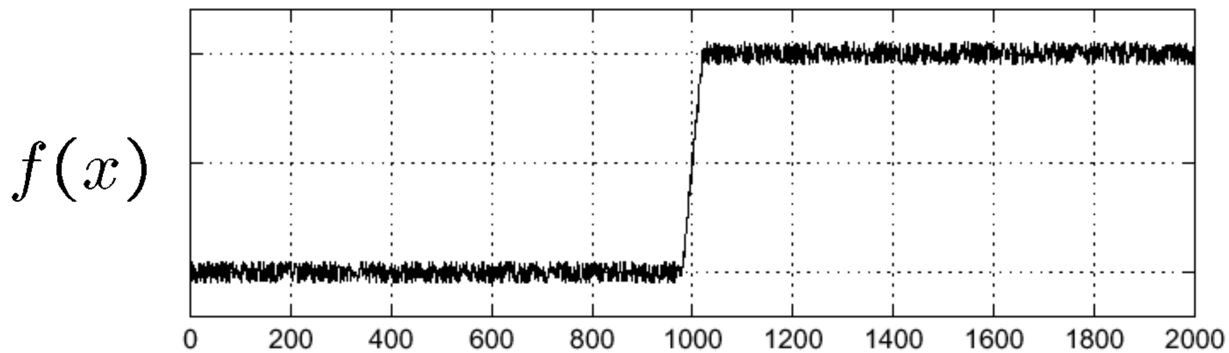


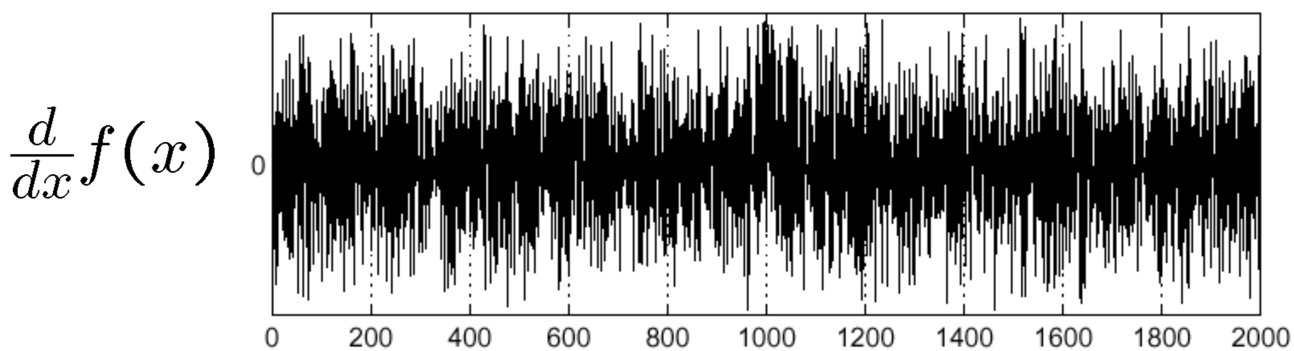
Source: L. Lazebnik

Effects of noise



Noisy input image

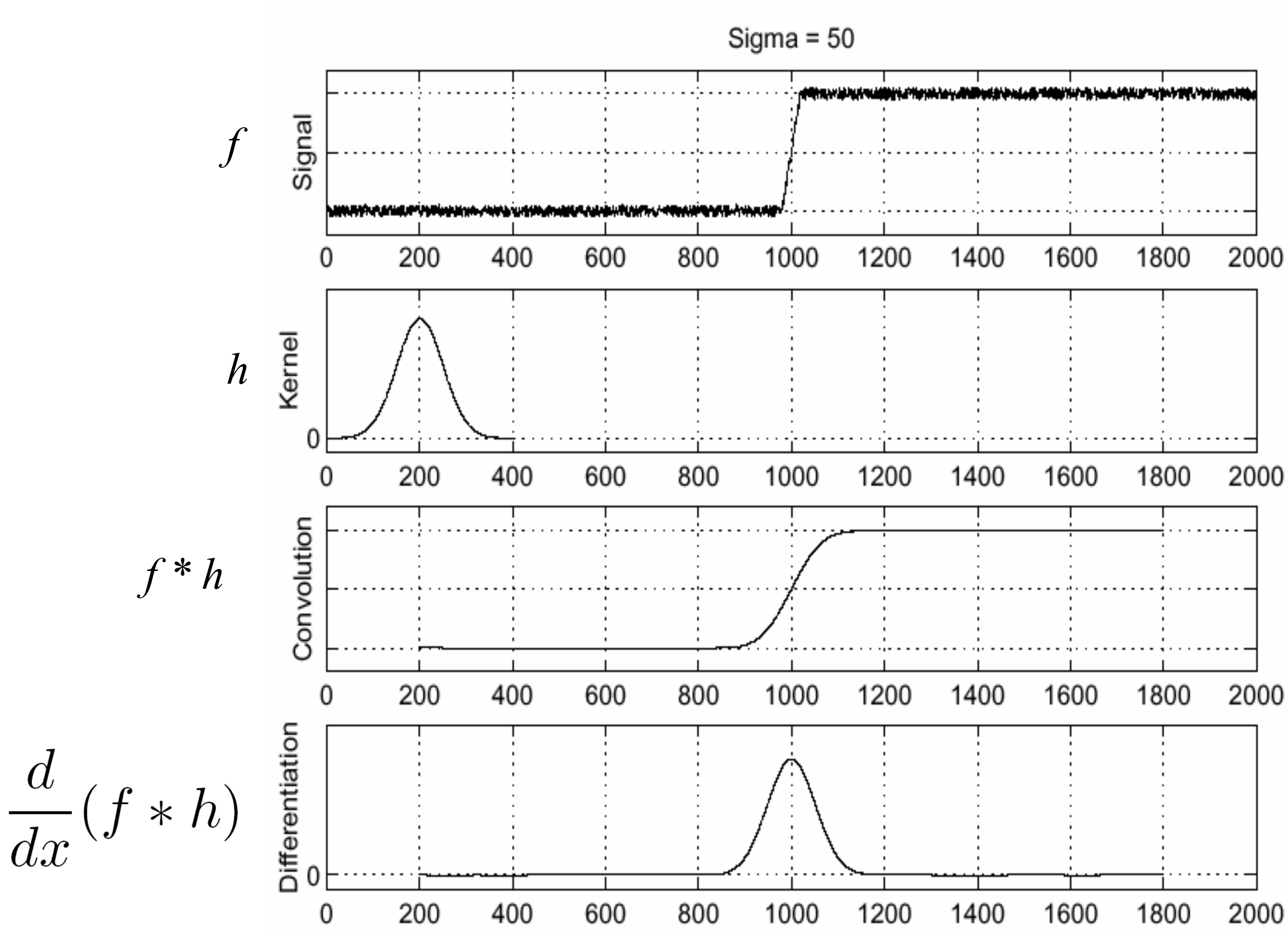




Where is the edge?

