

State and Finite State Machines

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CS 3410, Spring 2014

Computer Science

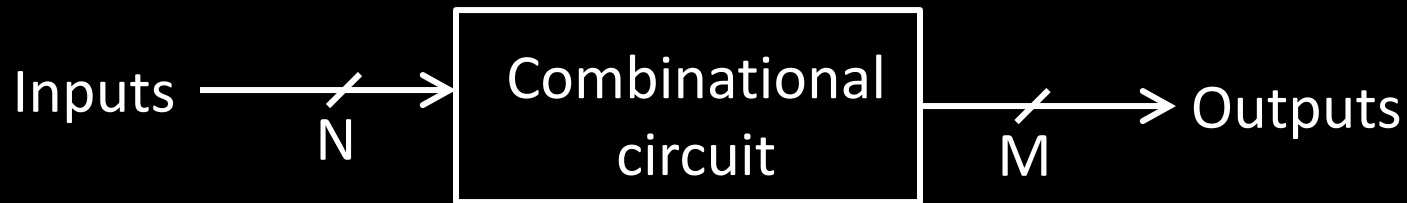
Cornell University

See P&H Appendix B.7, B.8, B.10, B.11

Stateful Components

Until now is combinatorial logic

- Output is computed when inputs are present
- System has no internal state
- Nothing computed in the present can depend on what happened in the past!



Need a way to record data

Need a way to build **stateful** circuits

Need a state-holding device

Finite State Machines

Goals for Today

State

- How do we store *one* bit?
- Attempts at storing (and changing) one bit
 - Set-Reset Latch
 - D Latch
 - D Flip-Flops
 - Master-Slave Flip-Flops
- Register: storing more than one bit, N-bits

Basic Building Blocks

- Decoders and Encoders

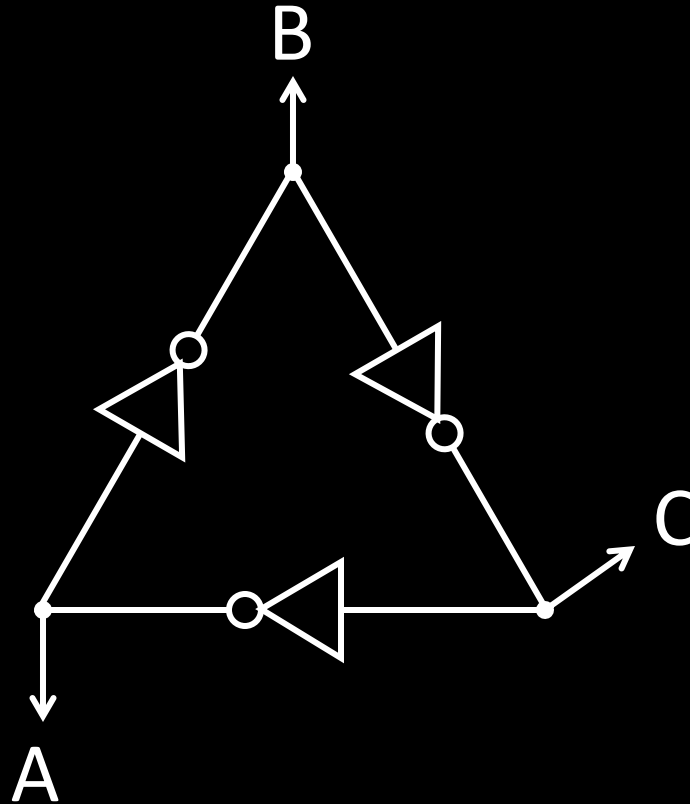
Finite State Machines (FSM)

- How do we design logic circuits with state?
- Types of FSMs: Mealy and Moore Machines
- Examples: Serial Adder and a Digital Door Lock

Goal

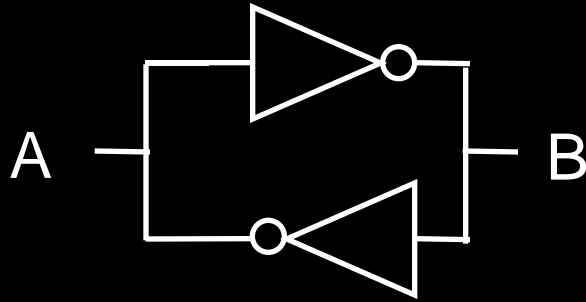
How do we store store *one* bit?

