CS 4110

Programming Languages & Logics

Lecture 23
Polymorphism

27 October 2014

Announcements

- Today: Foster office hours 4-5pm
- Wednesday: Mota guest lecture

Roadmap

Over the last few lectures, we've developed a simple type system for λ-calculus, extensions for handling a number of language features, and we proved normalization.

Today we'll develop a substantial extension of the simply-typed λ-calculus by making the type system polymorphic.

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- *Subtype polymorphism* allows values of type τ to masquerade as values of type τ', provided that τ is a subtype of τ'.
- *Ad-hoc polymorphism*, also called overloading, allows the same function name to be used with functions that take different types of parameters.
- *Parametric polymorphism* refers to code that is written without knowledge of the actual type of the arguments; the code is parametric in the type of the parameters.

Example

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$$\text{doubleInt} \triangleq \lambda f\!:\!\mathbf{int} \to \mathbf{int}.\ \lambda x\!:\!\mathbf{int}.\ f\,(f\,x)$$

Now suppose we want the same function for booleans, or functions...

$$\begin{aligned}\text{doubleBool} &\triangleq \lambda f\!:\!\mathbf{bool} \to \mathbf{bool}.\ \lambda x\!:\!\mathbf{bool}.\ f\,(f\,x)\\ \text{doubleFn} &\triangleq \lambda f\!:\!(\mathbf{int} \to \mathbf{int}) \to (\mathbf{int} \to \mathbf{int}).\ \lambda x\!:\!\mathbf{int} \to \mathbf{int}.\ f\,(f\,x)\\ &\vdots\end{aligned}$$

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Every major piece of functionality in a program should be implemented in just one place in the code. When similar functionality is provided by distinct pieces of code, the two should be combined into one by abstracting out the varying parts.

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In the doubling functions, the varying parts are the types.

We need a way to abstract out the type of the doubling operation, and later instantiate it with different concrete types.

Polymorphic λ-Calculus

Invented indepedently in 1972-1974 by a computer scientist John Reynolds and a logician Jean-Yves Girard (who called it System F).

Commonly used as a basis for studying type system extensions

Key feature: function abstraction and application at the type level!

Notation:

- $\Lambda X.\, e$: type abstraction
- $e[\tau]$: type application

Example:
$\lambda X.\, \lambda x : X.\, x$

Polymorphic λ-Calculus

Syntax

$$e ::= n \mid x \mid \lambda x\!:\!\tau.\, e \mid e_1\, e_2 \mid \Lambda X.\, e \mid e\,[\tau]$$
$$v ::= n \mid \lambda x\!:\!\tau.\, e \mid \Lambda X.\, e$$

Polymorphic λ-Calculus

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$$e ::= n \mid x \mid \lambda x\!:\!\tau.\, e \mid e_1\; e_2 \mid \Lambda X.\, e \mid e\,[\tau]$$
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Dynamic Semantics

$$E ::= [\cdot] \mid E\, e \mid v\, E \mid E\,[\tau]$$

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$$\frac{e \to e'}{E[e] \to E[e']} \qquad \frac{}{(\lambda x\!:\!\tau.\, e)\; v \to e\{v/x\}}$$

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Typing Judgment

Type Syntax

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Typing Judgment: $\Delta, \Gamma \vdash e : \tau$

- Γ a mapping from variables to types
- Δ a set of types in scope
- e an expression
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$$\frac{\Delta \cup \{X\}, \Gamma \vdash e : \tau}{\Delta, \Gamma \vdash \Lambda X.\, e : \forall X.\, \tau}$$

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In the polymorphic λ-calculus, however, we can type this expression using a polymorphic type:

$$\vdash \quad \lambda x : \forall X.\, X \to X.\, x\, [\forall X.\, X \to X]\, x \, : \, (\forall X.\, X \to X) \to (\forall X.\, X \to X)$$

However, all expressions in polymorphic λ-calculus still halt

Example: Products

We can encode products in polymorphic λ-calculus without adding any additional types!

The encodings are based on the (untyped) Church encodings:

$$\tau_1 \times \tau_2 \triangleq \forall R.\, (\tau_1 \to \tau_2 \to R) \to R$$

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Similarly, we can encode sums in polymorphic λ-calculus without adding any additional types!

Again, the encodings are based on the (untyped) Church encodings:

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$$
\mathrm{void} \triangleq \forall R.\, R
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The following theorem states this translation is adequate:

Theorem (Adequacy)

For all expressions e and e', we have $e \rightarrow e'$ iff $\mathit{erase}(e) \rightarrow \mathit{erase}(e')$.

Type Inference

The type inference (or "type reconstruction") problem asks whether, for a given untyped λ-calculus expression e' there exists a well-typed System F expression e such that $erase(e) = e'$

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See Chapter 23 of Pierce for further discussion, as well as restrictions for which type reconstruction is decidable.