Source: S. Seitz

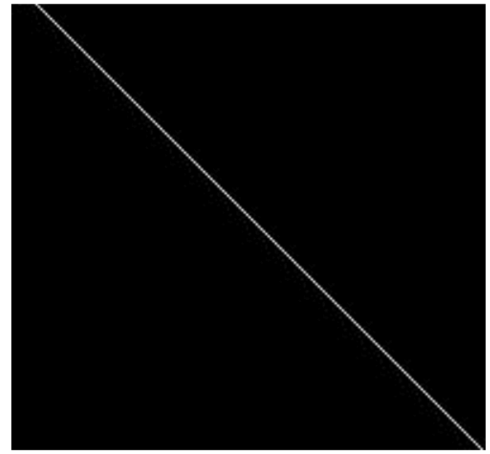
Solution: smooth first



To find edges, look for peaks in $\frac{d}{dx}(f * h)$

Source: S. Seitz

Image with Edge



Edge Location

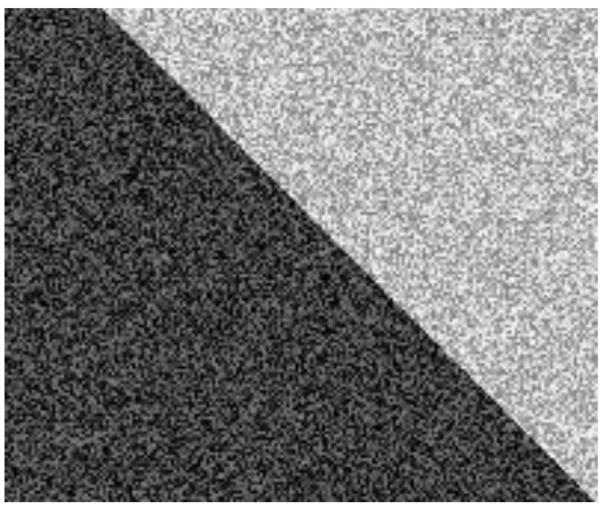
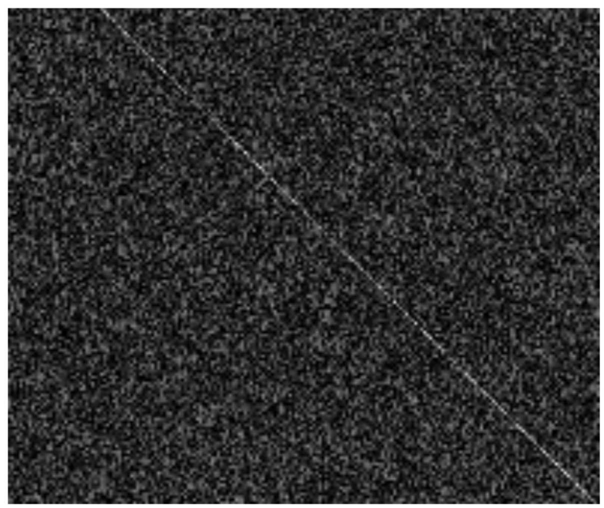
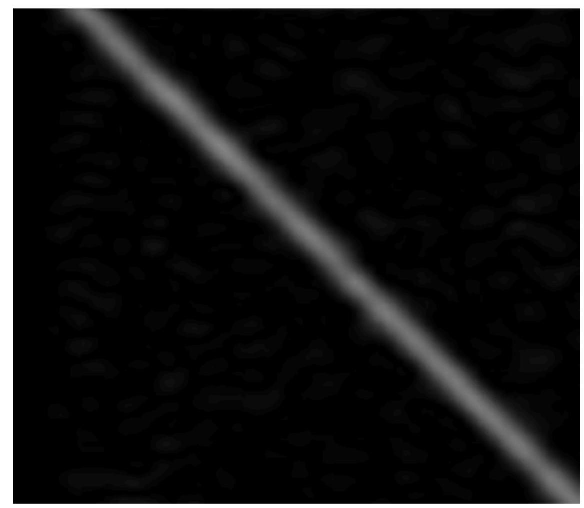


Image + Noise



Derivatives detect
edge *and* noise

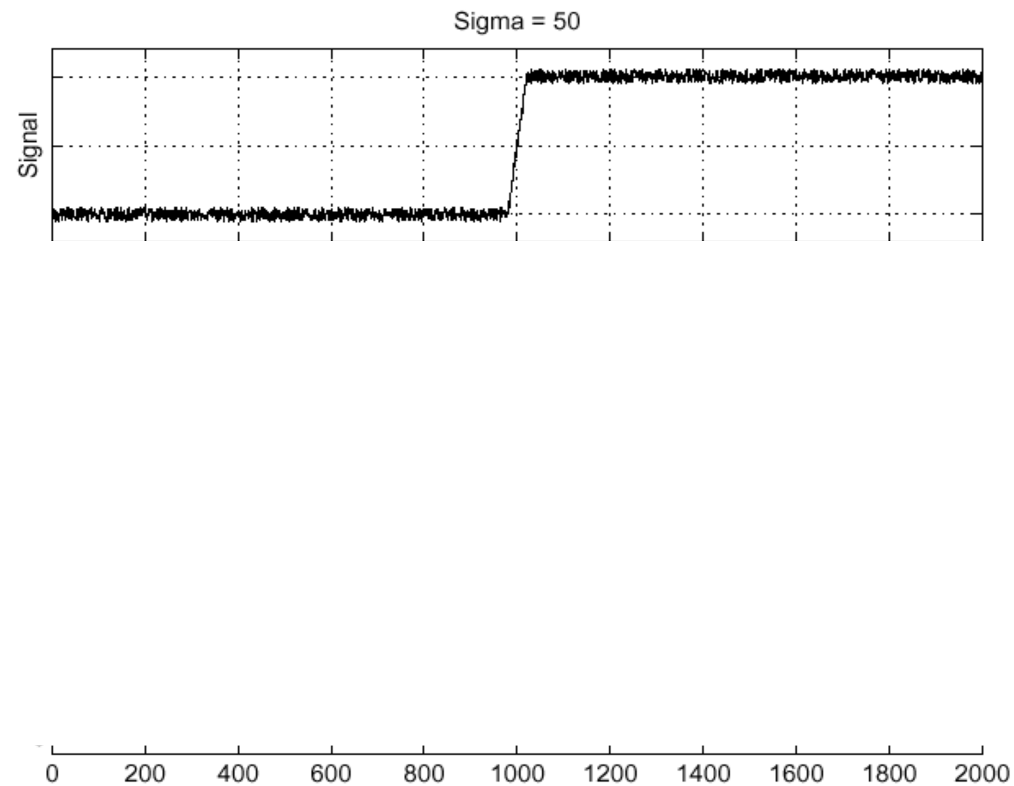


Smoothed derivative removes
noise, but blurs edge

Associative property of convolution

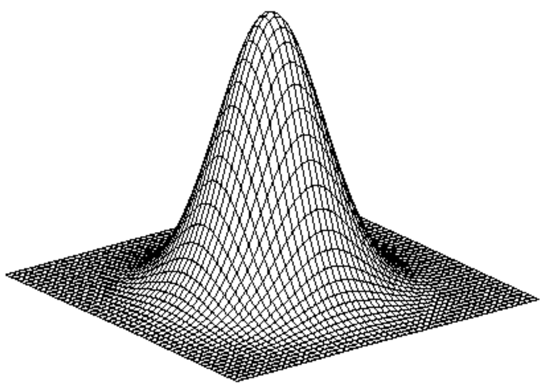
- Differentiation is a convolution
- Convolution is associative: $\frac{d}{dx}(f * h) = f * \frac{d}{dx}h$
- This saves us one operation:

f



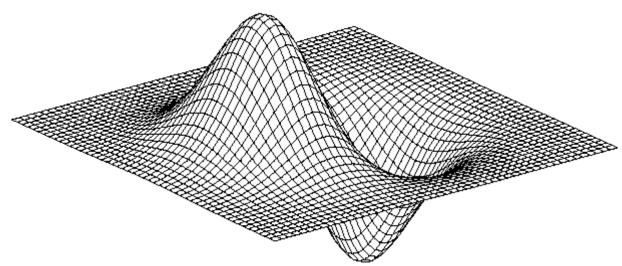
Source: S. Seitz

2D edge detection filters



Gaussian

$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



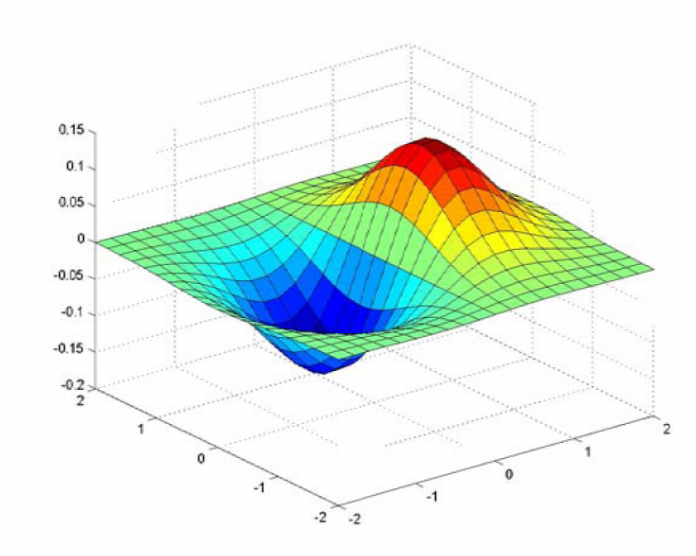
derivative of Gaussian (x)

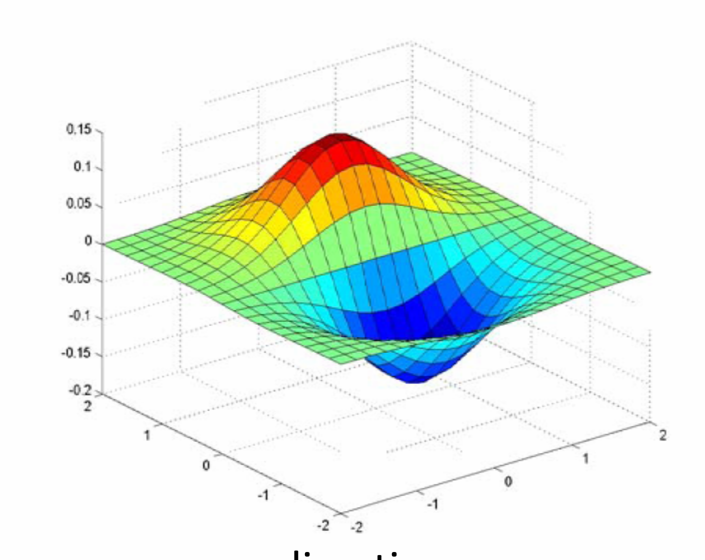
$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

$$\nabla G_\sigma(\boldsymbol{x}) = (\frac{\partial G_\sigma}{\partial x}, \frac{\partial G_\sigma}{\partial y})(\boldsymbol{x}) = [-x \ \ -y] \quad \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

$$\nabla^2 G_\sigma(\boldsymbol{x}) = \frac{1}{\sigma^3}\left(2 - \frac{x^2+y^2}{2\sigma^2}\right)\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

