CS4220 Assignment 8 Due: 5/7/14 (Wed) at 11pm

You must work either on your own or with one partner. You may discuss background issues and general solution strategies with others, but the solutions you submit must be the work of just you (and your partner). If you work with a partner, you and your partner must first register as a group in CMS and then submit your work as a group. The assignment is worth 10 points.(One point may be deducted for poor style.

Topics: Nonlinear Least Squares

1 Mars from Earth According to Ptolemy

Assume the availability of functions

```
  function [xE,yE] = Earth(t)
% (xE,yE) is the location of Earth at time t (days).

  function [xM,yM] = Mars(t)
% (xE,yE) is the location of Earth at time t (days).
```

Your job is to determine epicycle parameters P_1, θ_1, P_2, r_2, θ_2, P_3, r_3 and θ_3 so that if

$$\begin{aligned} x(t) &= \cos(2\pi t/P_1 + \theta_1\pi/180) + r_2\cos(2\pi t/P_2 + \theta_2\pi/180) + r_3\cos(2\pi t/P_3 + \theta_3\pi/180) \\ y(t) &= \sin(2\pi t/P_1 + \theta_1\pi/180) + r_2\sin(2\pi t/P_2 + \theta_2\pi/180) + r_3\sin(2\pi t/P_3 + \theta_3\pi/180) \end{aligned}$$

then the direction of the vector

$$Q(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

is "as close as possible" to the direction of

$$R(t) = \begin{bmatrix} x_M(t) - x_E(t) \\ y_M(t) - y_E(t) \end{bmatrix}$$

over a specified time interval $[t_1, t_2]$. (Assume t_1 and t_2 are integers.) To acquire intuition about this problem, play with the given script `A8.m`

In this epicycle model, think of the Earth at the center of a unit disk whose period of revolution is P_1. Centered on the rim of this disk is a second disk whose radius is r_2 and whose period of revolution is P_2. Finally, centered on the rim of the second disk is a third disk whose radius is r_3 and whose period of revolution is P_3. On the rim of the third disk is Mars and so $Q(t)$ is a vector that points to Mars from the "observation point" (0,0). In addition to the radii and "rpm" values associated with each circle, we have "phase angles" θ_1, θ_2, and θ_3 associated with each rotating disk.

For an objective function, use

$$\phi(P_1, \theta_1, P_2, r_2, \theta_2, P_3, r_3, \theta_3) = \sum_{t=t_1}^{t_2} \sin(\phi_t)^2$$

where ϕ_t is the angle between the 2-vectors $Q(t)$ and $R(t)$. Make effective use of `lsqnonlin`. Getting a good initial guess is important. It might be handy to use the fact that the two planets have approximate periods 365 and 697 (days).

Encapsulate everything in the function `ThreeCircle` whose specification is as follows:

```
  function [P1,theta1,P2,r2,theta2,P3,r3,theta3,err] = ThreeCircle(t1,t2)
% t1 and t2 are integers that satisfy 0<=t1 and t1+7<=t2<=5000.
% Let sigma(t) be the sine of the angle between the vector [xM(t)-xE(t); yM(t)-yE(t)]
% where
```

```
%           [xM(t);yM(t)] is the location of Mars at time t
%           [xE(t);yE(t)] is the location of Earth at time t.
% and the vector [x(t);y(t)] where
%    x(t) =    cos(2*pi*t/P1 + theta1*pi/180) +
%           r2*cos(2*pi*t/P2 + theta2*pi/180) +
%           r3*cos(2*pi*t/P3 + theta3*pi/180)
%    y(t) =    sin(2*pi*t/P1 + theta1*pi/180) +
%           r2*sin(2*pi*t/P2 + theta2*pi/180) +
%           r3*sin(2*pi*t/P3 + theta3*pi/180)
% The epicycle parameters P1,theta1,P2,r2,theta2,P3,r3,theta3 are chosen to
% minimize sigma(t1)^2 + sigma(t1+1)^2 + ... + sigma(t2)^2.
% err is the maximum of |sigma(t1)|,...,|sigma(t2)|
```

Submit your implementation of `ThreeCircle` to CMS. The assignment is worth 10 points: four points for setting up the correct function that has to be passed to `lsqnonlin` and making its evaluation efficient, four points for how well you reasoned about the starting value, and two points for the quality of the fit

For your information, vector cross-products can be used to compute the sine of an angle between two vectors. In particular, if

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad \text{and} \qquad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

are unit 2-norm vectors, then the sine of the angle between them is given by $u_1 v_2 - u_2 v_1$.

It is always a good idea to look at the behavior of the function that you are trying to fit. Below is a plot of how much Mars (the "red dot" in the simulation) advances each day in radians across the background of the fixed stars.