Last Name: ______________________ First Name: ____________ Cornell NetID: ______

CS 4620 Midterm, March 21, 2017
SOLUTION

1. [30 points] **Meshes**

   Suppose we are given the geometry shown in Figure 1, and the following position buffer containing floating point data:

   {-1.0, -1.0, 0.0, 0.0, -1.0, 0.0, 1.0, -1.0, 0.0, 0.0, 0.0, 0.0, 1.0, 0.0, 0.0, 0.0, 1.0, 0.0}

   We are also given the following color buffer, which stores color values in RGB format:

   {0.0, 0.0, 0.0, 1.0, 0.0, 0.0, 0.0, 1.0, 0.0, 0.0, 0.0, 1.0, 1.0, 1.0, 1.0}

   Suppose we were to store a color at each vertex of each triangle in the mesh. Each pixel on a triangle could then be determined by linearly interpolating the colors at each of its vertices weighted by barycentric coordinates. Figure 2 shows an example of this technique.

   (a) Write the index list that corresponds to the mesh shown in Figures 1 and 2. Each entry of the list should be in the form i/j, where i is an index into the position buffer above, and j is an index into the color buffer above. For instance, "1/2" means a vertex with position (0.0, -1.0, 0.0) and color (0.0, 1.0, 0.0). The index list should specify the triangle faces in the order labeled in Figure 1. When multiple orderings of indices are possible, chose the ordering that puts the lowest position index first. Use 0-based indexing. The triangles should be oriented such that the outward-facing normal is pointing out of the page.

   (b) If we were to modify the position buffer above, could we change the mesh's topology? Could we change its geometry? Explain your answer.

   Solution: Q1: The indices should be:

   {0/3, 1/4, 3/3, 1/4, 2/4, 3/1, 2/4, 4/3, 3/1, 3/0, 4/3, 5/0}

   This question is worth 20 points; 10 for correct position indices, and 10 for correct color indices.

   Any cycling of these indices within any group of 3 is technically correct; it is also correct if groups of 3 are rearranged. However, this is not what we asked for, so -1 for each for these errors.

-4 for listing at least one face clockwise.

Q2: Modifying the position buffer above would change the mesh's geometry, since the vertices of the mesh would move around in space. However, it would not change the mesh's topology, since its internal structure is preserved even when its vertices are moved.

This question is worth 10 points; 2 each for yes for geometry and no for topology, and 3 points each for reasonable explanations.

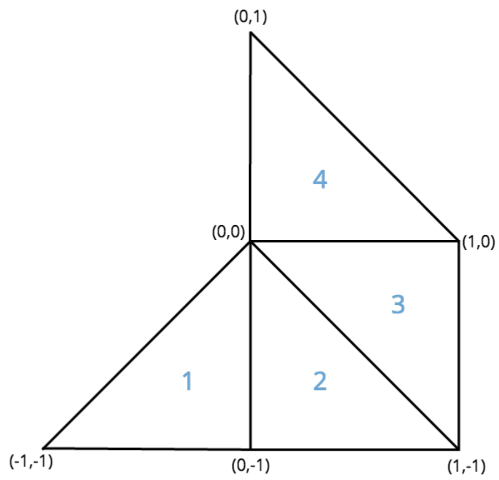


Figure 1: The target mesh configuration. All Z-coordinates of all positions are 0.

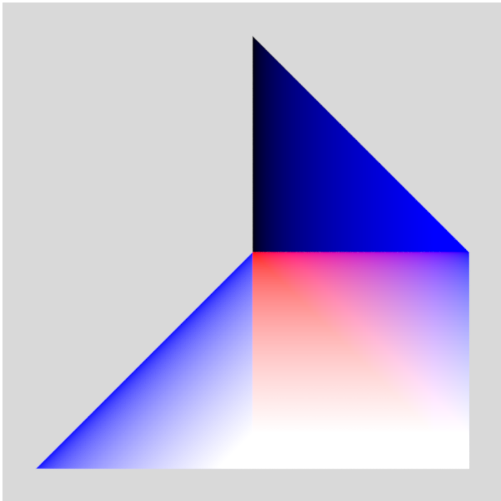


Figure 2: The target mesh, fully rendered. Color data is stored at each vertex, and is interpolated across each triangle using linear interpolation weighted by barycentric coordinates.