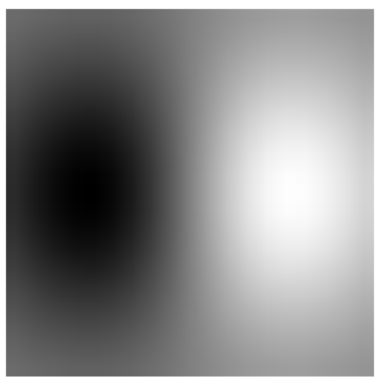
Derivative of Gaussian filter

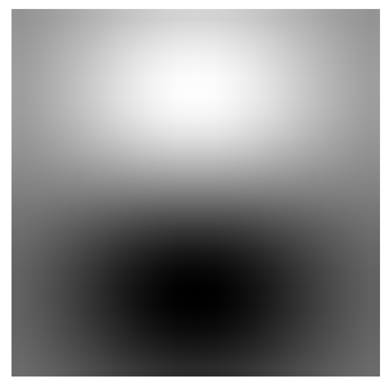




x-direction

y-direction





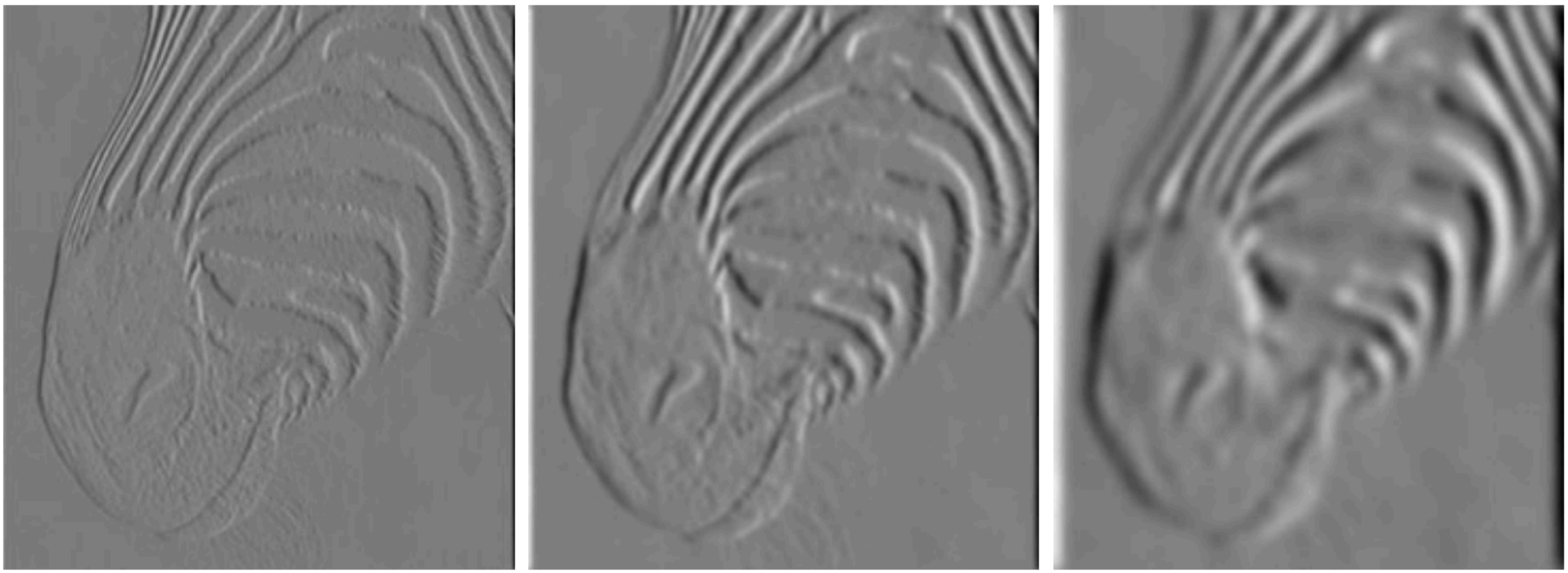


FIGURE 5.3: The scale (i.e., σ) of the Gaussian used in a derivative of Gaussian filter has significant effects on the results. The three images show estimates of the derivative in the x direction of an image of the head of a zebra obtained using a derivative of Gaussian filter with σ one pixel, three pixels, and seven pixels (**left** to **right**). Note how images at a finer scale show some hair, the animal's whiskers disappear at a medium scale, and the fine stripes at the top of the muzzle disappear at the coarser scale.

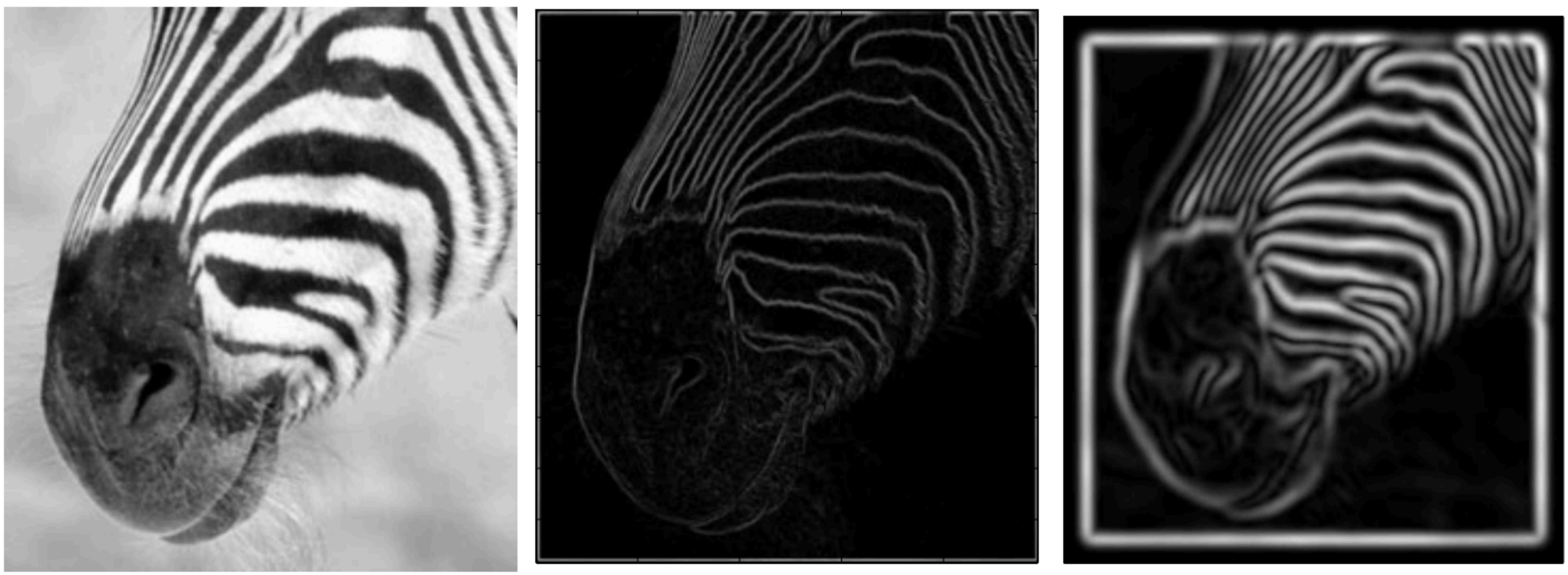


FIGURE 5.4: The gradient magnitude can be estimated by smoothing an image and then differentiating it. This is equivalent to convolving with the derivative of a smoothing kernel. The extent of the smoothing affects the gradient magnitude; in this figure, we show the gradient magnitude for the figure of a zebra at different scales. At the **center**, gradient magnitude estimated using the derivatives of a Gaussian with $\sigma = 1$ pixel; and on the **right**, gradient magnitude estimated using the derivatives of a Gaussian with $\sigma = 2$ pixel. Notice that large values of the gradient magnitude form thick trails.

The Sobel operator

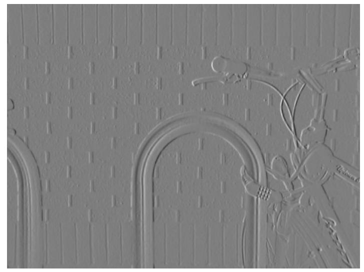
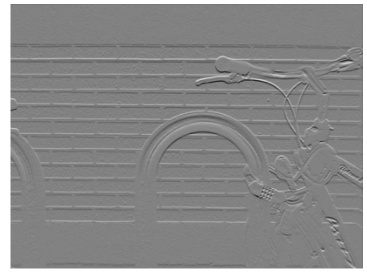
- Common approximation of derivative of Gaussian
  – A mask (not a convolution kernel)

$$\frac{1}{8}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array} \qquad \frac{1}{8}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

$$s_x \qquad\qquad\qquad s_y$$

- The standard defn. of the Sobel operator omits the 1/8 term
  – doesn't make a difference for edge detection
  – the 1/8 term **is** needed to get the right gradient magnitude

Sobel operator: example

Source: Wikipedia

Questions?

