

$$\begin{array}{c}
 \dots 1 \\
 \hline
 \vdash \{P\} \ c_1 \ \{R\} \\
 \dots 2 \\
 \hline
 \vdash \{R\} \ c_2 \ \{Q\} \\
 \text{SEQ} \\
 \hline
 \vdash \{P\} \ C \ \{Q\}
 \end{array}$$

$\textcircled{R=?}$

(2)

$$\begin{array}{c}
 \dots \\
 \hline
 \vdash \{I \wedge x > 0\} \ y := y * x ; \ x := x - 1 \ \{I\} \\
 \hline
 \vdash \{I\} \ c_2 \ \{I \wedge \neg(x > 0)\} \\
 \text{WHILE}
 \end{array}$$

$\textcircled{I=?}$

A blue arrow labeled 'R' points to the invariant $\{I\}$ in the while rule.

$$y = 1 \times n \times (n-1) \times \dots \times (x+1)$$

$$y = n! / x! \quad \wedge \quad x \geq 0$$

$$I \triangleq x! \cdot y = n! \quad \wedge \quad x \geq 0$$

①

$$\frac{}{\vdash \{ \} y := 1 \{ I \}} \text{ASSGN}$$

$$I[1/y] = x! \cdot 1 = n! \wedge x \geq 0$$

$$\vdash P \Rightarrow I[1/y]$$

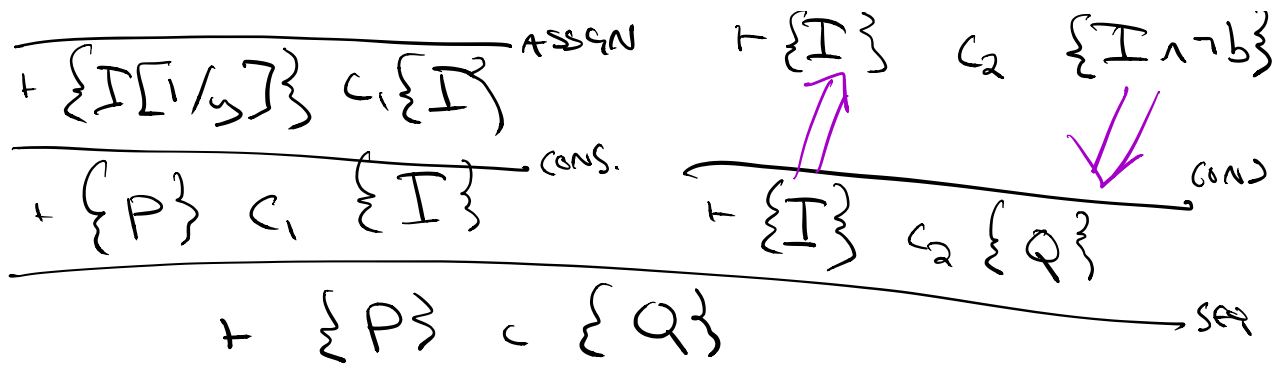
$$\vdash I \Rightarrow I \text{ CONSEQ}$$

$$\vdash \{ P \} y := 1 \{ I \}$$

$$x = n \wedge n > 0 \Rightarrow x! \cdot 1 = n! \wedge x \geq 0$$

RECAP

$$\frac{\frac{}{\vdash \{ I \wedge b \} \{ I \}}}{\vdash \{ I \wedge b \} \{ I \}} \text{WHILE}$$



$$I \wedge \neg b \Rightarrow Q$$

$$x!y = n! \wedge x \geq 0 \wedge \neg(x > 0) \Rightarrow y = n!$$

~~$$\vdash \{I \wedge b\} \quad y := y * x; \quad x := x - 1; \quad \{I\}$$~~

$$\begin{aligned} & \{P\} \\ \Rightarrow & \{x!y = n! \wedge x \geq 0\} \quad I = \dots \\ & y := 1 \\ & \{x!y = n! \wedge x \geq 0\} \\ & \text{while } x > 0 \text{ do } (\\ & \quad \{I \wedge x > 0\} \end{aligned}$$

$$\Rightarrow \{ (x-1)! (y+x)^{n!} \dots \}$$

$$y := y * x$$

$$\{ (x-1)! y^{n!} \dots \}$$

$$x := x - 1$$

$$\{ x! y^n \dots \}$$

)

$$\{ \text{I} \wedge \neg (x > 0) \}$$

$$\Rightarrow \{ y = n! \}$$