First Attempt: Unstable Devices



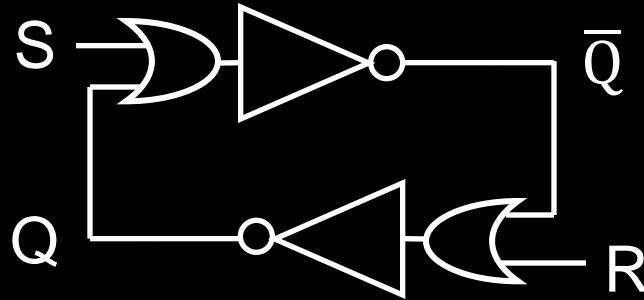
Second Attempt: Bistable Devices

- Stable and unstable equilibria?



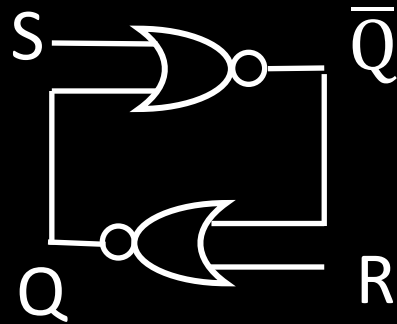
A Simple Device

Third Attempt: Set-Reset Latch



Can you store a value (with this circuit)?
Can you change its value?

Third Attempt: Set-Reset Latch



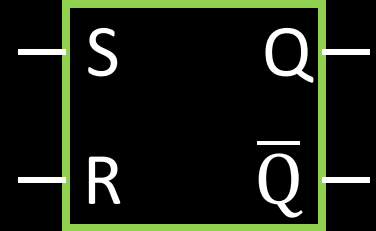
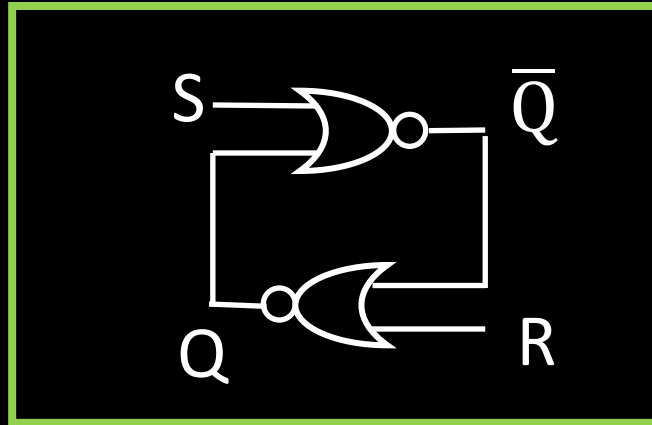
S	R	Q	\bar{Q}
0	0		
0	1		
1	0		
1	1		

Set-Reset (S-R) Latch

Stores a value Q and its complement

A	B	OR	NOR
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

Third Attempt: Set-Reset Latch



S	R	Q	\bar{Q}
0	0		
0	1		
1	0		
1	1		

Set-Reset (S-R) Latch

Stores a value Q and its complement

A	B	OR	NOR
0	0	0	1
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1	0	1	0
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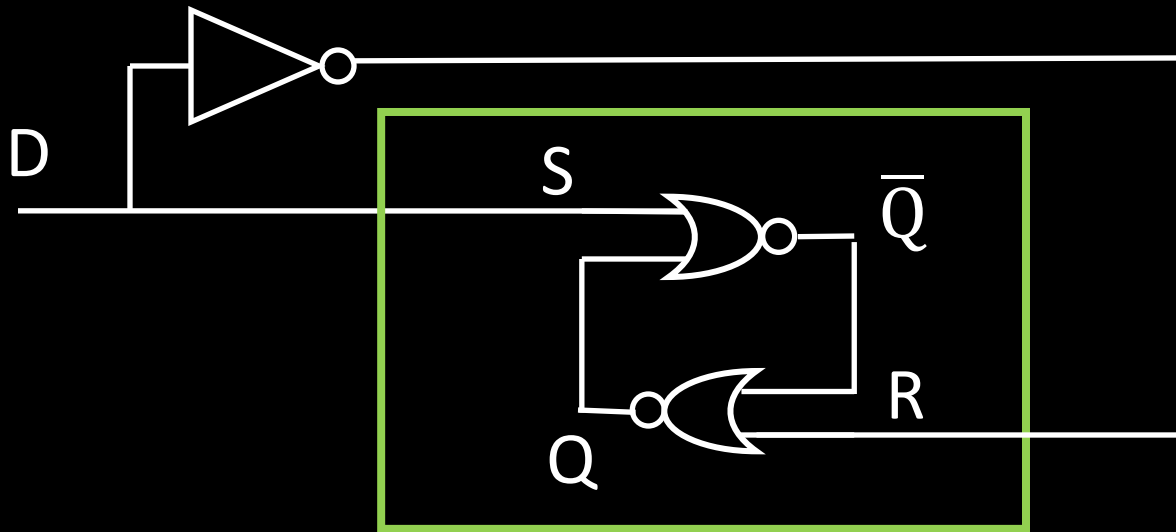
Takeaway

Set-Reset (SR) Latch can store one bit and we can change the value of the stored bit. But, SR Latch has a forbidden state.

