CS 465 Final Exam

Thursday 18 December 2003
2.5 hours

Problem 1: Gamma correction (6 pts)

The display on my Macintosh is calibrated to provide the transfer function $I = a^{1.8}$, where $a \in [0, 1]$ is the value associated with a pixel and I is the light intensity emitted by the display. The display on your Windows PC is calibrated to provide the transfer function $I = a^{2.2}$. Suppose I scan a photograph and adjust it to look just right on my display, then e-mail it to you. Assuming no software on either system is doing anything special about gamma correction, when you look at it on your display:

1. will it look too dark, too light, or correct?

2. will the colors look more saturated, less saturated, or correct?

Note that color saturation can be determined by the ratios of the color channels: if the ratios are close to 1 the color is nearly gray (not saturated), and if they are far from 1 the color is more saturated.

Problem 2: Surfaces and ray intersection (15 pts)

1. What implicit surface does the following equation define for a unit vector $\hat{\mathbf{v}}$:

$$f_1(\mathbf{x}) = \|(\mathbf{x} - \mathbf{c}) - [(\mathbf{x} - \mathbf{c}) \cdot \hat{\mathbf{v}}]\hat{\mathbf{v}}\| - r$$

2. What implicit surface does this equation define:

$$f_2(\mathbf{x}) = \|M\mathbf{x}\| - 1$$

(a) when $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) when $M = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$

3. Assume we have a function `intersectUnitSphere(p, d)` that returns the t value of the first intersection of the ray $\mathbf{x} = \mathbf{p} + t\mathbf{d}$ with the sphere $\|\mathbf{x}\| = 1$, or $+\infty$ if there is no intersection. Give a procedure for intersecting a ray with the surface defined by f_2 above (also returning t or $+\infty$) for any M. Does your procedure work for both values of M mentioned in part 2?

Problem 3: Image filtering (15 pts)

If the following 1D discrete filters are used to define 2D filters and applied to images, which filter goes with which operation?

1.

2.

3.

4.

5.
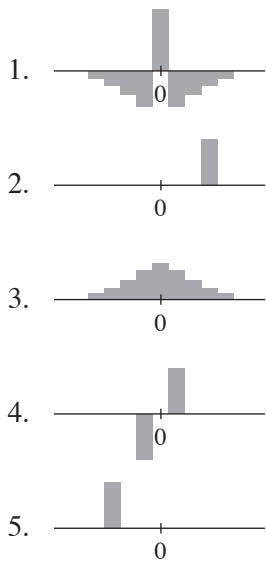
(a) blur
(b) sharpen
(c) shift left and down
(d) shift right and up
(e) differentiate (in some way)

Problem 4: Image reconstruction (12 pts)

If we use each of the following 1D reconstruction filters to reconstruct a continuous function $g(x)$ from a sequence of samples $f[i]$, for which filters will (a) $g(x)$ be C^0; (b) $g(x)$ be C^1; (c) $g(x)$ interpolate $f[i]$? (Make a 4 by 3 grid.)

1.

2.

3.

4.
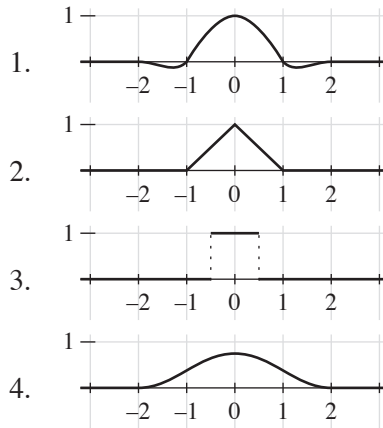
Problem 5: Transformations (16 pts)

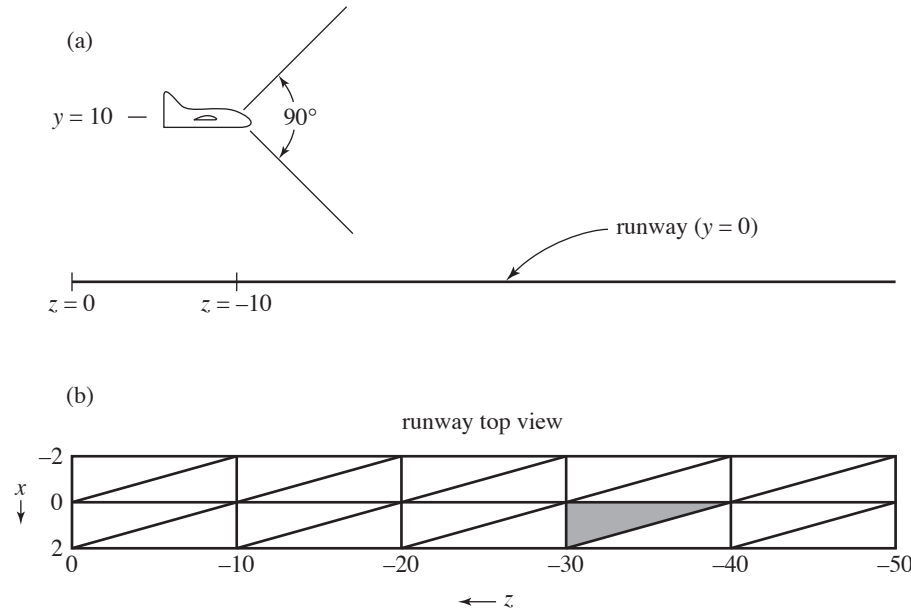Consider the following classes of elementary 3D transformations:

