$$e ::= x \mid \lambda x.\, e \mid e_1\ e_2$$

PARAM, BODY, FUNC, ARG

x: VARIABLES

$\lambda x.\, e$: FUNCTIONS — ABSTRACTION

$e_1\ e_2$: FUNCTION CALLS — APPLICATION

APPLIED λ-CALCULUS:

$$\mid n \mid e_1^{\,e_2} \mid e_1 \div e_2 \mid s \mid e_1 \cdot e_2$$

λ- TERM

$\lambda x.\, x$

$(\lambda x.\, x)\ (\lambda y.\, y)$

$\rightarrow \lambda y.\, y$

$(\lambda x.\, x\ \ x)\ (\lambda y.\, y)$

$\rightarrow (\lambda y.\, y)\ (\lambda y.\, y)$

$\rightarrow \lambda y.\, y$

$(\lambda x.\ x\ x)\ (\lambda y.\ y\ y)$

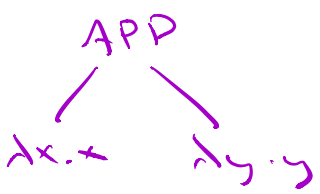
$\rightarrow (\lambda y.\ y\ y)\ (\lambda y.\ y\ y)$

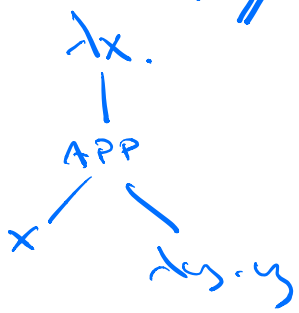
$\rightarrow (\lambda y.\ y\ y)\ (\lambda y.\ y\ y)$

