

$\beta$ -REDUCTION

$$\begin{array}{ccc} & \text{REFLEX} & \text{REFLEX} \\ & \downarrow & \downarrow \\ (\lambda x. x + x) & ((\lambda y. y) 5) & \\ \downarrow & & \downarrow \\ ((\lambda y. y) 5) + ((\lambda y. y) 5) & & (\lambda x. x + x) (5) \end{array}$$

$$(\lambda x. e_1) e_2 \stackrel{\text{B-EQUIVALENT}}{\longleftrightarrow} e_1 \{e_2/x\}$$

CALL-BY-VALUE

$$v ::= \lambda x. e$$

$$\boxed{e \rightarrow e'}$$

$$\frac{}{(\lambda x. e) v \rightarrow e \{v/x\}} \text{B}$$

$$\frac{e_1 \rightarrow e_1'}{e_1 e_2 \rightarrow e_1' e_2}$$

$$\frac{e \rightarrow e'}{v e \rightarrow v e'}$$

$$\begin{aligned} & ((\lambda x. \lambda y. y x) (5+2)) (\lambda x. x+1) \\ \rightarrow & ((\lambda x. \lambda y. y x) 7) (\lambda x. x+1) \\ \rightarrow & (\lambda y. y 7) (\lambda x. x+1) \\ \rightarrow & (\lambda x. x+1) 7 \\ \rightarrow & 7+1 \\ \rightarrow & 8 \end{aligned}$$

CALL-BY-NAME

APPLY FUNCTIONS A.S.A.P.

$$(\lambda x. e) e_2 \rightarrow e, \{e_1/x\} B$$

$$\frac{e_1 \rightarrow e_1'}{e_1 e_2 \rightarrow e_1' e_2}$$

$$\begin{aligned} & ((\lambda x. \lambda y. y x) (5+2)) (\lambda x. x+1) \\ \rightarrow & (\lambda y. y (5+2)) (\lambda x. x+1) \\ \rightarrow & (\lambda x. x+1) (5+2) \\ \rightarrow & (5+2) + 1 \\ \rightarrow & 7+1 \quad (\lambda x. x) (\lambda x. x) \\ \rightarrow & 8 \quad \lambda x. ((\lambda y. y) x) \end{aligned}$$