CS 4110

Programming Languages & Logics

Lecture 28
Existential Types

Namespaces

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It’s no fun to program in a language with a single, global namespace: C, FORTRAN, and PHP until depressingly recently.

Components of a large program have to worry about name collisions.

And components become tightly coupled: any component can use a name defined by any other.

Modularity

A *module* is a collection of named entities that are related.

Modules provide separate namespaces: different modules can use the same names without worrying about collisions.

Modules can:

- Choose which names to export
- Choose which names to keep hidden
- Hide implementation details

Existential Types

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$$\tau ::= \cdots \mid X \mid \forall X.\, \tau$$

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$$\tau ::= \cdots \mid X \mid \forall X.\, \tau$$

If we have $\forall$, why not $\exists$? What would *existential* type quantification do?

$$\tau ::= \cdots \mid X \mid \exists X.\, \tau$$

Existential Types

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$$
\begin{aligned}
&\exists\, \textbf{Counter}.\\
&\quad \{\, \mathsf{new} : \textbf{Counter},\\
&\quad\;\; \mathsf{get} : \textbf{Counter} \to \textbf{int},\\
&\quad\;\; \mathsf{inc} : \textbf{Counter} \to \textbf{Counter} \,\}
\end{aligned}
$$

Existential Types

Together with records, existential types let us *hide* the implementation details of an interface.

$$\begin{array}{l} \exists\ \textbf{Counter}. \\ \quad \{\ \text{new} : \textbf{Counter}, \\ \quad\ \ \ \text{get} : \textbf{Counter} \to \textbf{int}, \\ \quad\ \ \ \text{inc} : \textbf{Counter} \to \textbf{Counter}\ \} \end{array}$$

Here, the *witness type* might be **int**:

$$\begin{array}{l} \{\ \text{new} : \textbf{int}, \\ \ \ \ \text{get} : \textbf{int} \to \textbf{int}, \\ \ \ \ \text{inc} : \textbf{int} \to \textbf{int}\ \} \end{array}$$

Existential Types

Let's extend our STLC with existential types:

$$
\begin{aligned}
\tau ::= {} & \mathbf{int} \\
\mid {} & \tau_1 \to \tau_2 \\
\mid {} & \{\, l_1 : \tau_1, \ldots, l_n : \tau_n \,\} \\
\mid {} & \exists X.\, \tau \\
\mid {} & X
\end{aligned}
$$

Syntax & Dynamic Semantics

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where v has type $\tau\{\tau'/X\}$.

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A value has type $\exists\, X.\ \tau$ is a pair $\{\tau', v\}$
where v has type $\tau\{\tau'/X\}$.

We'll add new operations to construct and destruct these pairs:

$$\text{pack } \{\tau_1, e\} \text{ as } \exists\, X.\ \tau_2$$

$$\text{unpack } \{X, x\} = e_1 \text{ in } e_2$$

Syntax

$$
\begin{aligned}
e ::= {} & x \\
\mid {} & \lambda x\!:\!\tau.\, e \\
\mid {} & e_1\ e_2 \\
\mid {} & n \\
\mid {} & e_1 + e_2 \\
\mid {} & \{\, l_1 = e_1, \ldots, l_n = e_n \,\} \\
\mid {} & e.l \\
\mid {} & \mathsf{pack}\ \{\tau_1, e\}\ \mathsf{as}\ \exists X.\, \tau_2 \\
\mid {} & \mathsf{unpack}\ \{X, x\} = e_1\ \mathsf{in}\ e_2
\end{aligned}
$$

$$
\begin{aligned}
v ::= {} & n \\
\mid {} & \lambda x\!:\!\tau.\, e \\
\mid {} & \{\, l_1 = v_1, \ldots, l_n = v_n \,\} \\
\mid {} & \mathsf{pack}\ \{\tau_1, v\}\ \mathsf{as}\ \exists X.\, \tau_2
\end{aligned}
$$

Dynamic Semantics

$$
\begin{aligned}
E ::= {} & \ldots \\
& \mid \text{pack } \{\tau_1, E\} \text{ as } \exists\, X.\ \tau_2 \\
& \mid \text{unpack } \{X, x\} = E \text{ in } e
\end{aligned}
$$

$$
\frac{}{\text{unpack } \{X, x\} = (\text{pack } \{\tau_1, v\} \text{ as } \exists\, Y.\ \tau_2) \text{ in } e \rightarrow e\{v/x\}\{\tau_1/X\}}
$$

Static Semantics

$$\frac{\Delta, \Gamma \vdash e : \tau_2\{\tau_1/X\}}{\Delta, \Gamma \vdash \mathsf{pack}\ \{\tau_1, e\}\ \mathsf{as}\ \exists\, X.\, \tau_2 : \exists\, X.\, \tau_2}$$