2. [15 points] **Texturing**

Consider the following texture:

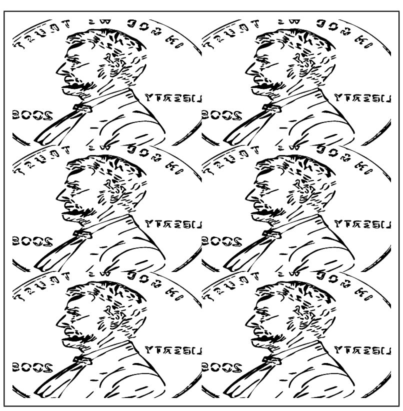
(0, 1) (1, 1)

(0, 0) (1, 0)

For each of the following images, label the four corners with UV coordinates that produce that result when the above texture is applied to a square. All of the corner UV coordinates are integers. (The borders are not part of the images.)

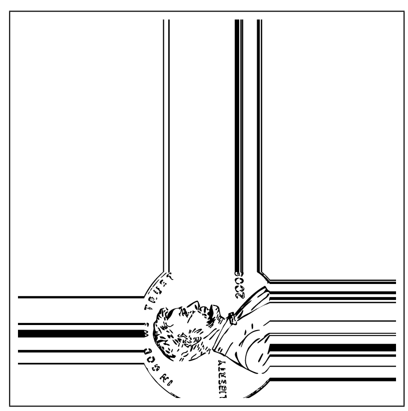
Also, write down which texture wrap mode would produce that result.

(a)



Wrap mode:

(b)



Wrap mode:

Solution:

(a)

(2, 3) (0, 3)

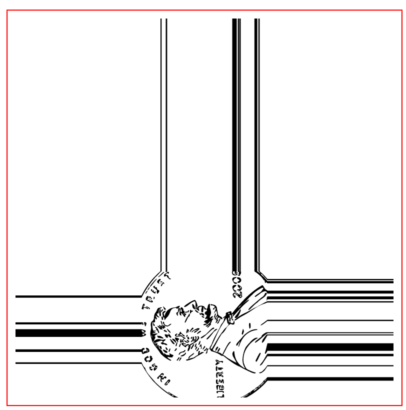
(2, 0) (0, 0)

Wrap mode: Repeat

Note that any integer translation of the UV coordinates is acceptable for this part.

(b)

(3, 2) (3, -1)

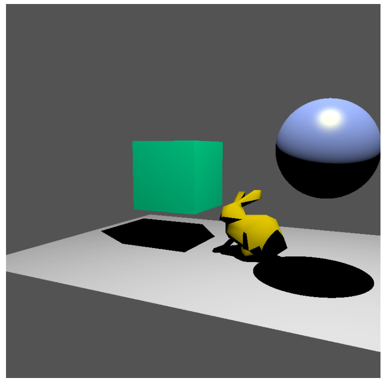


(0, 2) (0, -1)

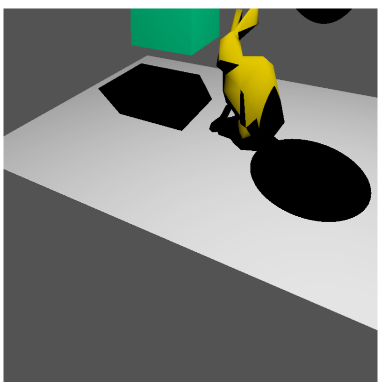
Wrap mode: Clamp

(a) 2 points: two repetitions on one axis, three on the other
   1 point: correct assignment of repetition counts to the axes
   2 point: correct horizontal mirroring
   2 points: correct texture wrap mode
(b) 2 points: correct rotation
   2 points: correct sizing (3x3)
   2 points: correct translation
   2 points: correct texture wrap mode

