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CS 4620 Final, May 15, 2017

SOLUTION

1. [20 points] **Overview**

   (a) True or False?

   1. ____ In a ray tracing algorithm, the outer loop is over pixels and the inner loop is over objects.
   2. ____ In a rasterization algorithm, the outer loop is over pixels and the inner loop is over objects.
   3. ____ In rasterization, each fragment corresponds to exactly one pixel.
   4. ____ In rasterization, each pixel corresponds to exactly one fragment.
   5. ____ In the Blinn-Phong shading model, the specular highlight increases in size as the exponent p increases.
   6. ____ In the Cook-Torrance shading model, the specular highlight increases in size as the roughness α increases.
   7. ____ In the Blinn-Phong shading model, the brightest point of the specular highlight increases in brightness as the highlight gets smaller.
   8. ____ In the Cook-Torrance shading model, the brightest point of the specular highlight increases in brightness as the highlight gets smaller.
   9. ____ Perspective transformations preserve straight lines.
   10. ____ Affine transformations preserve parallel lines.
   11. ____ Perspective transformations preserve parallel lines.
   12. ____ 3D Rigid body transformations commute.
   13. ____ 3D Nonuniform scale transformations commute.
   14. ____ To transform tangents properly, you must apply the inverse transpose of the matrix used to transform positions.

   (b) One-sentence answer:

   (a) What is the *varying* keyword used for in GLSL? Why is it needed in the language?

   (b) Classify the following splines as interpolating or approximating: Bézier, Catmull-Rom, B-Spline.

   (c) Briefly explain what it means for a spline to have the convex hull property.

Solution: 1. T, 2. F, 3. T, 4. F, 5. F, 6. T 7. F, 8. T, 9. T, 10. T, 11. F, 12. F, 13. T, 14. F [1 point each.]

(a) it is for declaring varying parameters. It is needed to provide a way to pass data from the vertex to the fragment shaders. [2 points: 1 for what, 1 for why.]

(b) Bézier interpolates endpoints but not internal points. Catmull-Rom: interpolating. B-spline: approximating. [2 points. 1 for getting 2/3 right. OK to just say Bezier is approximating.]

(c) The convex hull property means that all points on each spline segment are inside the convex hull of the control points that determine that segment. [2 points. 1 for getting at the notion of bounding/containment.]

2. [20 points] **Meshes**

Consider the following file in OBJ format as we have discussed and used in class.

```
v -0.5 0.0 -0.5
v -0.5 0.0 0.5
v 0.5 0.0 -0.5
v 0.5 0.0 0.5
v -0.5 5.0 -0.5
v -0.5 5.0 0.5
v 0.5 5.0 -0.5
v 0.5 5.0 0.5
v 0.0 6.5 0.0
f 1 3 5
f 5 3 7
f 3 4 7
f 7 4 8
f 4 2 8
f 8 2 6
f 2 1 6
f 6 1 5
f 5 7 9
f 7 8 9
f 8 6 9
f 6 5 9
```

(a) What familiar landmark does this mesh resemble?

(b) This mesh does not conform to a convention established in this class. Identify the problem and explain exactly what changes to the file could be made to fix it.

(c) What vertex normal would be computed at vertex 1 if you computed smooth-shading normals for this surface using the algorithm of Assignment 1? (Even though it would not really be appropriate to use smooth normals for a shape like this....)

Solution:

(a) It's a clock tower [6 points]

(b) The faces are consistenly oriented clockwise. The change is to interchange any two vertex indices in each face. [7 points: 3 for problem, 4 for fix. 3 for "bottom not closed."]

(c) The normal is $2(-1, 0, 0) + 1(0, 0, -1) = (-2, 0, -1)/\sqrt{5}$ [7 points; -2 for not normalizing properly. 2/7 for (-1,-1,-1) which misses the point of how it is computed. Reversed normal is fine (using original faces rather than corrected.)]

3. [20 points] **GLSL**

   In Ray 1 you were provided with a shader that produced an RGB color according to the point's normal direction in world space coordinates. In this problem you will implement the same shader in GLSL. Specifically:

   - The vertex shader should do the standard transformation to produce `gl_Position`, as well as calculating any other values you may need.
   - The fragment shader should produce `gl_FragColor` according to the interpolated world space normal that would be used for shading. The {red, green, blue} channel should be equal to 0.5 * (the normal's {x, y, z} component + 1.0) respectively, and the fragment should be fully opaque.

