$$(\lambda x:\tau_1\to\tau_2.\ x\ \ x)\ (\lambda x:\tau_3\to\tau_4.\ x\ \ x)$$

τ $\quad$ τ $\quad$ $\tau_1\to\tau_2$

$\tau_1 \to \tau_2$

$$\tau_1 = \tau_1 \to \tau_2$$

"PROOF"

INDUCT ON e.

$e_1\ e_2$ ASSUMING $\vdash e:\tau$.

$$\vdash e_1 : \tau' \to \tau$$
$$\vdash e_2 : \tau'$$

BY IHOP:

SAFETY:

$$e_1 \to^* v_1 \qquad \vdash v_1 : \tau' \to \tau$$
$$e_2 \to^* v_2 \leftarrow \tau'$$

BY CANONICAL FORMS:

$$v_1 = \lambda x:\tau'.\ e'$$

WTS

τ

$$e \to^* v_1\ v_2 \xrightarrow{\beta} e'\{v_2/x\} \to^* v_3$$

EVIL IHOP

PROOF!

INDUCT ON e.

CASE x $\quad \Gamma \vdash x : \tau$

INVERSION $\Rightarrow \Gamma(x) = \tau$.

$x = x_i$ (FOR SOME i)

$\tau = \tau_i$

$$e\{v_1/x_1\} \cdots \{v_k/x_k\} = v_i$$

$\{v_i/x_i\}$

$$R_{\tau_i}(v_i)$$

$$R_{\tau_i}(\quad)$$, AS DESIRED.

CASE $()$

$\tau = \text{unit}$

$$e\{\quad\}\{\quad\}\cdots\{\quad\} = ()$$

$$R_{\text{unit}}(())$$

CASE $e = \lambda x : \tau' . e'$

BY INVERSION,

$$\tau = \tau' \to \tau''$$

$$\Gamma, x : \tau' \vdash e' : \tau''$$

LET e'' BE SOME EXPR

$R_{\tau'}(e'')$.

- $\vdash e'' : \tau'$
- e'' HALTS

$$\exists v'' \quad e'' \to^* v''$$

SO $R_{\tau'}(v'')$

<u>BY IHOP</u>, $\leftarrow$ TO e'

$$R_{\tau''}(e'\{v_1/x_1\} \ldots \{v_k/x_k\})$$

$$R_{\tau''}(e \ e'')$$

$$R_{\tau' \to \tau''}(e\{\quad\} \ldots \{\quad\})$$

<u>$e_1 \ e_2$</u>