CS 4620 Prelim 2

Thursday 5 December 2013—90 minutes

Problem 1: Shader Programming (30 pts)

For each of the following types of rendering: (a) List the uniform variables, vertex attributes, and varying variables that you would have to declare in a GLSL program. (b) Describe in words the operations that would be done by the vertex program. (c) Describe in words the operations that would be done by the fragment program.

1. Per-fragment (Phong) shading with texture-mapped diffuse color and Blinn-Phong highlights.
2. The program used to generate image 1.2 shown on the projector.
3. The program used to generate image 1.3 shown on the projector.

In all cases we are using exactly two light sources. In the texture mapped cases, the diffuse color of the surface comes directly from the texture map. Ignore ambient lighting for the purposes of this problem.

Example: for per-vertex (Gouraud) diffuse shading, the answer could be:

(a) Uniforms: diffuse color; 2 light positions; 2 light colors; modelview matrix; normal matrix; projection matrix. Attributes: vertex position, vertex normal. Varyings: surface color.

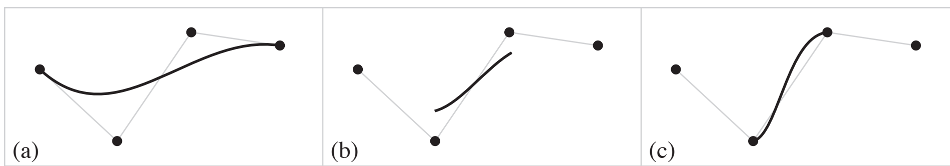
(b) Transform vertex position by modelview matrix, transform normal by normal matrix, compute surface color using diffuse shading; further transform vertex position by projection matrix.

(c) Copy surface color to output.

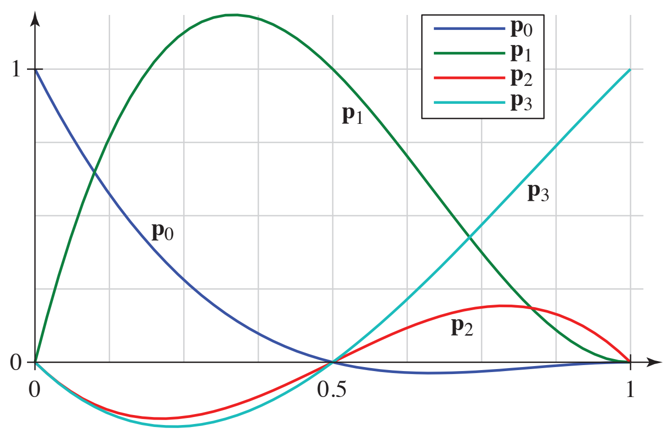
Problem 2: Spline curves (30 pts)

1. Consider a cubic Bézier spline with control points $(0, 0)$, $(9, 0)$, $(9, 0)$, and $(9, 9)$. Calculate the position and tangent vector of the spline for $t = 0$, $\frac{1}{3}$, $\frac{2}{3}$, and 1. (The length of the tangent vector is unimportant; it just needs to be tangent to the curve.)

2. In the following figure, we used the same four control points to define spline segments using the B-spline, the cubic Bézier spline, and the Catmull-Rom spline. Which one is which?



3. Someone has invented the M-spline ("mystery spline"), a cubic spline in which a segment is influenced by four control points $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$. The M-spline basis functions look like this:



   (a) At what t values, if any, does the spline pass through one of its control points?

   (b) At what t values, if any, is the spline tangent to the line between two of the points?

   In each case specify which points go with which t values.

For reference:

$$M_{\text{Bsp}} = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \qquad M_{\text{Bez}} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Problem 3: Quaternions (25 pts)

1. For each of the following matrices, give the corresponding quaternion, if any.

   (a) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

2. For each of the following quaternions, describe the corresponding rotation, if any, in terms of an axis and an angle.

   (a) $\mathbf{i}$

   (b) $1 + \mathbf{i}$

   (c) 1

   (d) $(1 + \mathbf{i}\sqrt{3})/2$

   (e) slerp$(\mathbf{i}, \mathbf{j}, 0.5)$

Problem 4: Short answer (15 pts)

Answer each of the following questions in one to three precise and accurate sentences. If you're writing more than 100 words per question you're working too hard.

1. Humans have trichromatic color vision. What does this mean and what is its significance in designing color displays?
2. What is the significance of an alpha channel in a color image? Give an example of an application where it would be useful to have an alpha channel and explain why.
3. If we compare the box filter and a bicubic filter for the purposes of image resampling, give one advantage of each over the other.