- scale(s_x, s_y, s_z)

- rotate-x(θ) — rotate about x axis counterclockwise by θ

- rotate-y(θ)

- rotate-z(θ)

- translate(t_x, t_y, t_z)

Each of the following sequences of transformations happens to reduce to a single transformation from one of these classes. Find the equivalent elementary transformation for each sequence.

1. scale(2, 1, 1), then scale(1, 3, 4)

2. scale(2, 1, 1), then rotate-y(90°), then scale(3, 1, 1), then rotate-y(-90°)

3. rotate-x(90°), then rotate-y(90°), then rotate-z(90°)

4. rotate-z(90°), then translate(1, 0, 0), then rotate-z(-90°)

Problem 6: Rasterization (18 pts)

You are working on a flight simulator. On approach to landing, the plane is flying level over the runway, as illustrated in diagram (a). The runway is made out of triangles as illustrated in diagram (b).

(a)

$y = 10$ — 90°

runway ($y = 0$)

$z = 0$ $z = -10$

(b)

runway top view

x

-2

0

2

0 -10 -20 -30 -40 -50

$\leftarrow z$

Assume the camera generating the cockpit view has a field of view of 90°, and the image is 1000 by 1000 pixels.

1. What are the pixel coordinates of the three vertices of the shaded triangle? (To one-pixel accuracy—don't worry about pixel centers vs. pixel corners.)

2. What are c_x and c_y, the x and y per-pixel increments the rasterizer would use to evaluate the three barycentric coordinates of the triangle?

The runway is texture mapped with a texture that covers exactly the area of the runway illustrated in diagram (b). The texture coordinate u runs from 0 to 1 across the runway from left to right (from the plane's viewpoint), and the texture coordinate v runs from 0 to 1 along the runway in the direction of the plane's travel.

3. What quantities would be interpolated by the rasterizer for the purposes of texture mapping the shaded triangle, and what are their values at the vertices?

For the triangle to be rendered, the following operations have to happen somewhere: transformation, lighting, texturing, and hidden surface removal.

4. If the runway is rendered using diffuse Gouraud shading, what attributes will need to be interpolated by the rasterizer (in addition to the barycentric coordinates and depth), and how will the above operations be divided between the vertex and fragment stages of the graphics pipeline?

5. If we decide to switch to Phong shading to better simulate the glare from the pavement, what attributes will need to be interpolated by the rasterizer (in addition to the barycentric coordinates and depth), and how will the above operations be divided between the vertex and fragment stages of the graphics pipeline?

Problem 7: Color (18 pts)

1. Which of hue, saturation, and luminance is affected by each of the following operations? (Make a 4 by 3 grid.)

   (a) mixing white with a colored paint

   (b) replacing the RGB color (r, g, b) with (ar, ag, ab)

   (c) changing Y in the (x, y, Y) color model

   (d) changing y in the (x, y, Y) color model

2. Suppose we are developing a color display for Martians that have, like humans, three types of color receptors. Their S receptors have uniform sensitivity from 400 to 500 nm and zero sensitivity elsewhere; their M receptors are uniformly sensitive from 500 to 600 nm; and the L receptors are sensitive from 600 to 700 nm:
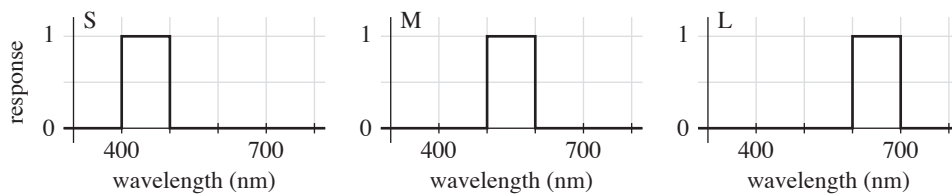
Which of these spectra is/are metameric (for Martians) to gray and why?