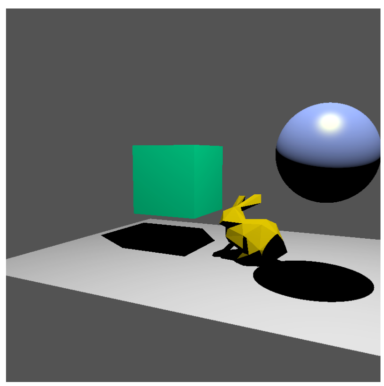
3. [25 points] **Ray Tracing Bugs**



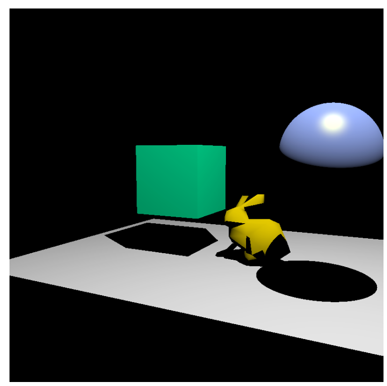
The image here is a correct ray traced rendering of a scene consisting of a single light, a box, a bunny mesh, a sphere, and a plane. The sphere has Blinn-Phong shading, and all other surfaces have Lambertian. We have modified the ray tracer to introduce several single-line bugs, one for each image you see below. For each case, match the image with the bug described on the next page, and give a brief description of how the bug leads to the visible changes in the image. Each image corresponds with exactly one bug, and the first one is completed as an example.



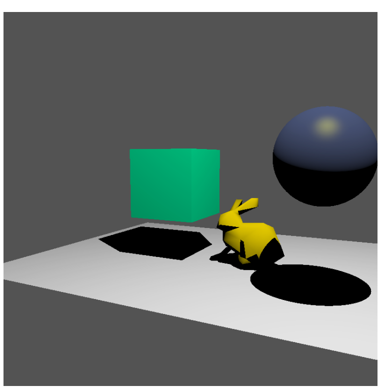
(a)



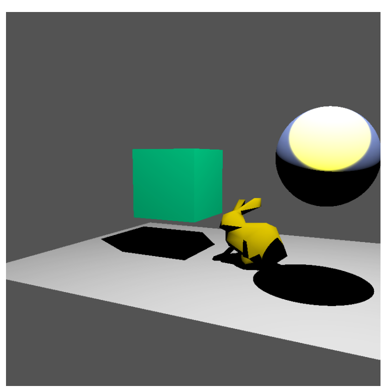
(b)



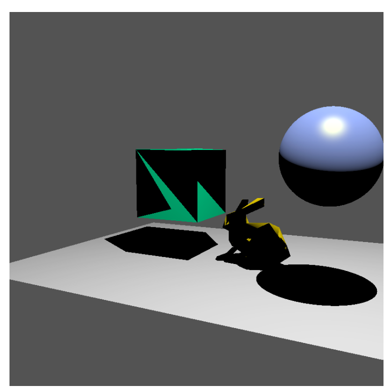
(c)



(d)



(e)



(f)

0. The triangle intersection code ignores vertex normals provided in the input mesh.

   This bug corresponds to image (b). Without vertex normals, the ray tracer is using the triangle normals, leading to a faceted appearance.

1. The half vector for Blinn-Phong shading is not normalized.

2. Rays that don't intersect any object are not handled properly.

3. The camera is generating incorrect ray directions.

4. The light intensity is not factored into Blinn-Phong shading.

5. Ray intersection is not properly determining the first object hit.

Solution: Any reasonable explanation will do. 5 points total for each question 1-5
+2 points for correct answer
+3 points for reasonable explanation
If the student provides an incorrect answer but gives a reasonable explanation for their answer, then give them 1 point.

0. b) Explanation provided in exam
1. e) The specular highlight is too bright
2. c) The background will appear as color 0 (black) because it is not sampled when a ray misses.

3. a) The camera is not generating rays properly so the image appears to be stretched by the UVs.
4. d) Only the sphere is Phong shaded so it appears dimmer.
5. f) The ray is detecting the last surface hit instead of the first.

4. [24 points] **Transforms** [1]

(a) Rewrite the following 3D affine transformation

$$\begin{bmatrix} 3\frac{\sqrt{2}}{2} & -3\frac{\sqrt{2}}{2} & 0 & 0 \\ 5\frac{\sqrt{2}}{2} & 5\frac{\sqrt{2}}{2} & 0 & 1 \\ 0 & 0 & 7 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

as a product TSR, where T is a translation, S a scaling, and R a rotation.

Solution:

$$T = \begin{bmatrix} 1 & & & 0 \\ & 1 & & 1 \\ & & 1 & 2 \\ & & & 1 \end{bmatrix}, \qquad S = \begin{bmatrix} 3 & & & \\ & 5 & & \\ & & 7 & \\ & & & 1 \end{bmatrix}, \qquad R = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & & \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

Grading: 2 points for each matrix.

