

$$\begin{array}{c}
 (\lambda x : \tau_1 \rightarrow \tau_2. x \quad x) \quad (\lambda x : \tau_3 \rightarrow \tau_4. x \quad x) \\
 \uparrow \quad \quad \quad \uparrow \\
 \tau_1 \rightarrow \tau_2
 \end{array}$$

$$\tau_1 = \tau_1 \rightarrow \tau_2$$

"PROOF"

INDUCT ON e .

e_1, e_2 ASSUMING $\vdash e : \tau$.

$$\vdash e_1 : \tau' \rightarrow \tau$$

$$\vdash e_2 : \tau'$$

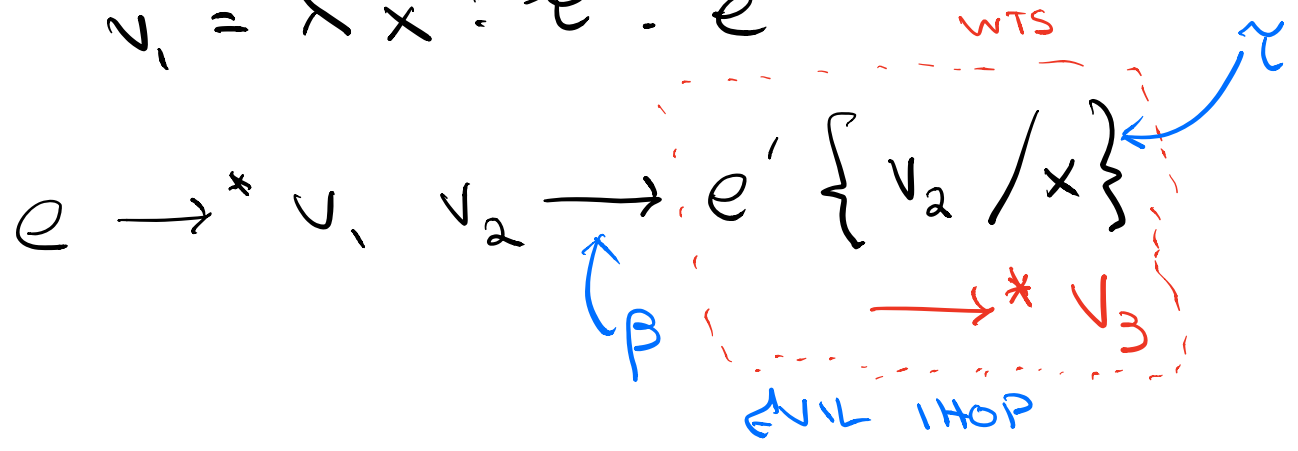
By IHOP:

$$e_1 \rightarrow^* v_1 \quad \text{SAFETY:} \quad \vdash v_1 : \tau' \rightarrow \tau$$

$$e_2 \rightarrow^* v_2 \quad \tau'$$

By CANONICAL FORMS:

$$v_1 = \lambda x : \tau'. e'$$



PROOF!

INDUCT ON e .

CASE $x \quad \Gamma \vdash x : \tau$

INVERSION $\Rightarrow P(x) = \tau$.

$x = x_i$ (FOR SOME i)

$\tau = \tau_i$

$e \{ v_1 / x_1 \} \dots \{ v_k / x_k \} = v_i$

\nearrow

\leftarrow v_i / x_i

$R_{\tau_i}(v_i)$

$R_{\tau_i}(\quad)$, AS DESIRED.

CASE $()$

$\tau = \text{unit}$

$e \{ \quad \} \{ \quad \} \dots \{ \quad \} = ()$

$R_{\text{unit}}(())$

CASE $e = \lambda x : \tau'. e'$

BY INVERSION,

$$\tau = \tau' \rightarrow \tau''$$

$$\Gamma, x : \tau' \vdash e' : \tau''$$

Let e'' BE SOME EXPR
 $R_{\tau'}(e'')$.

- $\vdash e'' : \tau'$
- e'' HALTS

$$\exists v'' \quad e'' \rightarrow^* v''$$

so $R_{\tau'}(v'')$

By IHOP, \leftarrow to e'

$$R_{\tau''}(e' \{v_1 / x_1\} \dots \{v_k / x_k\})$$

$$R_{\tau''}(e \ e'')$$

$$R_{\tau' \rightarrow \tau''}(e \ \{ \} \dots \{ \})$$

$e_1 \ e_2$