   Vertex shader:

```
uniform mat4 modelMatrix;
uniform mat4 viewMatrix;
uniform mat4 modelViewMatrix;
uniform mat4 projectionMatrix;

uniform mat3 normalMatrix; // = inverse transpose of modelMatrix

attribute vec3 position;
attribute vec3 normal;

// Declare any varyings here:



void main() {
    // Implement vertex shader here.










}
```

Fragment shader:

```
// Declare any varyings here:



void main() {
    // Implement fragment shader here.











}
```

Solution:

Vertex shader:

```
uniform mat4 modelMatrix;
uniform mat4 viewMatrix;
uniform mat4 modelViewMatrix;
uniform mat4 projectionMatrix;

uniform mat3 normalMatrix; // = inverse transpose of modelMatrix

attribute vec3 position;
attribute vec3 normal;

// Declare any varyings here:
// 2 points
varying vec3 vNormal;

void main() {
    // Implement vertex shader here.
    // 4 points
    gl_Position = projectionMatrix * modelViewMatrix * vec4(position, 1.0);

    // 3 points
    vNormal = normalize(normalMatrix * normal);
}
```

Fragment shader:

```
// Declare any varyings here:
// 2 points
varying vec3 vNormal;

void main() {
    // Implement fragment shader here.

    // 3 points
    vec3 N = normalize(vNormal);

    // 3 points
    vec3 color = (N + 1.0) * 0.5;

    // 3 points
    gl_FragColor = vec4(color, 1.0);
}
```

4. [20 points] **Splines**

You are working with an artist to design an animation of magic vines rapidly climbing a castle wall in pursuit of the heroine, who is scrambling over the wall to escape an evil witch.

The artist would like to control the path along which a vine grows by specifying a series of points. The vine's path should pass through the location of each point exactly, so you decide to use cubic Catmull-Rom splines. As a reminder, the matrix defining a Catmull-Rom spline segment is:

$$M_{\text{CR}} = \frac{1}{2}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}.$$

In this matrix the top row corresponds to t^3 and the bottom row to t^0.

(a) How many points must the artist specify to obtain a single, C^1-continuous path that contains k Catmull-Rom spline segments? Give your answer in terms of k.

(b) The artist specifies the following points for one vine's path:

(0, -2, 0), (0, 0, 0), (1, 1, 0), (-1, 2, 2), (0, 4, 0)

The vine starts growing at the start of frame 0, and stops growing at the start of frame 24; in that time, the vine grows along the full Catmull-Rom spline specified by the above points. Where is the tip of the vine at the beginning of frame 18? You may write your answer in terms of matrix products (but the entries in the matrices should be numbers, not symbolic expressions).

(c) The animation framework you are using supports motion blur, which can increase the quality of the final animation. However, the motion blur tool needs to know the velocity of animated objects in order to correctly render them. Using the same vine path specified in part (b), what is the vector velocity of the tip of the vine at the beginning of frame 18? You may again write your answer in terms of matrix products.

Solution:

(a) You need $k + 3$ control points to define k segments. [4 points; 1 for $k + 1$.]

(b) The given frame number works out to $t = 1.5$. This segment uses the second through last control points, so the point in question can be written:

$$\begin{bmatrix} \frac{1}{8} & \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix} M_{\text{CR}} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 2 & 2 \\ 0 & 4 & 0 \end{bmatrix}$$

[10 points: 3 for getting the right t value, 3 for getting the right control points, 3 for knowing how to use the spline matrix, 1 for getting the right answer.]

(c) This problem is like the previous one, except that the vector of monomials needs to be differentiated.