(b) The following transformation is a rotation in 3D:

$$\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

What is the axis of rotation? By how many degree counterclockwise?

Solution:

Let us call the first rotation R_0. It is a rotation by 30° counterclockwise around the z-axis. The middle rotation, denoted by R_1, is a rotation around the x-axis by 45° counterclockwise. It is surrounded by
The last rotation is just R_0^{-1}.
So, the product is rotation around what R_0^{-1} maps to the x-axis. This is the vector $(\sqrt{3}/2, 1/2, 0)$. The rotation is 45° counterclockwise.

Grading: 4 points of axis, and 2 points for the angle.

[1] In case anyone is blanking on these: $\cos 30° = \sin 60° = \frac{\sqrt{3}}{2}$; $\cos 60° = \sin 30° = \frac{1}{2}$; $\cos 45° = \sin 45° = \frac{\sqrt{2}}{2}$.

(c) A unit sphere $S = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$ is transformed by the following affine transformation:

$$\begin{bmatrix} 1 & & & \\ & 2 & & \\ & & 2 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & 2 \\ & 1 & & 5 \\ & & 1 & 6 \\ & & & 1 \end{bmatrix}$$

Consider the point $\mathbf{a} = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ on S. Where is $\mathbf{a}$ after the transformation? What is the normal vector at $\mathbf{a}$ after the transformation?

Solution:

After the transformation $\mathbf{a}$ is given by:

$$\begin{bmatrix} 1 & & \\ & 2 & \\ & & 2 \end{bmatrix} \left(\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} + \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} \right) = \begin{bmatrix} 2 + 1/\sqrt{3} \\ 2(5 + 1/\sqrt{3} \\ 2(6 + 1/\sqrt{3} \end{bmatrix} = \begin{bmatrix} 2 + 1/\sqrt{3} \\ 10 + 2/\sqrt{3}) \\ 12 + 2/\sqrt{3}) \end{bmatrix}$$

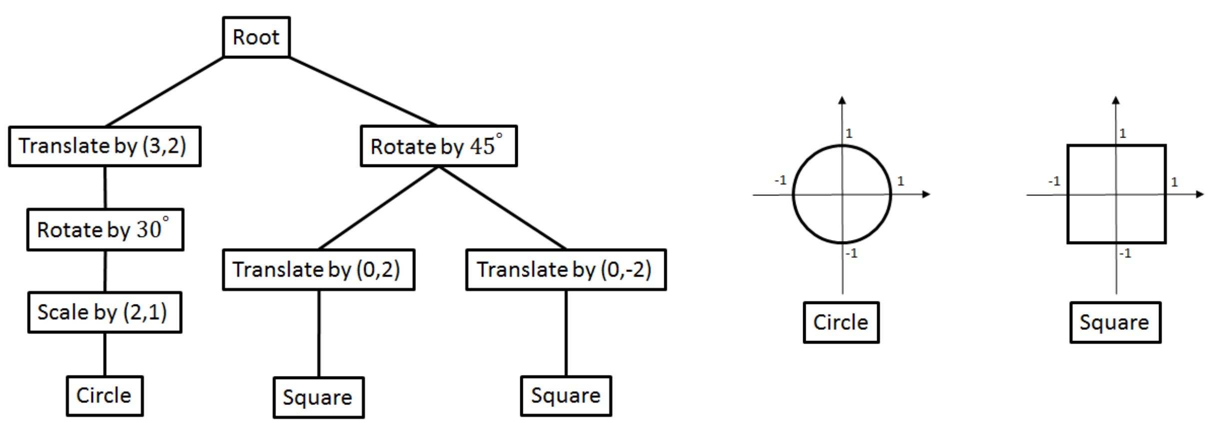
The normal vector is proprotional to:

$$\left(\begin{bmatrix} 1 & & \\ & 2 & \\ & & 2 \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \right)^{-T} \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3}) \\ 1/\sqrt{3}) \end{bmatrix} = \begin{bmatrix} 1 & & \\ & 0.5 & \\ & & 0.5 \end{bmatrix} \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3}) \\ 1/\sqrt{3}) \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} \\ 1/(2\sqrt{3}) \\ 1/(2\sqrt{3}) \end{bmatrix}$$