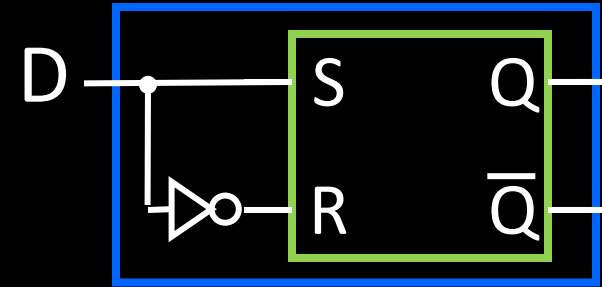
Next Goal

How do we avoid the forbidden state of S-R Latch?

Fourth Attempt: (Unclocked) D Latch



Fill in the truth table?



D	Q	\bar{Q}
0		
1		

A	B	OR	NOR
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

Takeaway

Set-Reset (SR) Latch can store one bit and we can change the value of the stored bit. But, SR Latch has a forbidden state.

(Unclocked) D Latch can store and change a bit like an SR Latch while avoiding the forbidden state.

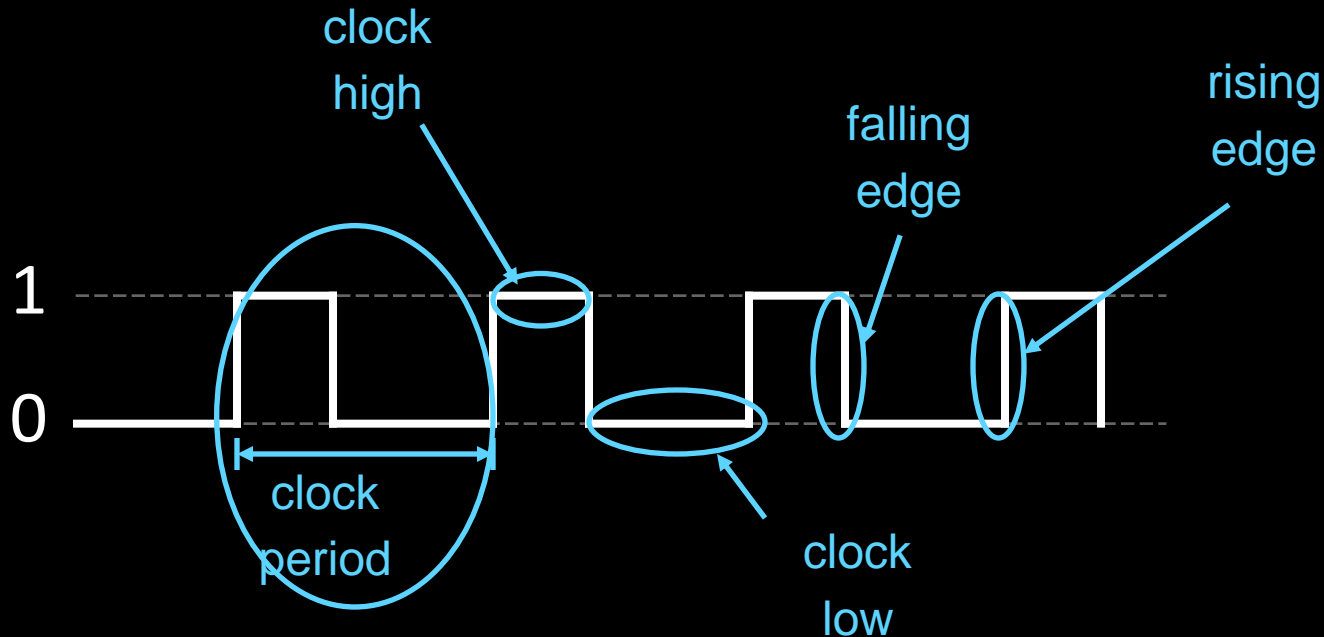
Next Goal

How do we coordinate state changes to a D Latch?