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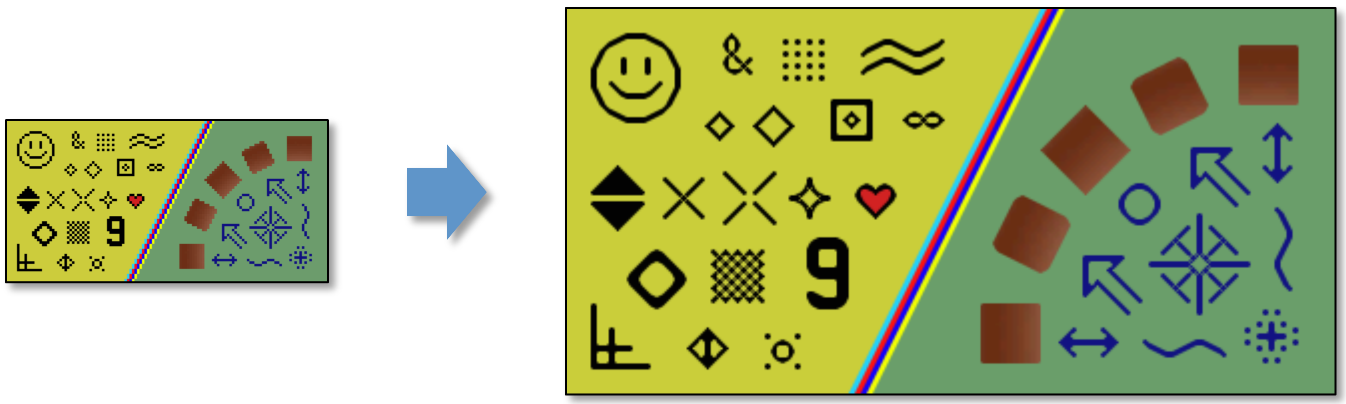
Image Resampling



Image

This image is too big to fit on the screen. How can we generate a half-sized version?

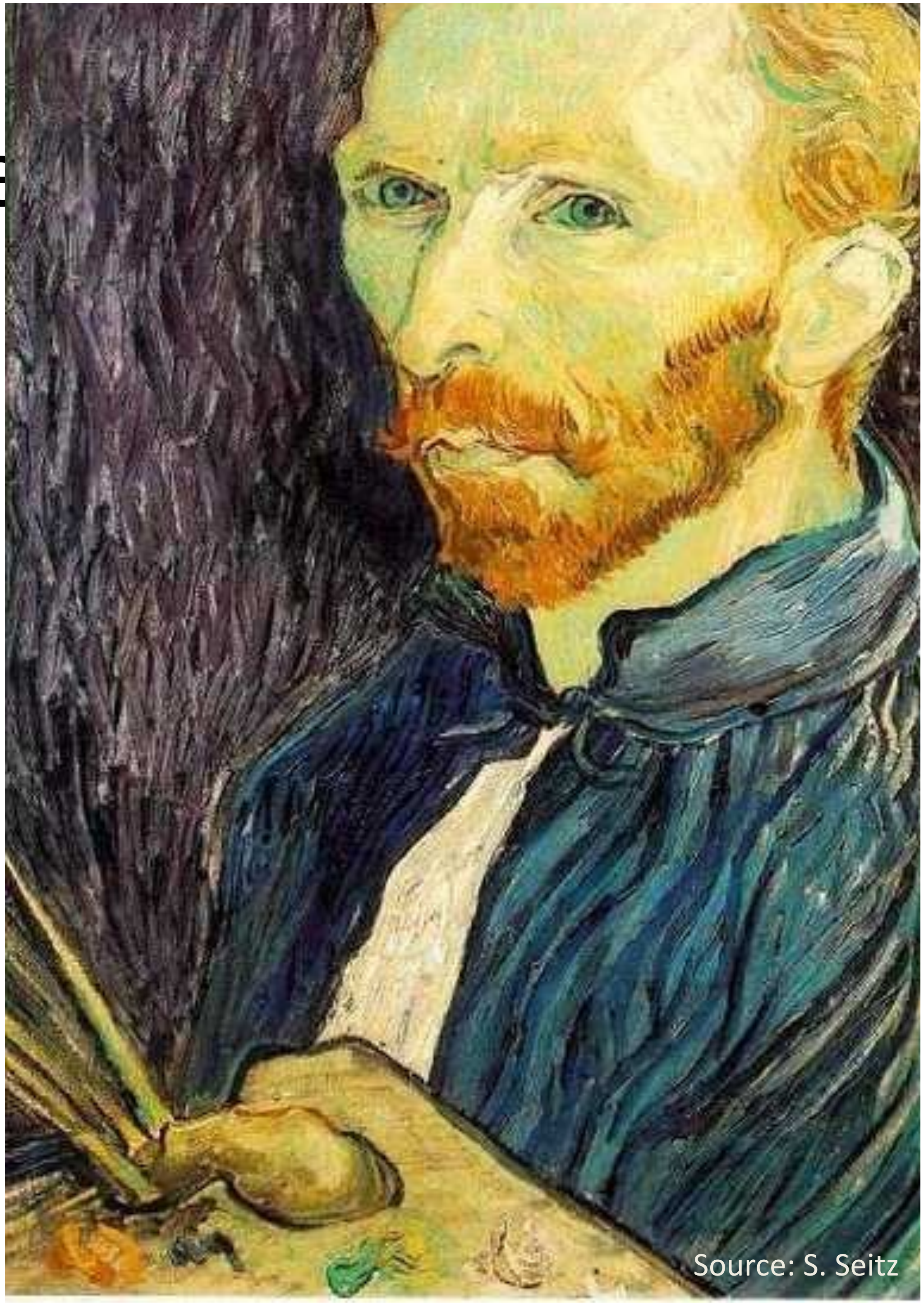
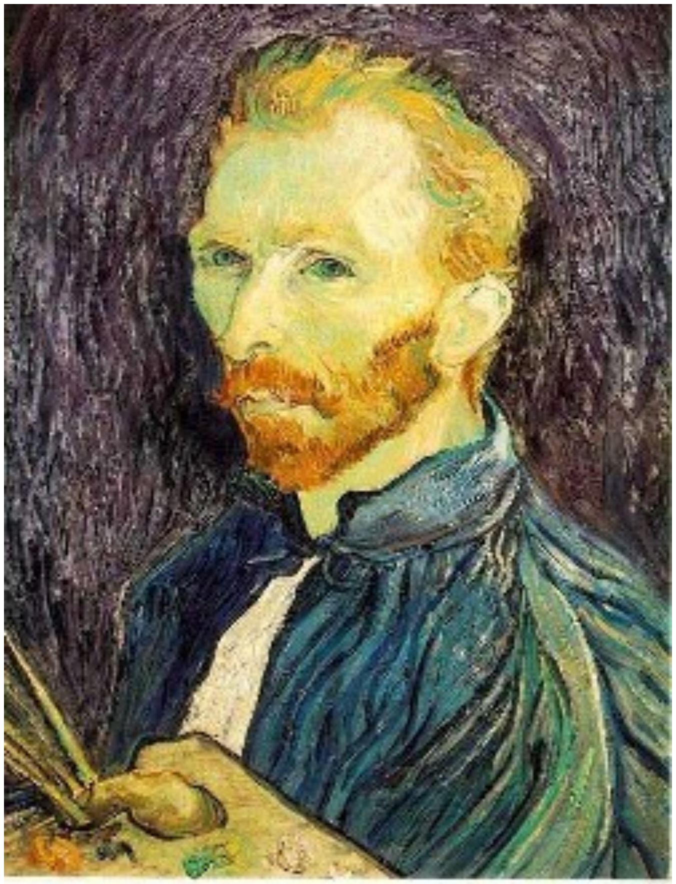


Image sub-sampling



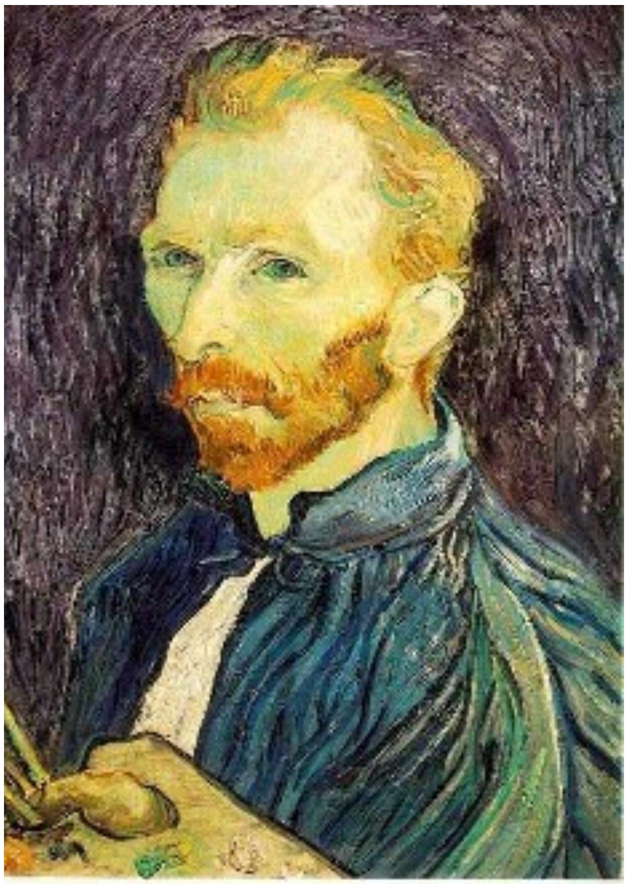
1/4

1/16

Throw away every other row and column to create a 1/2 size image
- called *image sub-sampling*