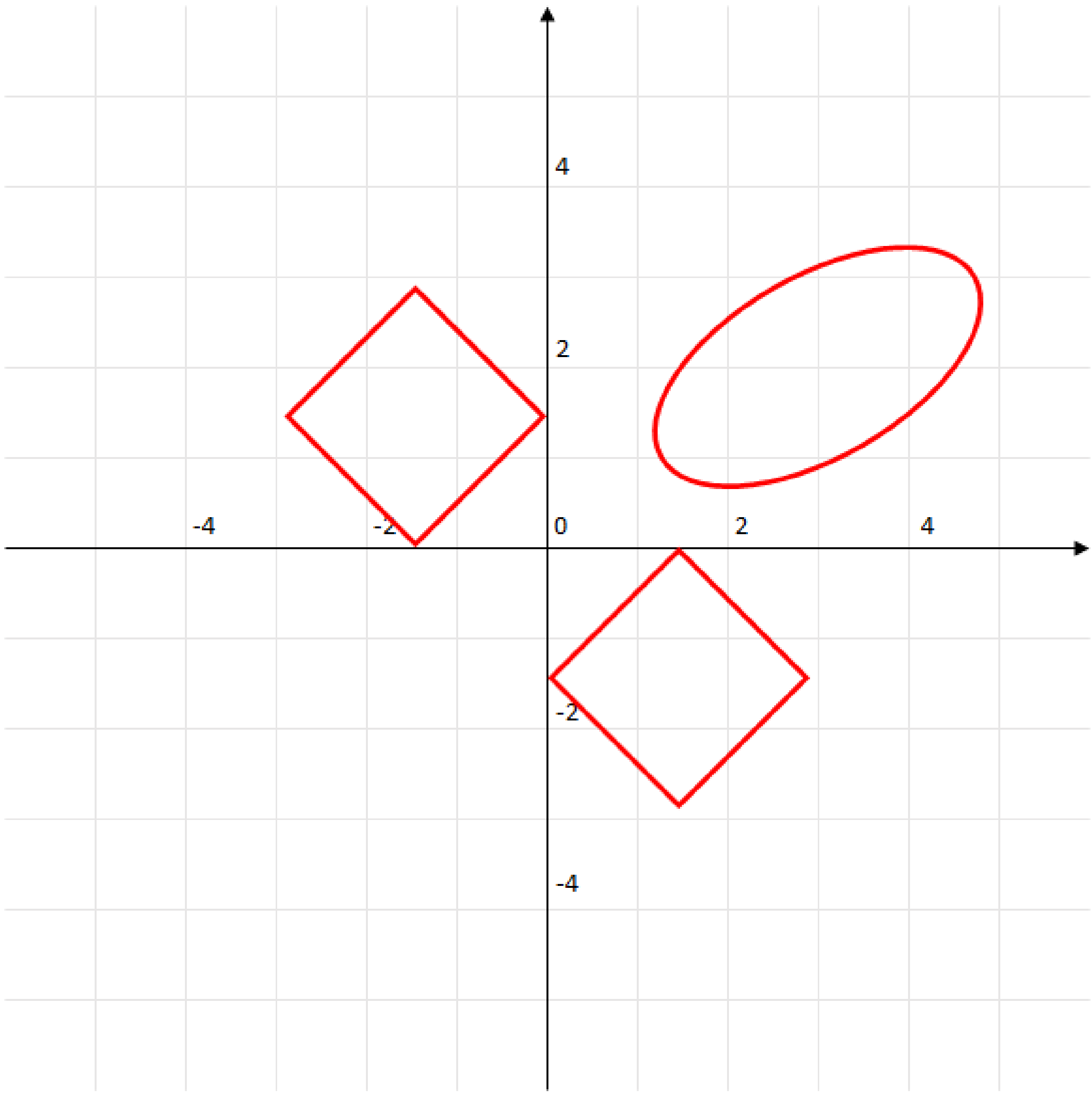
In other words, the normal vector is given by normalize$(2, 1, 1) = (2/\sqrt{6}, 1/\sqrt{6}, 1/\sqrt{6})$.

Grading: 3 points for the position and 3 points for the normal.

(d) Consider the scene graph given on the left. The "Circle" and the "Square" nodes denote the shapes given on the right. Draw the transformed shapes according to the scene graph in the space provided on the following page. Great precision in drawing is not needed; if your lines are within 1/4 unit of the right location, that is sufficient.



Solution:



Grading: 2 points for each figure.