CS 4220 / MATH 4260: Homework 4

Instructor: Anil Damle
Due: April 17, 2019

Policies

You may discuss the homework problems freely with other students, but please refrain from looking at their code or writeups (or sharing your own). Ultimately, you must implement your own code and write up your own solution to be turned in. Your solution, including plots and requested output from your code should be typeset and submitted via the CMS as a pdf file. Additionally, please submit any code written for the assignment via the CMS as well. This can be done by either including it in your solution as an appendix, or uploading it as a zip file via the CMS.

Note, this HW is shorter and focused on root finding since the second project (focused on optimization) will be available soon.

Question 1:

Implement Newton's method and the Secant method. For each of the following use a sensible convergence criteria (describe it) and compute a root of the function. Illustrate the order of convergence (and the rate if linear) exhibited by the method and discuss if you observe what you expect.

(a) $f(x) = x^2$

(b) $f(x) = \sin x + x^3$

(c) $f(x) = \sin \frac{1}{x}$ for $x \neq 0$

Question 2:

For any positive integer p prove Newton's method used to find a root of $f(x) = x^p$ converges for any initial guess. Furthermore, give the analytic rate of convergence including constants (*i.e.*, explicitly bound or express the error at the step $k+1$ in terms of the error at step k).

Question 3 (an interesting ungraded problem):

Suppose we have a function $f : \mathbb{R}^n \to \mathbb{R}$ with a local minimizer x^* such that for any direction $\delta x \in \mathbb{R}^n$ (say with $\|\delta x\|_2 = 1$) there exists an $\epsilon > 0$ such that $f(x^* + \alpha(\delta x)) > f(x^*)$ for all $\alpha \in (-\epsilon, \epsilon)$. Does this guarantee that x^* is a strict local minimizer of $f(x)$? Why or why not?