Source: S. Seitz

Image sub-sampling



1/2

1/4 (2x zoom)

1/16 (4x zoom)

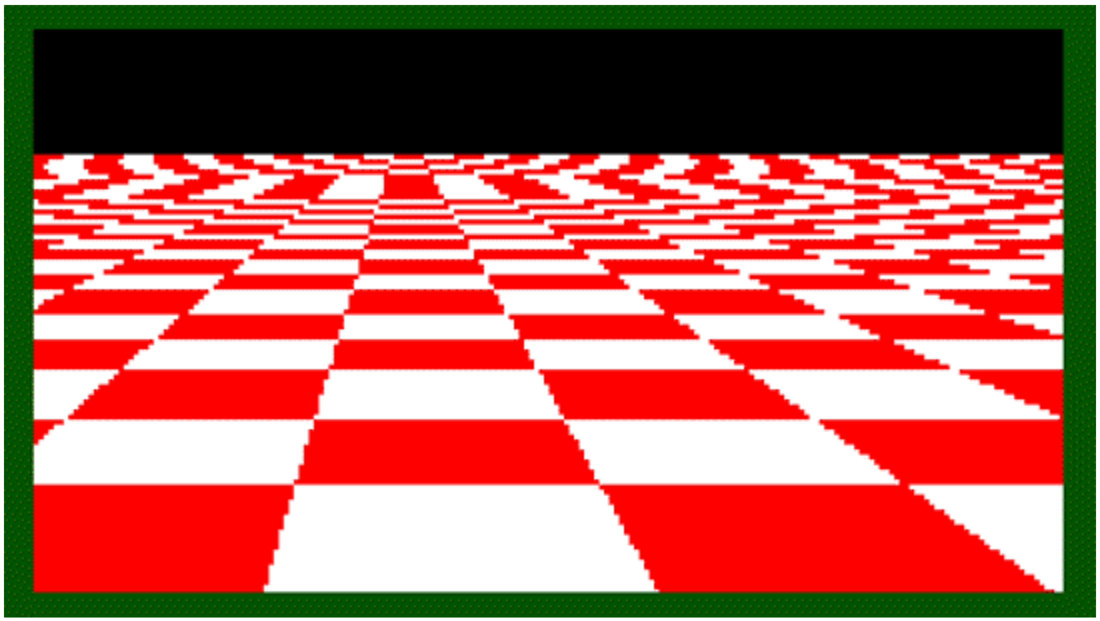
Why does this look so crufty?

Source: S. Seitz

Image sub-sampling

Source: F. Durand

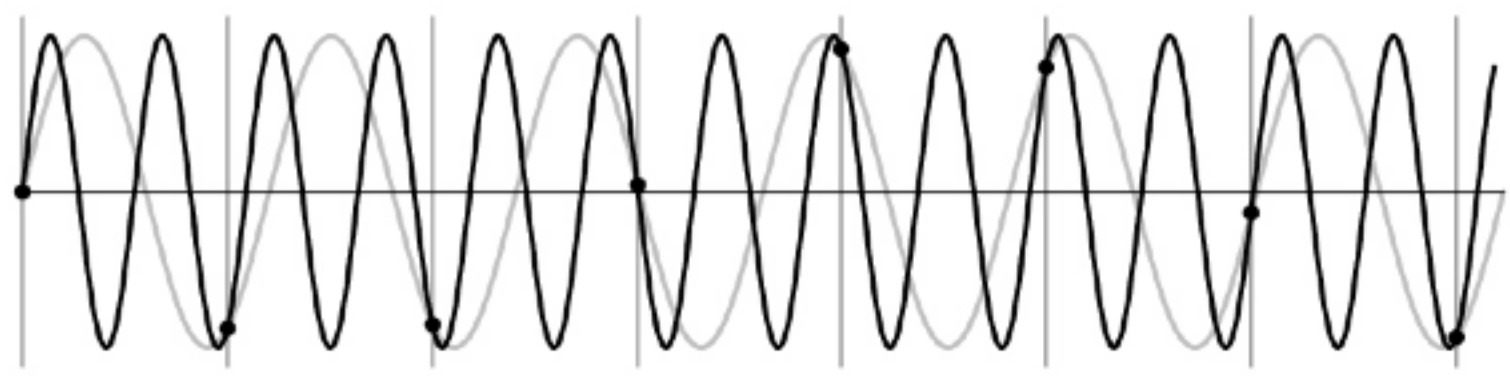
Even worse for synthetic images



Source: L. Zhang

What is aliasing?

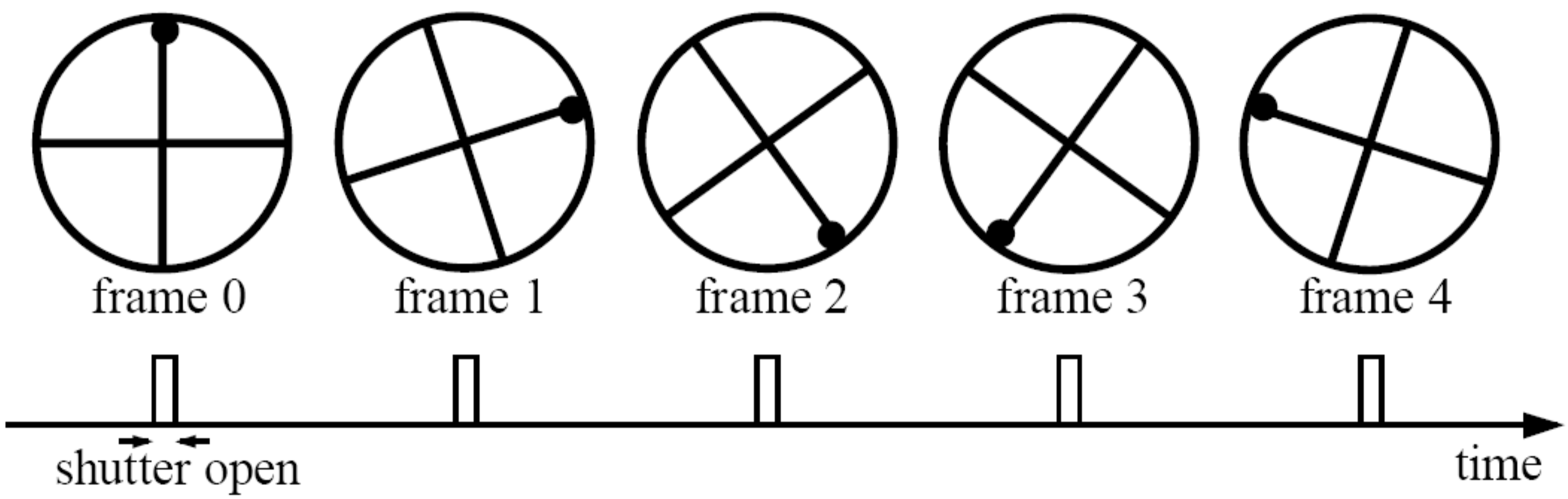
- What if we “missed” things between the samples?
- Simple example: undersampling a sine wave
  - unsurprising result: information is lost
  - surprising result: indistinguishable from lower frequency
  - also was always indistinguishable from higher frequencies
  - *aliasing*: signals “traveling in disguise” as other frequencies



Wagon-wheel effect

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):

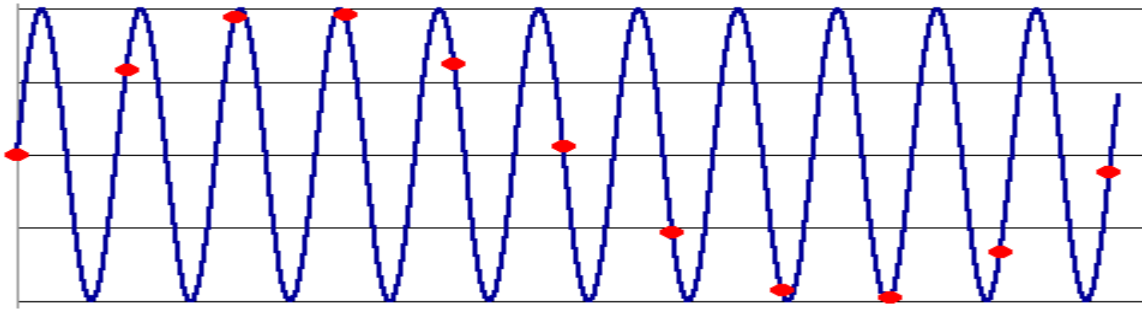


Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

(See http://www.michaelbach.de/ot/mot_wagonWheel/index.html)