Clocks

Clock helps coordinate state changes

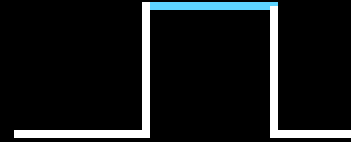
- Usually generated by an oscillating crystal
- Fixed period; frequency = $1/\text{period}$



Clock Disciplines

Level sensitive

- State changes when clock is high (or low)



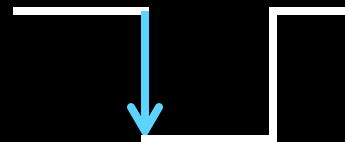
Edge triggered

- State changes at clock edge

positive edge-triggered



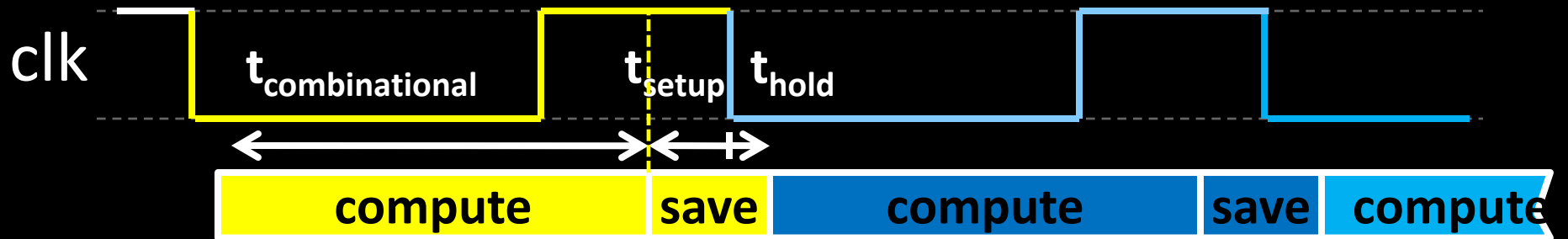
negative edge-triggered



Clock Methodology

Clock Methodology

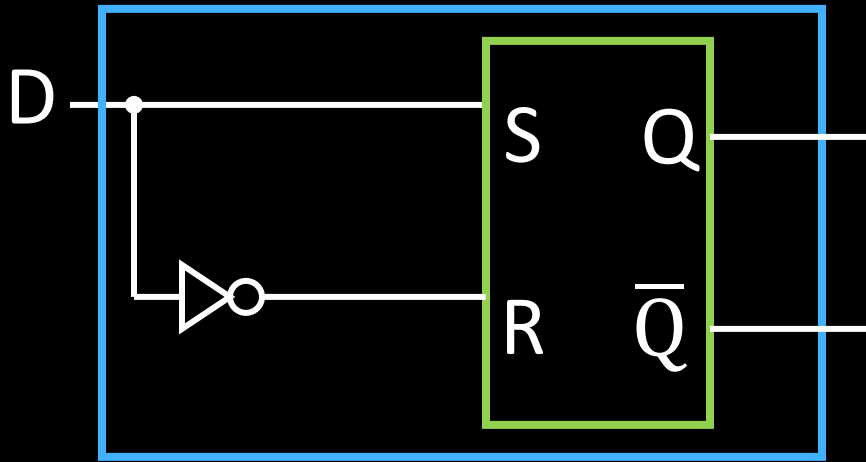
- Negative edge, synchronous



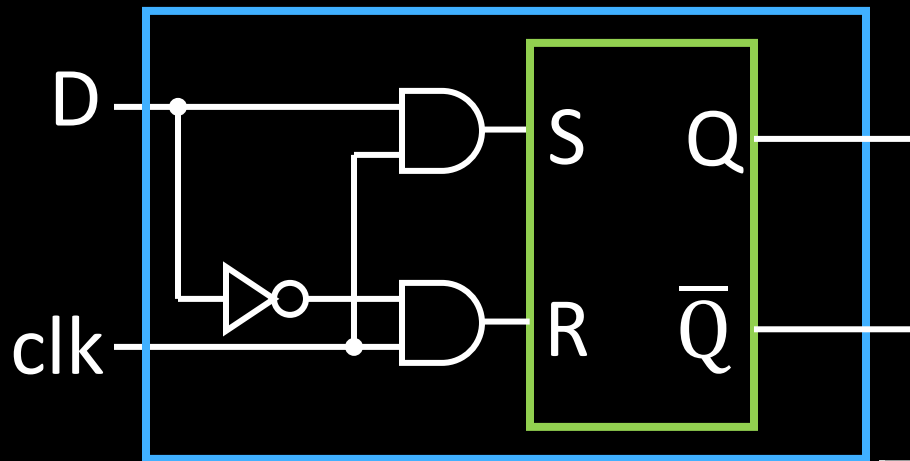
- Edge-Triggered: Signals must be stable near falling clock edge