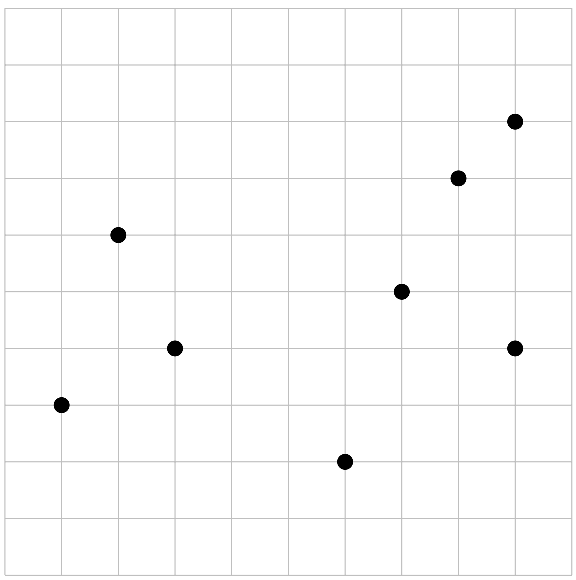
$$\begin{bmatrix} \frac{3}{4} & 1 & 1 & 0 \end{bmatrix} M_{\text{CR}} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 2 & 2 \\ 0 & 4 & 0 \end{bmatrix}$$

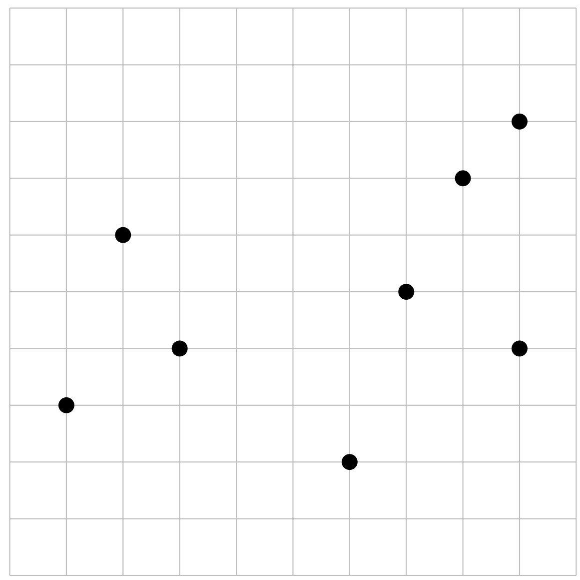
[6 points: 4 for knowing how to use the spline matrix to find a derivative, 2 for getting the right answer. Correctly worked out answer based on already-penalized wrong t value is OK.]

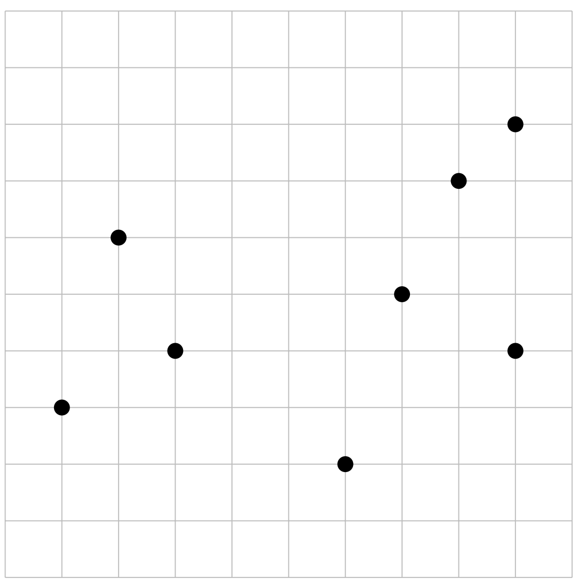
5. [20 points] **Bounding volume hierarchy**

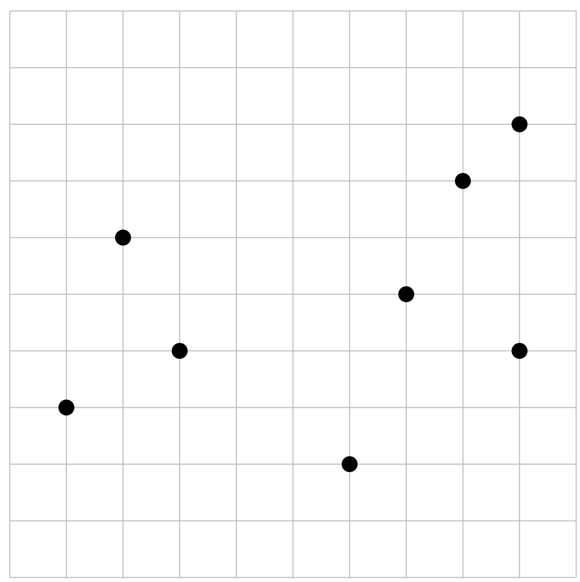
   Draw a 2D axis-aligned bounding box hierarchy for the following set of points, using the top-down strategy with median splitting that was used in Ray 2. The widest axis should be split each time. Leaf nodes should contain up to two (2) points each. Drawn lines do not have to be perfect; they will be treated as being on the nearest grid line.