Source: L. Zhang

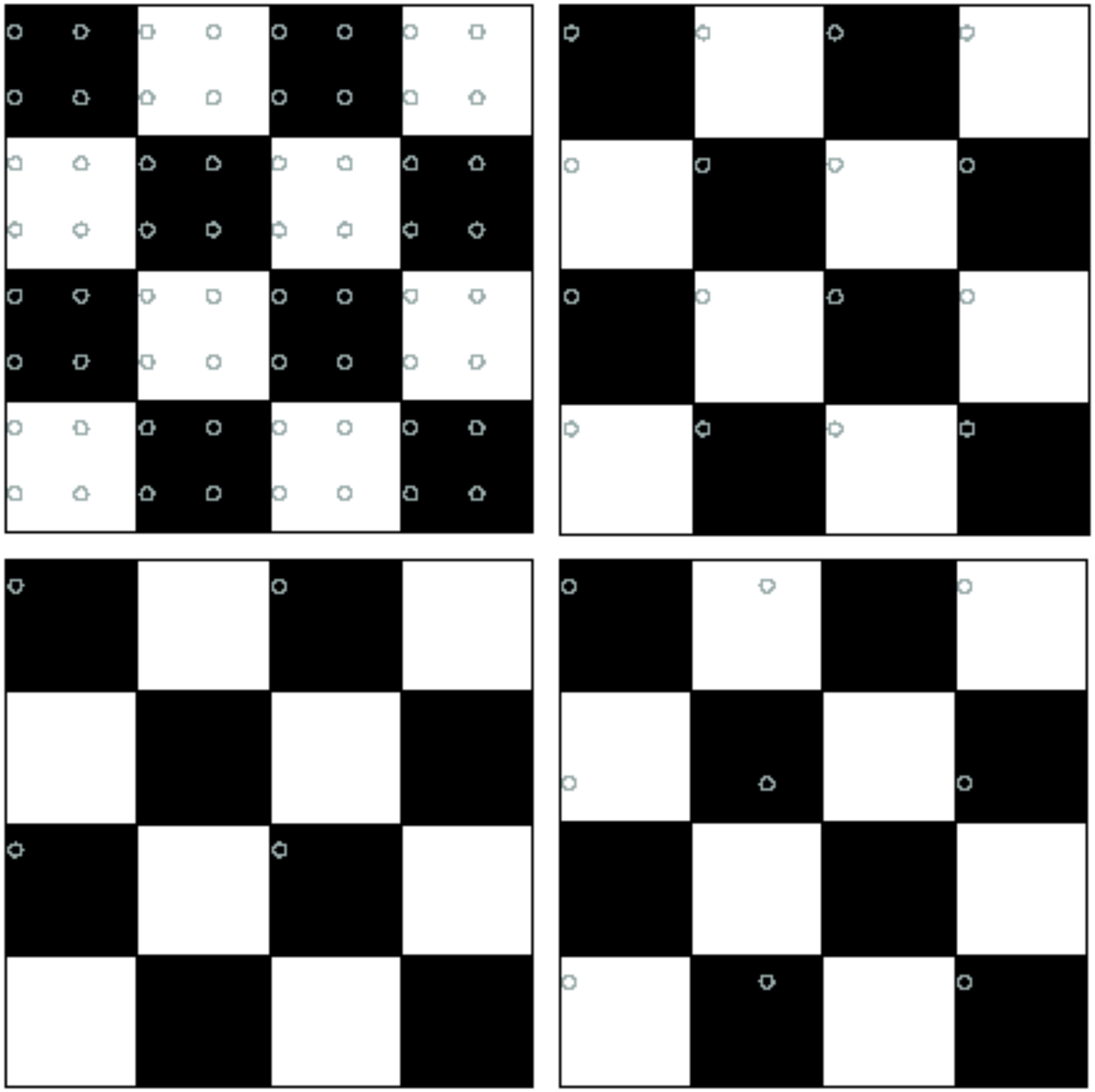
Aliasing



- Occurs when your sampling rate is not high enough to capture the amount of detail in your image
- Can give you the wrong signal/image—an *alias*
- To do sampling right, need to understand the structure of your signal/image
- Enter Monsieur Fourier…
- To avoid aliasing:
  - sampling rate ≥ 2 * max frequency in the image
    - said another way: ≥ two samples per cycle
  - This minimum sampling rate is called the **Nyquist rate**

Source: L. Zhang

Nyquist limit – 2D example



Good sampling

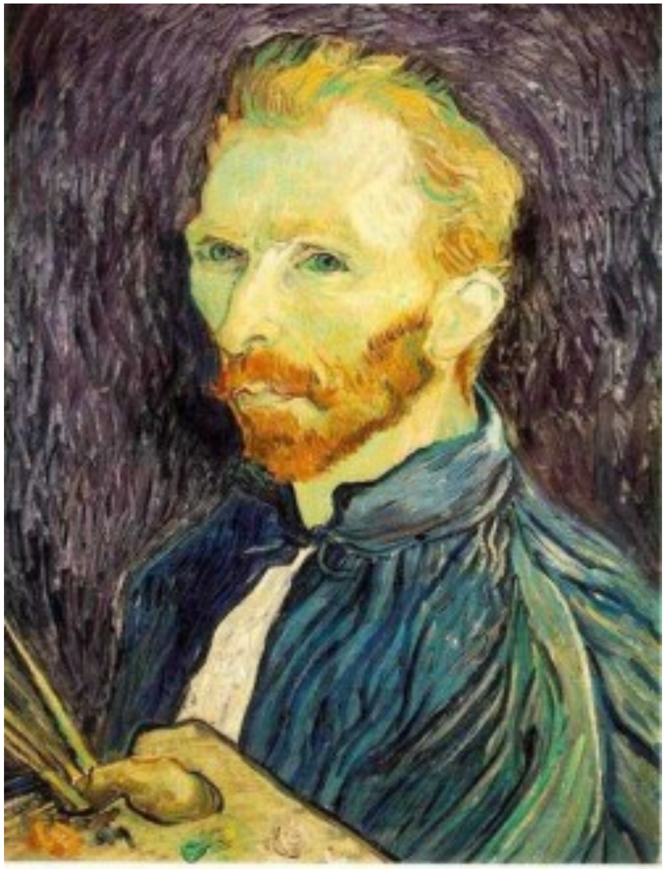
Bad sampling

Aliasing

- When downsampling by a factor of two
  - Original image has frequencies that are too high

- How can we fix this?

Gaussian pre-filtering



Gaussian 1/2

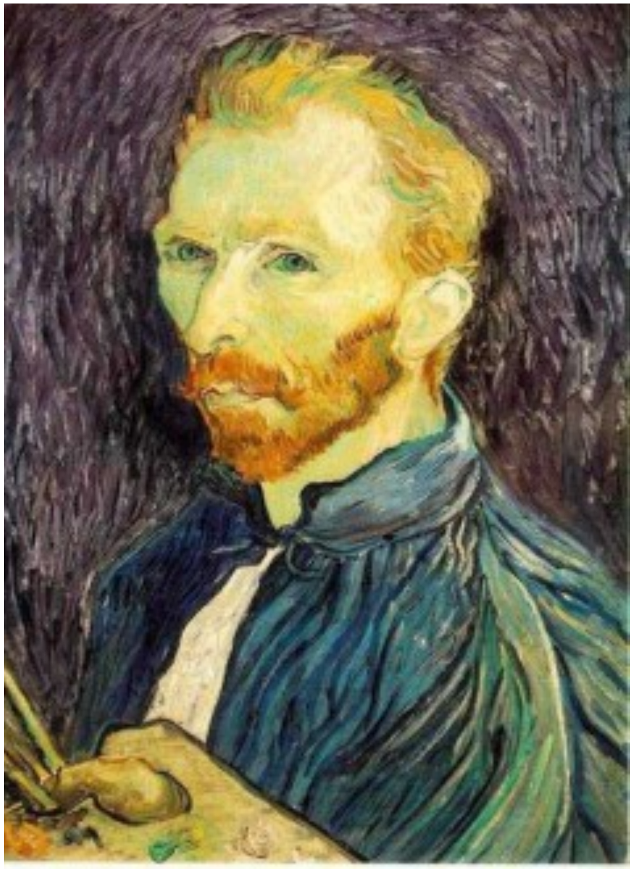
G 1/4

G 1/8

- Solution: filter the image, *then* subsample

Source: S. Seitz

Subsampling with Gaussian pre-filtering



Gaussian 1/2

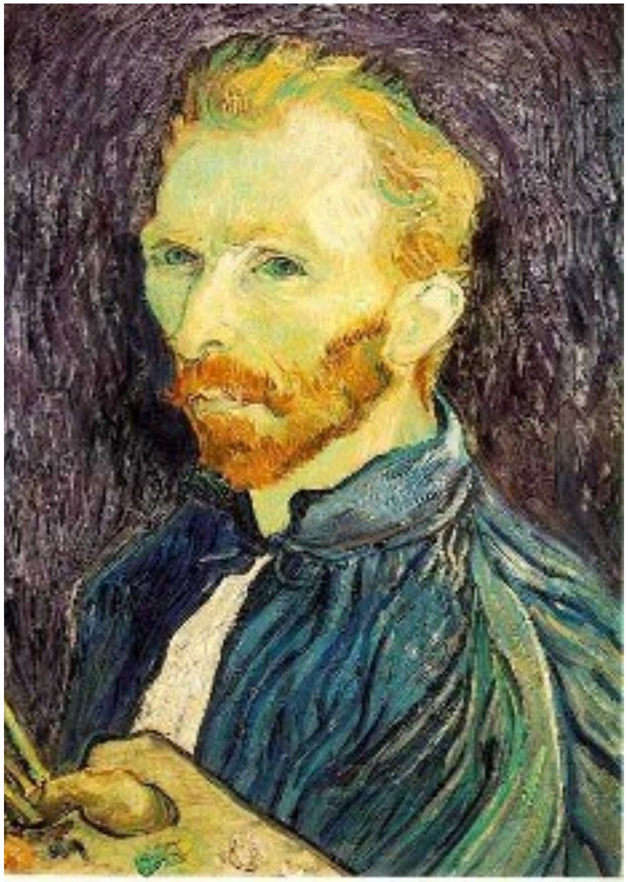
G 1/4

G 1/8

- Solution: filter the image, *then* subsample

Source: S. Seitz

Compare with...



