PRESERVATION

$\vdash e : \tau$ AND $e \to e'$ $\Rightarrow$ $\vdash e' : \tau$

INDUCT ON $e \to e'$.

$P(e \to e') \triangleq \forall \tau.$ IF $\vdash e : \tau$ THEN $\vdash e' : \tau$

ADD $\dfrac{n = n_1 + n_2}{n_1 + n_2 \to n}$

HERE, $e = n_1 + n_2$ AND $e' = n$.

(WHERE $n = n_1 + n_2$)

$$\dfrac{\dfrac{}{\vdash n_1 : \text{int}} \quad \dfrac{}{\vdash n_2 : \text{int}}}{\vdash n_1 + n_2 : \underbrace{\text{int}}_{\tau_1}} \text{T-ADD} \qquad \dfrac{}{\vdash n : \underbrace{\text{int}}_{\tau_2}} \text{INT}$$

So $\tau_1 = \tau_2$, AS DESIRED.

β

$$(\lambda x : \tau'. e_1)\, v \to e_1\{v/x\}$$

HERE, $e =$ $(\lambda x : \tau'. e_1)\, v$ AND $e' =$ $e_1\{v/x\}$

BY ASSUMPTION, $\vdash e : \tau$

BY INVERSION:

$$\dfrac{\dfrac{\dfrac{x : \tau' \vdash e_1 : \tau}{\vdash \lambda x : \tau'. e_1 : \tau' \to \tau}\ \text{T-ABS} \qquad \vdash v : \tau'}{\vdash (\lambda x : \tau'. e_1)\, v : \tau}\ \text{T-APP}$$

SO: $x : \tau' \vdash e_1 : \tau$ AND $\vdash v : \tau'$ } $\Rightarrow$ "SUBSTITUTION LEMMA"

WTS $\vdash e_1\{v/x\} : \tau$ }

$$\text{CONTEXT}\quad \frac{e_0 \to e_0'}{E[e_0] \to E[e_0']}$$

By IHOP: $\vdash e_0 : \tau' \Rightarrow \vdash e_0' : \tau'$.

GIVEN $\vdash E[e_0] : \tau$.

WTS $\vdash E[e_0'] : \tau$.

"CONTEXT LEMMA"

□

PROGRESS

$\vdash e : \tau \Rightarrow e$ IS A VALUE OR $e \to e'$

INDUCT ON THE TYPE JUDGMENT.

$$\text{T-VAR}\quad e = x \quad \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

VACUOUS.

T-UNIT, T-INT, T-ABS

e IS A VALUE.

$$\text{T-ADD}\quad \frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}}$$

IHOPs: e_1 IS A VALUE OR $e_1 \to e_1'$

e_2 ... OR $e_2 \to e_2'$

- e_1 IS NOT A VALUE

$$\frac{e_1 \to e_1'}{e_1 + e_2 \to e_1' + e_2}\ \text{CONTEXT}$$

- O.W. e_1 IS A VALUE
  IF e_2 IS NOT A VALUE:

$$\frac{e_2 \to e_2'}{e_1 + e_2 \to e_1 + e_2'}\ \text{CONTEXT}$$

- BOTH VALUES

"CANONICAL FORMS"

$e_1 = n_1 \qquad e_2 = n_2$

$$\frac{}{e_1 + e_2 \to n}\ \text{WHERE } n = n_1 + n_2$$

T-APP

$e = e_1\ e_2$

AND $\vdash e_1 : \tau' \to \tau$

$\vdash e_2 : \tau'$

WHOP(e_1) AND WHOP(e_2)

- e_1 IS NOT A VALUE

- O.W. e_1 IS A VALUE

  ASSUME e_2 IS NOT.

- e_1 AND e_2 ARE VALUES

$$e_1 = \lambda. \ldots\ldots \quad \text{C.F.}$$

$$\frac{}{e_1\ e_2 \to e'\{e_2/x\}}\ \beta$$