   Four copies of the same diagram have been provided. You only need to complete one; if one copy becomes illegible as a result of corrections, cross it out and use another.

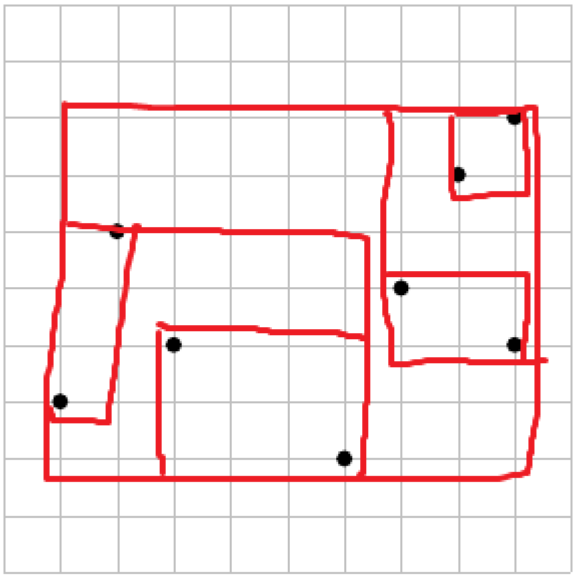








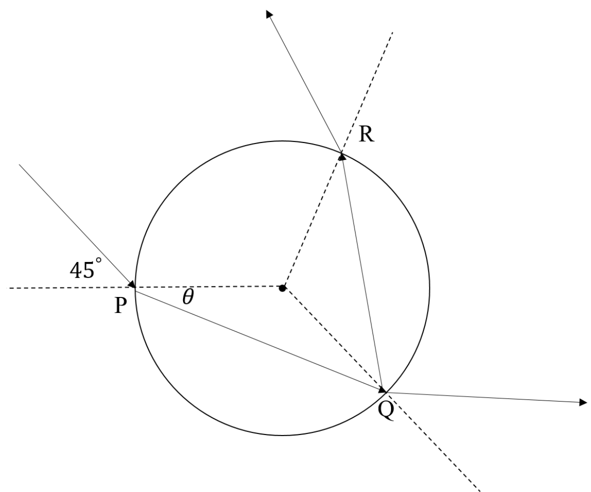
Solution:



[5 points per internal node: 2 for bbox, 3 for correct split. 1 for each leaf node, for correct bbox. 1 freebie. Subtrees are evaluated based on input from previous split. Consistenly wrong splitting only costs -3.]

6. [20 points] **Refraction**

Snell's law: Suppose we have a glass sphere, with refractive index $\sqrt{2}$. Outside the ball is air with refractive index 1.0. A ray is incident to the glass sphere at point P with incident angle 45 degrees. Here is a diagram in the plane through the ray and the sphere's center:



Note: this drawing is not to scale.

(a) Using Snell's law, what is the refraction angle θ?

(b) After refraction at P, if the ray refracts again and returns back to the air at point Q, how many degrees does the ray direction rotate (counter-clockwise) during this refraction-refraction path? That is, what is the angle between the ray direction of the ray exiting at Q and the ray that hit the sphere originally at P?

(c) If the ray refracts into the sphere, reflects internally, and then refracts back to air at point R, how many degrees has the ray direction rotated relative to that initial incident ray?

Solution:

(a) 30 degrees [8 points]

(b) 30 degrees [6 points]

(c) $15 + 120 + 15 = 150$ degrees [6 points]