all

ABOUT PARENTHESES

$(\lambda x.\ (x)\ ((\lambda y.\ y)))$





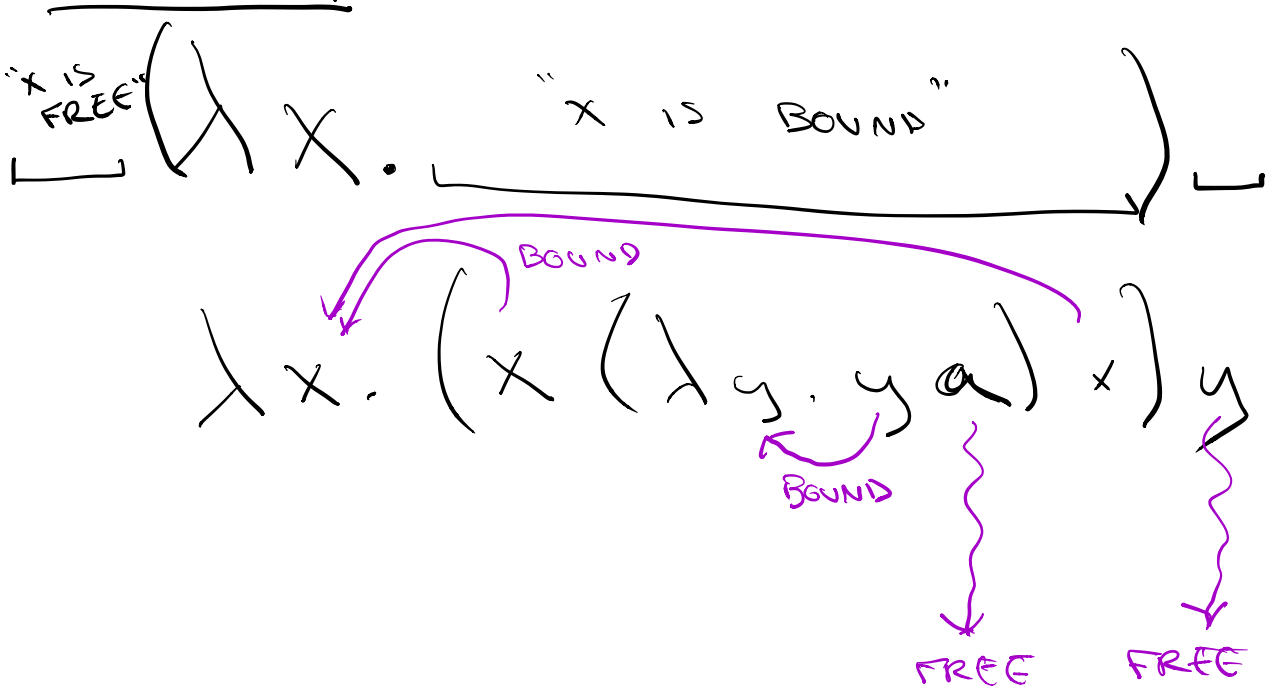
- λ "BODIES" ARE AS "BIG" AS POSSIBLE

$\lambda x.\ ((x\ \ y)\ \ 2)$

- APPLICATIONS ARE LEFT-ASSOC

$(((e_1\ \ e_2)\ \ e_3)\ \ e_4)\ e_5$

BINDING



CLOSED TERM : ALL BOUND VARS

OPEN TERM : AT LEAST ONE FREE

$\lambda y.\ (\lambda x.\ x)\ y$

$\lambda x.\ x \quad \lambda y.\ y$

α-EQUIVALENT

$=_\alpha$

$\lambda x.\ x + x$

$\lambda x.\ 2 * x$

$\lambda x.\, x =_\alpha \lambda y.\, y$ $\quad\quad$ $\lambda x.\, x \neq \lambda y.\, y$

α-RENAMING

$\lambda x.\, \lambda y.\, x$

$\rightsquigarrow \lambda z.\, \lambda y.\, z$

$\neq_\alpha \lambda y.\, \lambda y.\, y =_\alpha \lambda z.\, \lambda y.\, y$

$(\lambda f.\, f\ 42)\ (\lambda x.\, x \div 2)$

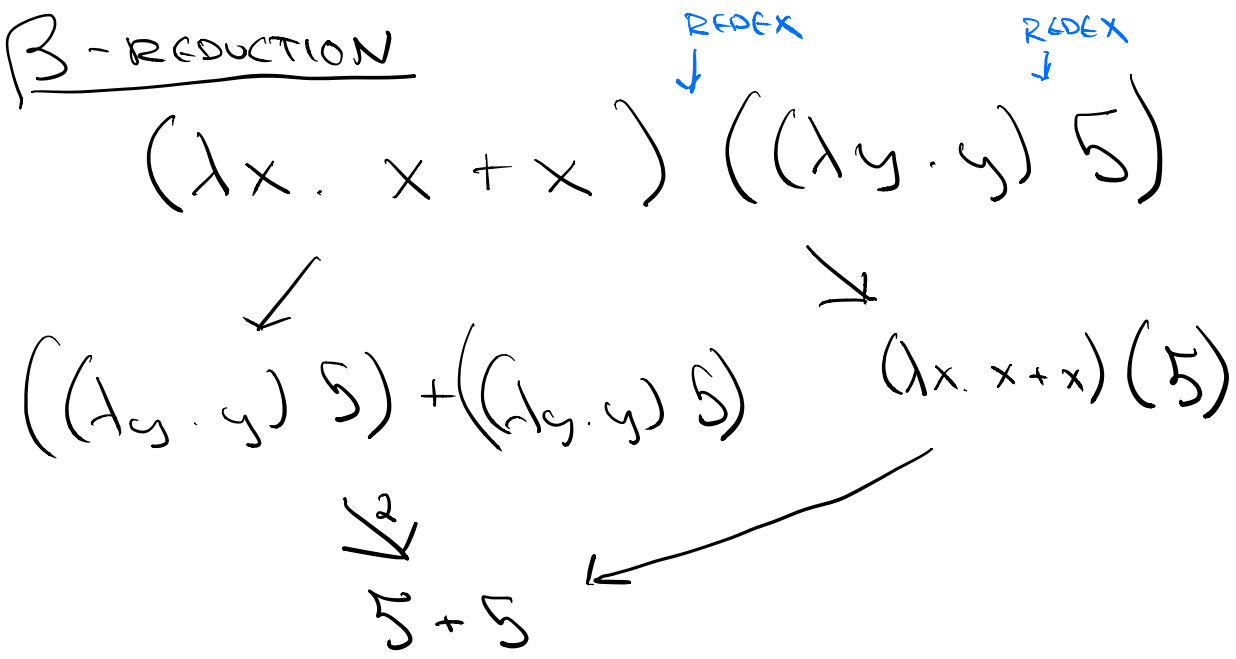
$(\lambda x.\, x\ y)\ (\quad)$

CONTAIN x's

$(\lambda x.\, e_1)\ e_2$ ← REDEX

β-EQUIVALENT

$\rightarrow e_1\{e_2/x\}$ ← SUBSTITUTE x FOR e_2 IN e_1


β-REDUCTION
REDEX
REDEX
(λx. x + x) ((λy. y) 5)
((λy. y) 5) + ((λy. y) 5)
(λx. x + x) (5)
2
5 + 5