1/2

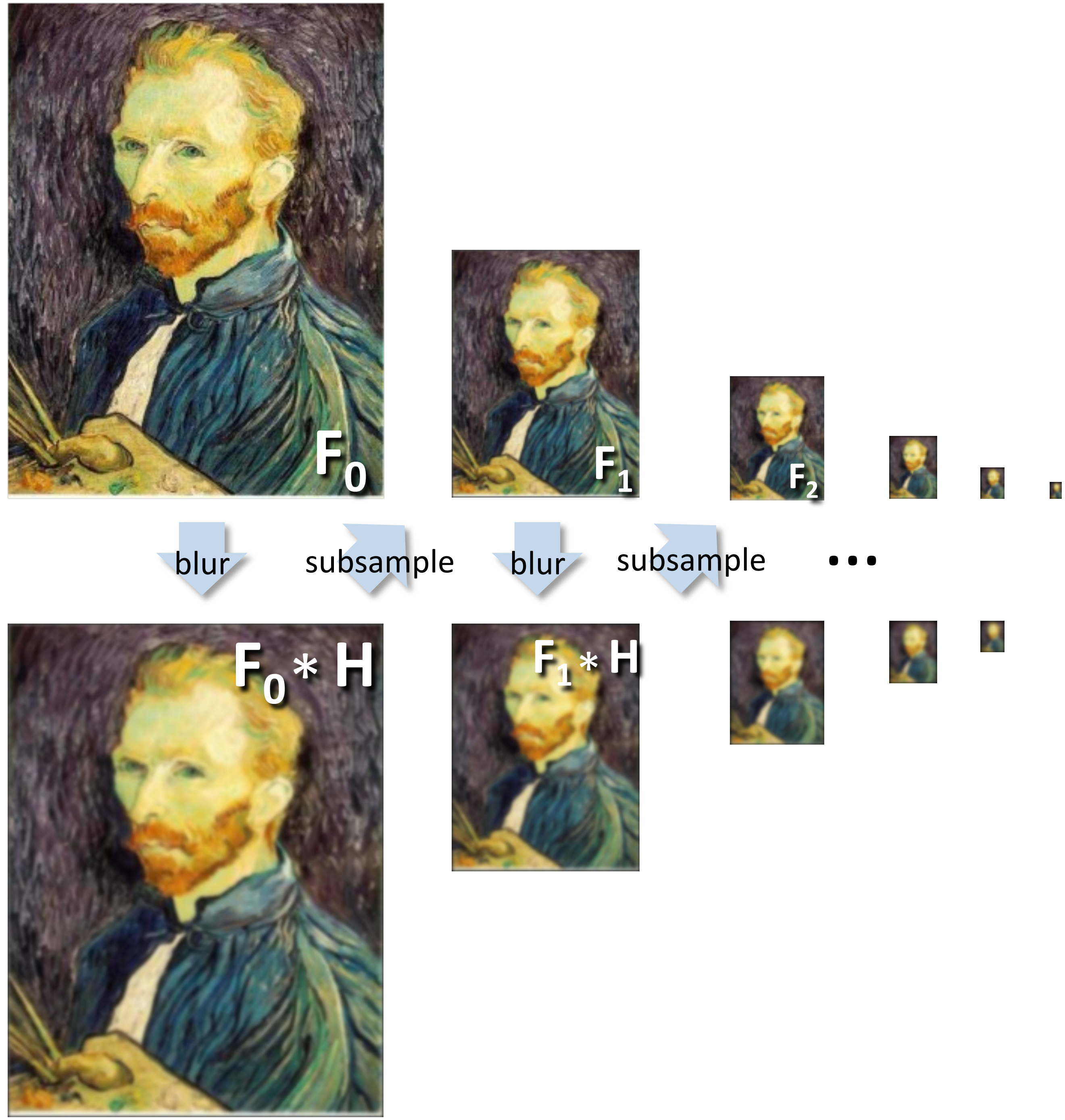
1/4 (2x zoom)

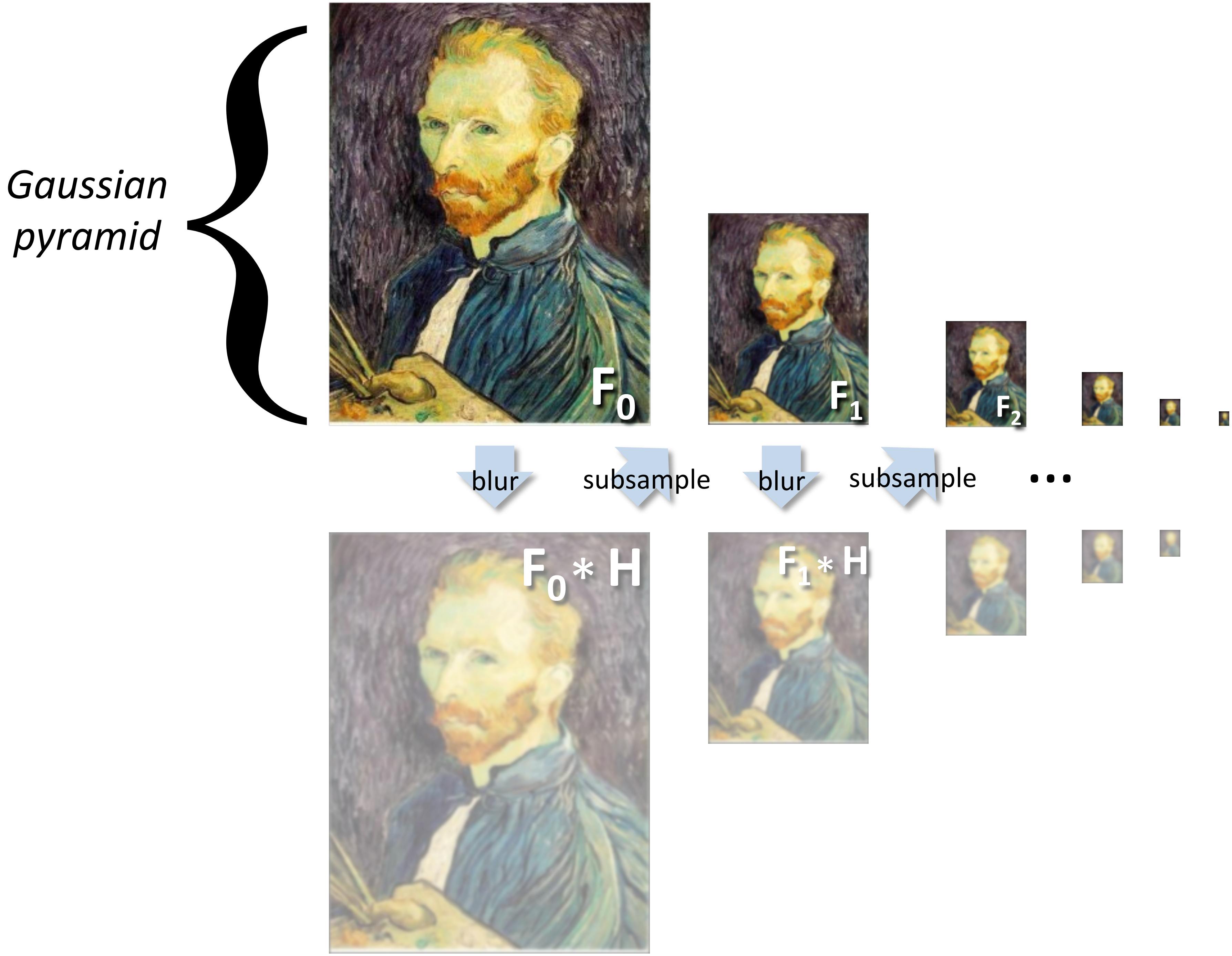
1/8 (4x zoom)