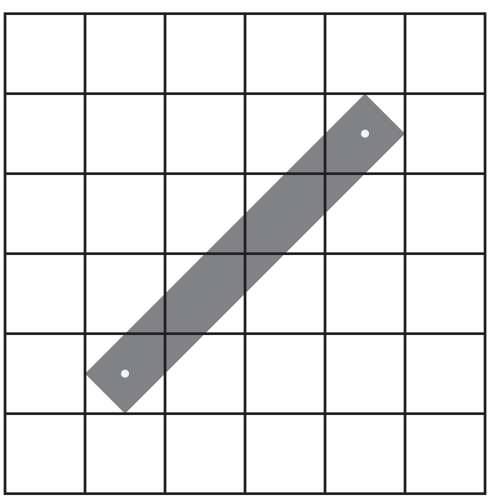
7. [20 points] **Antialiasing**

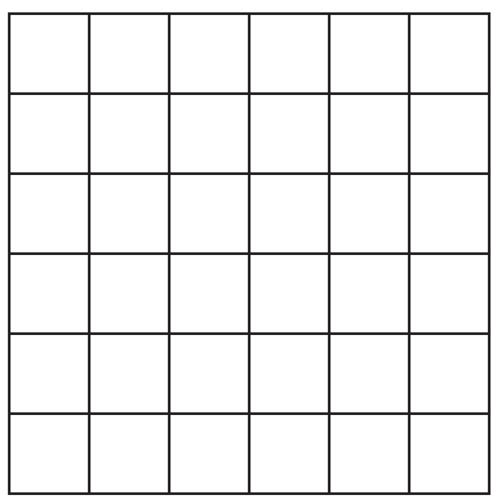
   (a) Why is aliasing a problem?

   (b) When does aliasing arise?

   (c) How does filtering alleviate aliasing?

   (d) Suppose we are rendering a black line segment on a white background in a 6x6 grayscale image; it is $1/\sqrt{2}$ pixels in width, and goes from pixel coordinates $(1, 1)$ to $(4, 4)$ as illustrated below. The line has intensity 0, and the background has intensity 1. In the blank grid to the right, fill in the pixel values of the antialiased image that would be produced by exact box filtering (also known as unweighted area averaging; it is what the Ray 2 antialiasing would compute in the limit as the number of samples goes to infinity). You don't have to fill in pixels with value 0 or 1.





Solution:

   (a) Acceptable answers include:

- It introduces features to images that are not supposed to be there.
- It creates non-smooth patterns in images that are disturbing to the human eyes.
- It makes images jaggy.

[4 points]

(b) When we display images that are created by point sampling signals that have details of frequency higher than the Nyquist rate.

An answer is acceptable if it contains the "point sampling" and "high frequency" or "fine detail"keywords and makes some kine of sense. [4 points]

(c) Filtering reduces high-frequency details in the original signal.

An acceptable answer should be about "reducing high frequency" or "removing fine details." [4 points]

(d) The answer has values 1/4 and 7/8 along the line and 3/8 at the ends. [8 points: 3 for attempting to compute area averages, 3 for having the right general pattern of intensities, 2 for getting the right numbers.]

8. [20 points] **Monte Carlo Integration**

We would like to compute the definite integral

$$\int_{-1}^{1} x^2 \; dx$$

with Monte Carlo integration.

(a) What is the value of the definite integral? Show your work.

(b) If we sample x from the interval $[-1, 1]$ uniformly. For any $x \in [-1, 1]$, what is the probability density $p(x)$ with which x gets sampled?

(c) We would like to create an estimator $g_1(x)$ for $\int_{-1}^{1} x^2 \; dx$ using the uniform sampling distribution. Write the expression for $g_1(x)$:

(d) Instead of sampling uniformly, we now sample x with probability density $q(x) = |x|$. Write the expression for the estimator $g_2(x)$ for $\int_{-1}^{1} x^2 \; dx$ that uses this sampling distribution.

Solution:

(a)

$$\int_{-1}^{1} x^2 \; dx = \left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{1^3}{3} - \frac{(-1)^3}{3} = \frac{2}{3}.$$

[3 points]

(b)

$$p(x) = \frac{1}{1-(-1)} = \frac{1}{2}.$$

[5 points: 3 for indicating awareness of probability density concept, 2 for right answer.]

(c)

$$g_1(x) = \frac{x^2}{p(x)} = \frac{x^2}{1/2} = 2x^2.$$

[6 points: 3 for attempting to compute f/p; 2 for doing it pretty much right; 1 for having normalization correct.]

(e)

$$g_2(x) = \frac{x^2}{q(x)} = \frac{x^2}{|x|} = |x|.$$

[6 points: 3 for attempting to compute f/p; 2 for doing it pretty much right; 1 for having normalization correct.]