Th $\forall b, c.$

while b do c

$\sim$ if b then
(c; while b do c)
else skip

Pf.

WTS: $\forall \sigma, \sigma'$,

$\langle \sigma,$ while b do c $\rangle \Downarrow \sigma'$

$\iff$

$\langle \sigma,$ if ... $\rangle \Downarrow \sigma'$

SHOW $\Rightarrow$ AND $\Leftarrow$.

FIRST, $\Rightarrow$:

ASSUME

AND PROVE

INDUCT ON THE DERIVATION OF:

$\langle \sigma,$ while ... $\rangle \Downarrow \sigma'$

For $D \Vdash \langle \sigma, \hat{c} \rangle \Downarrow \sigma'$,

$P(D) = \hat{c} = \text{while } b \text{ do } c$

$\Rightarrow \langle \sigma, \text{if } \ldots \rangle \Downarrow \sigma'$

Cases

SKIP, ASSN, SEQ, IFT, IFF

Vacuous. $\hat{c} \neq \text{while } b \text{ do } c$