Static Semantics

$$\frac{\Delta, \Gamma \vdash e : \tau_2\{\tau_1/X\}}{\Delta, \Gamma \vdash \mathsf{pack}\ \{\tau_1, e\}\ \mathsf{as}\ \exists\, X.\ \tau_2 : \exists\, X.\ \tau_2}$$

$$\frac{\Delta, \Gamma \vdash e_1 : \exists\, X.\ \tau_1 \quad \Delta \cup \{X\}, \Gamma, x : \tau_1 \vdash e_2 : \tau_2 \quad \Delta \vdash \tau_2\ \mathsf{ok}}{\Delta, \Gamma \vdash \mathsf{unpack}\ \{X, x\} = e_1\ \mathsf{in}\ e_2 : \tau_2}$$

The side condition $\Delta \vdash \tau_2$ ok ensures that the existentially quantified type variable X does not appear free in τ_2.

Example

$$
\begin{array}{l}
\text{let } \mathit{counterADT} = \\
\quad \text{pack } \{\, \textbf{int}, \\
\qquad\qquad \{\, \text{new} = 0, \\
\qquad\qquad\quad \text{get} = \lambda i : \textbf{int}.\, i, \\
\qquad\qquad\quad \text{inc} = \lambda i : \textbf{int}.\, i + 1 \,\}\, \} \\
\quad \text{as} \\
\qquad \exists\, \textbf{Counter}. \\
\qquad\qquad\quad \{\, \text{new} : \textbf{Counter}, \\
\qquad\qquad\quad\;\; \text{get} : \textbf{Counter} \to \textbf{int}, \\
\qquad\qquad\quad\;\; \text{inc} : \textbf{Counter} \to \textbf{Counter} \} \\
\text{in} \ldots
\end{array}
$$

Example

Here's how to use the existential value *counterADT*:

$$\begin{array}{l}\text{unpack}\ \{T, c\} = \mathit{counterADT}\ \text{in} \\ \text{let}\ y = c.\text{new in} \\ c.\text{get}\ (c.\text{inc}\ (c.\text{inc}\ y))\end{array}$$

Representation Independence

We can define alternate, equivalent implementations of our counter...

$$
\begin{array}{l}
\text{let } \mathit{counterADT} = \\
\quad \text{pack } \{\{x : \textbf{int}\}, \\
\qquad\qquad \{\ \text{new} = \{x = 0\}, \\
\qquad\qquad\ \ \text{get} = \lambda r : \{x : \textbf{int}\}.\ r.x, \\
\qquad\qquad\ \ \text{inc} = \lambda r : \{x : \textbf{int}\}.\ r.x + 1\ \}\ \} \\
\quad \text{as} \\
\qquad \exists \textbf{Counter}. \\
\qquad\qquad \{\ \text{new} : \textbf{Counter}, \\
\qquad\qquad\ \ \text{get} : \textbf{Counter} \rightarrow \textbf{int}, \\
\qquad\qquad\ \ \text{inc} : \textbf{Counter} \rightarrow \textbf{Counter}\} \\
\text{in } \ldots
\end{array}
$$

Existentials and Type Variables

In the typing rule for unpack, the side condition $\Delta \vdash \tau_2$ ok prevents type variables from "leaking out" of unpack expressions.

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This rules out programs like this:

let $m =$

$\quad$ pack $\{\textbf{int}, \{a = 5, f = \lambda x\!:\!\textbf{int}.\ x + 1\}\}$ as $\exists\, X.\ \{a\!:\!X, f\!:\!X \to X\}$

in

unpack $\{T, x\} = m$ in $x.f\, x.a$

where the type of $x.f\, x.a$ is just T.

Encoding Existentials

We can encode existentials using universals!

The idea is to use a Church encoding where an existential value is a function that takes a type and then calls a continuation.

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The idea is to use a Church encoding where an existential value is a function that takes a type and then calls a continuation.

$$\exists X.\, \tau \quad \triangleq \quad \forall Y.\, (\forall X.\, \tau \to Y) \to Y$$

$$\text{pack } \{\tau_1, e\} \text{ as } \exists X.\, \tau_2 \quad \triangleq \quad \Lambda Y.\, \lambda f : (\forall X. \tau_2 \to Y).\, f\, [\tau_1]\, e$$

$$\text{unpack } \{X, x\} = e_1 \text{ in } e_2 \quad \triangleq \quad e_1\, [\tau_2]\, (\Lambda X. \lambda x : \tau_1.\, e_2)$$

where e_1 has type $\exists X. \tau_1$ and e_2 has type τ_2