Source: S. Seitz

Gaussian pre-filtering

- Solution: filter the image, *then* subsample

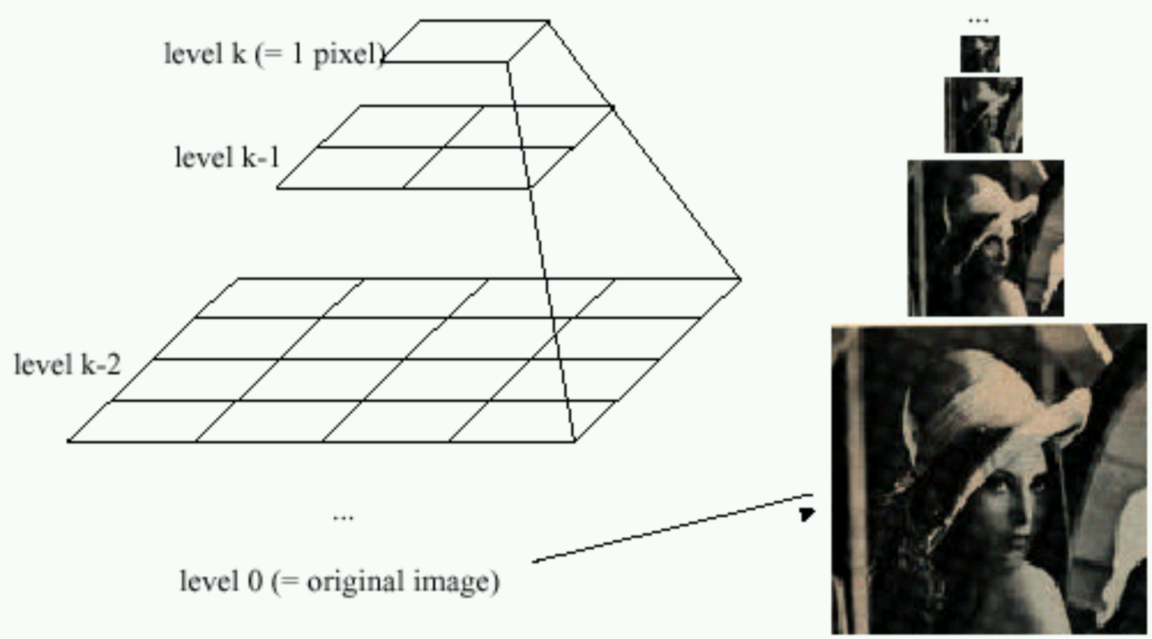




Gaussian pyramid
F0
F1
F2
blur
subsample
blur
subsample
...
F0 * H
F1 * H

Gaussian pyramids [Burt and Adelson, 1983]

Idea: Represent NxN image as a “pyramid” of 1×1, 2×2, 4×4,…, $2^k \times 2^k$ images (assuming $N=2^k$)

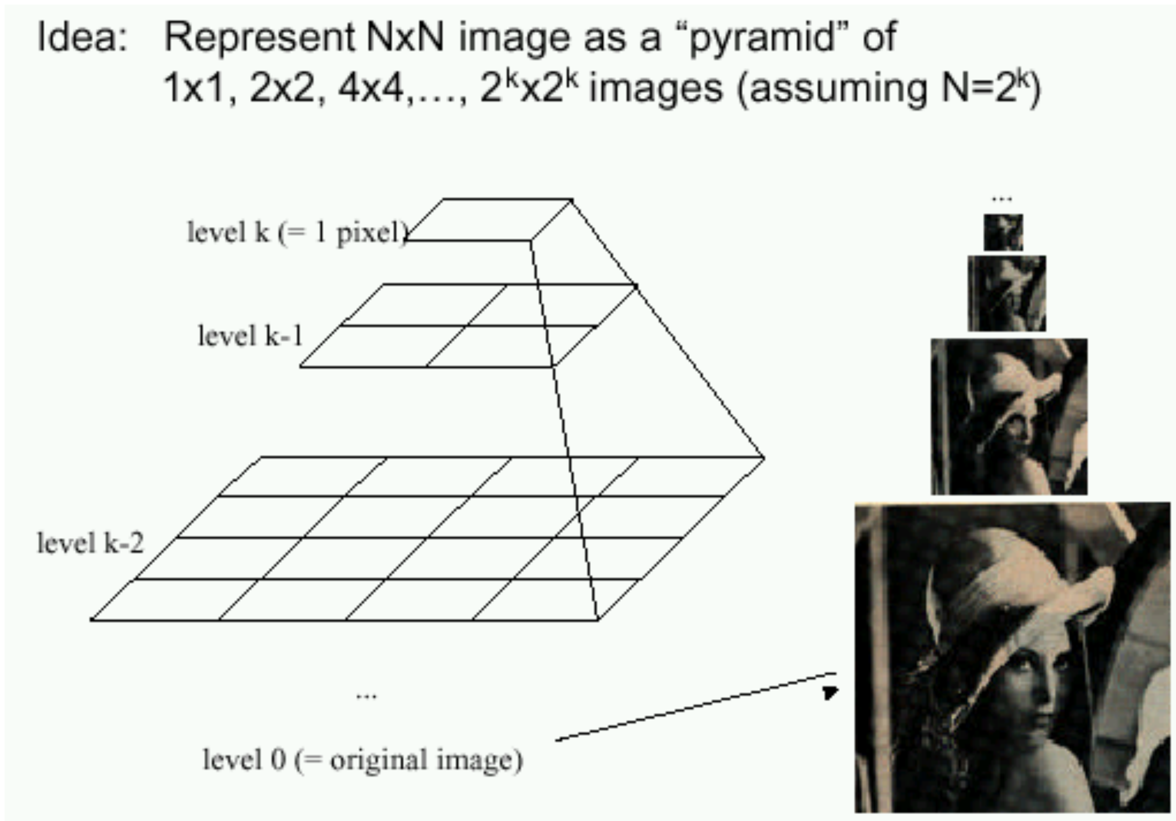


- In computer graphics, a *mip map* [Williams, 1983]
- A precursor to *wavelet transform*

Gaussian Pyramids have all sorts of applications in computer vision

Source: S. Seitz

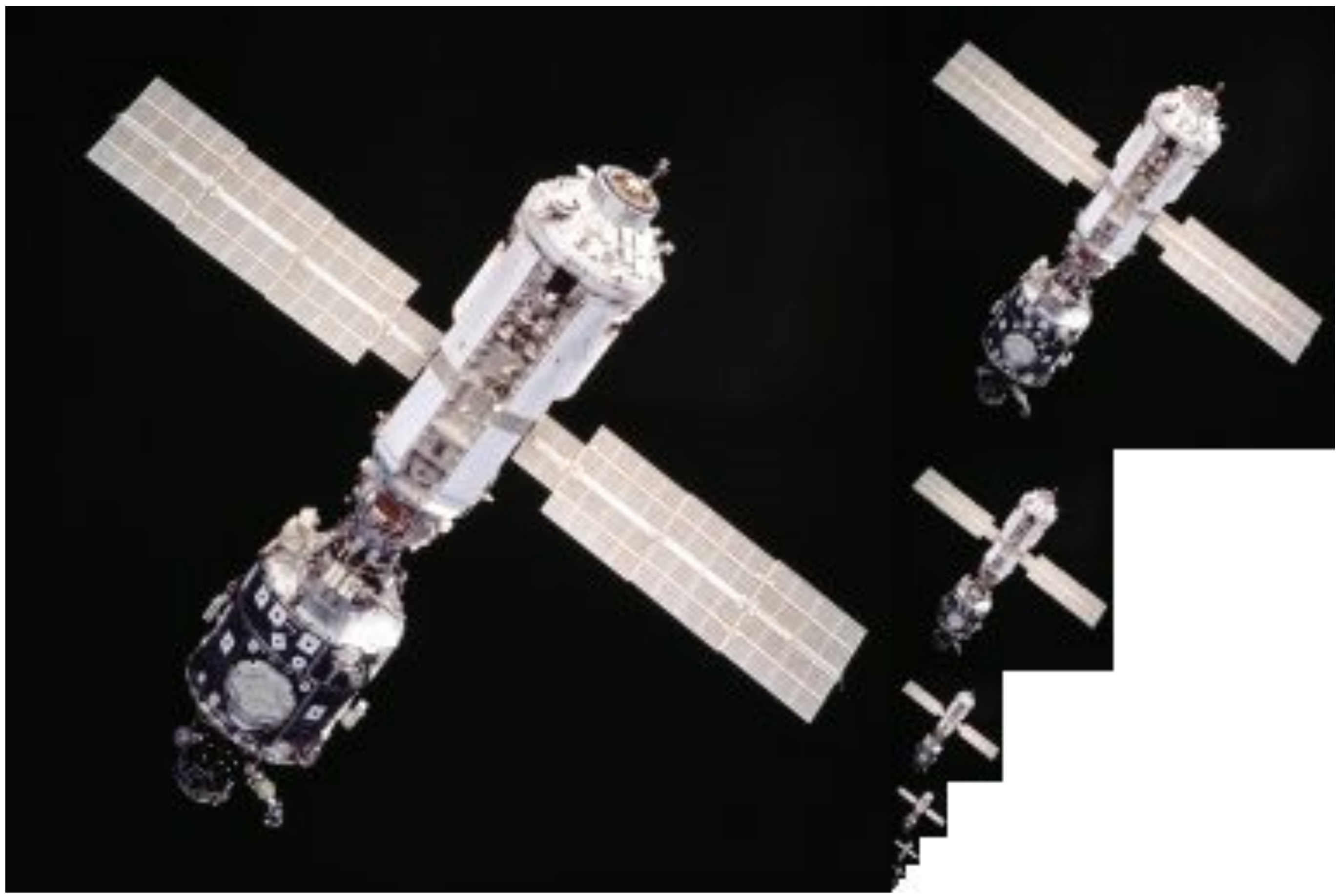
Gaussian pyramids [Burt and Adelson, 1983]



- How much space does a Gaussian pyramid take compared to the original image?

Source: S. Seitz

Gaussian Pyramid



Questions?