TH $\forall b, c.$

$$\texttt{while } b \texttt{ do } c \sim \texttt{if } b \texttt{ then } (c; \texttt{ while } b \texttt{ do } c) \texttt{ else skip}$$

PF.

WTS: $\forall \sigma, \sigma',$

$$\langle \sigma, \texttt{while } b \texttt{ do } c \rangle \Downarrow \sigma' \iff \langle \sigma, \texttt{if } \ldots \rangle \Downarrow \sigma'$$

SHOW $\Rightarrow$ AND $\Leftarrow$.

FIRST, $\Rightarrow$:

ASSUME

AND PROVE.

INDUCT ON THE DERIVATION OF:

$$\langle \sigma, \texttt{while } \ldots \rangle \Downarrow \sigma'$$

FOR $D \Vdash \langle \sigma, \hat{c} \rangle \Downarrow \sigma'$,

$P(D) = \hat{c} = \text{while } b \text{ do } c$
$\Rightarrow \langle \sigma, \text{if} \ldots \rangle \Downarrow \sigma'$

CASES

SKIP, ASSGN, SEQ, IFT, IFF

VACUOUS. $\hat{c} \neq \text{while } b \text{ do } c$

WHILE-F

THE DERIVATION IS:

$$\dfrac{\dfrac{\vdots}{\langle \sigma, b \rangle \Downarrow \text{false}}}{\langle \sigma, \text{while } b \text{ do } c \rangle \Downarrow \sigma} \text{ WHILE-F}$$

WE HAVE:

- A DERIVATION $\langle \sigma, b \rangle \Downarrow \text{false}$.
- $\sigma = \sigma'$

DERIVE:

$$\dfrac{\dfrac{\vdots}{\langle \sigma, b\rangle \Downarrow \text{false}} \qquad \dfrac{}{\langle \sigma, \text{skip}\rangle \Downarrow \sigma}\,\text{SKIP}}{\langle \sigma, \text{if } b \text{ then } (c;\ \text{while } b \text{ do } c) \text{ else skip}\rangle \Downarrow \sigma}\,\text{IF-F}$$

WHILE-T

We have:

- $\langle \sigma, b\rangle \Downarrow \text{true}$
- $\langle \sigma, c\rangle \Downarrow \sigma''$ CAREFUL!
- $\langle \sigma'', \text{while } b \text{ do } c\rangle \Downarrow \sigma'$

Derive:

$$\dfrac{\dfrac{\vdots}{\langle \sigma, b\rangle \Downarrow \text{true}} \qquad \dfrac{\dfrac{\vdots}{\langle \sigma, c\rangle \Downarrow \sigma''} \qquad \dfrac{\vdots}{\langle \sigma'', \text{while } b \text{ do } c\rangle \Downarrow \sigma'}}{\langle \sigma, c;\ \text{while } b \text{ do } c\rangle \Downarrow \sigma'}}{\langle \sigma, \text{if } \cdots \rangle \Downarrow \sigma'}\,\text{IF-T}$$

$\square$

Th Determinism.

Pf Induct on derivation of

$\langle \sigma, c \rangle \Downarrow \sigma'$

to show

$\underbrace{\langle \sigma, c \rangle \Downarrow \sigma'' \Rightarrow \sigma' = \sigma''}_{P(\langle \sigma, c \rangle \Downarrow \sigma')}.$

Cases:

- $$\frac{}{\langle \sigma, \text{skip} \rangle \Downarrow \sigma} \text{ SKIP}$$

  Here, $c = \text{skip}$ and $\sigma = \sigma'$.

  By inversion,

  $$\frac{}{\langle \sigma, \text{skip} \rangle \Downarrow \sigma''} \text{ SKIP}$$

  so $\sigma'' = \sigma$.

  So $\sigma'' = \sigma'$ by transitivity.

- ASSGN

- $$\frac{\begin{array}{c}\vdots \\ \langle \sigma, c_1 \rangle \Downarrow \hat{\sigma}\end{array} \quad \begin{array}{c}\vdots \\ \langle \hat{\sigma}, c_2 \rangle \Downarrow \sigma'\end{array}}{\langle \sigma, c_1; c_2 \rangle \Downarrow \sigma'} \text{ SEQ}$$

HERE, $c = c_1 ; c_2$.

BY INVERSION:

$$\dfrac{\dfrac{\vdots}{\langle \sigma, c_1 \rangle \Downarrow \hat{\hat{\sigma}}} \quad \dfrac{\vdots}{\langle \hat{\hat{\sigma}}, c_2 \rangle \Downarrow \sigma''}}{\langle \sigma, c_1 ; c_2 \rangle \Downarrow \sigma''} \text{SEQ}_1$$

BY IHOP ON 1^{ST} SUBDERIVATION $\hat{\sigma} = \hat{\hat{\sigma}}$. THIS SATISFIES THE PREMISE OF THE IHOP ON 2^{ND} SUBDERIVATION, SO $\sigma' = \sigma''$.

WHILE-T

$$\dfrac{\dfrac{\vdots}{\langle \sigma, b \rangle \Downarrow \text{true}} \quad \dfrac{\vdots}{\langle \sigma, \hat{c} \rangle \Downarrow \hat{\sigma}} \quad \dfrac{\vdots}{\langle \hat{\sigma}, \text{while} \ldots \rangle \Downarrow \sigma'}}{\langle \sigma, \text{while } b \text{ do } \hat{c} \rangle \Downarrow \sigma'} \text{WHILE-T}$$

HERE, $c = \text{while } b \text{ do } \hat{c}$.

BY INVERSION: (AND DETERMINISM OF $\Downarrow_{bool}$)

$$\dfrac{\dfrac{\vdots}{\langle \sigma, b \rangle \Downarrow \text{true}} \quad \dfrac{\vdots}{\langle \sigma, \hat{c} \rangle \Downarrow \hat{\hat{\sigma}}} \quad \dfrac{\vdots}{\langle \hat{\hat{\sigma}}, \text{while} \ldots \rangle \Downarrow \sigma''}}{\langle \sigma, \text{while } b \text{ do } \hat{c} \rangle \Downarrow \sigma''} \text{WHILE-T}$$

BY IHOP 1: $\hat{\sigma} = \hat{\hat{\sigma}}$.

BY THAT AND LHOP 2: $\sigma'' = \sigma'$.