- Positive edge synchronous

Fifth Attempt: D Latch with Clock



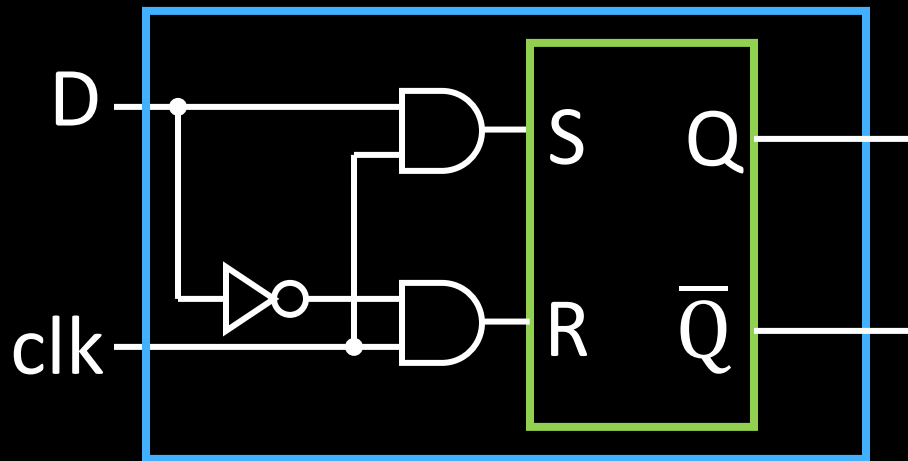
Fifth Attempt: D Latch with Clock



Fill in the truth table

clk	D	Q	\bar{Q}
0	0		
0	1		
1	0		
1	1		

Fifth Attempt: D Latch with Clock

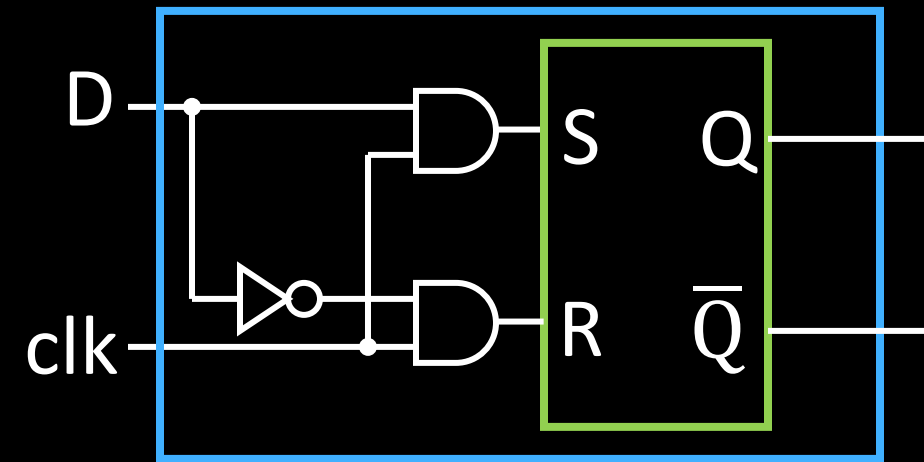


Fill in the truth table

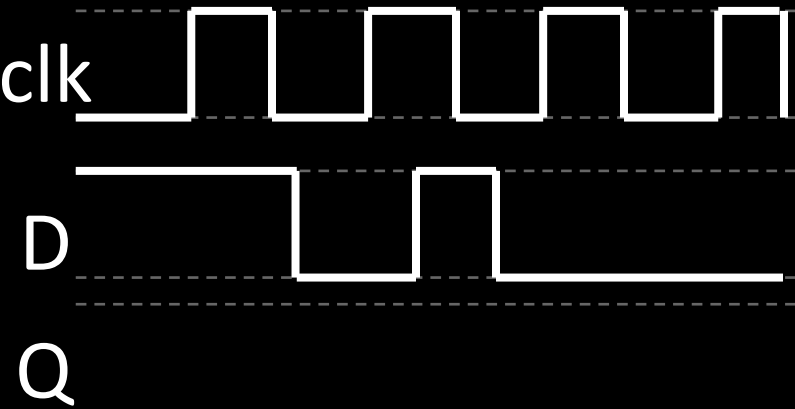
S	R	Q	\bar{Q}	
0	0	Q	\bar{Q}	hold
0	1	0	1	reset
1	0	1	0	set
1	1	forbidden		

clk	D	Q	\bar{Q}
0	0		
0	1		
1	0		
1	1		

Fifth Attempt: D Latch with Clock

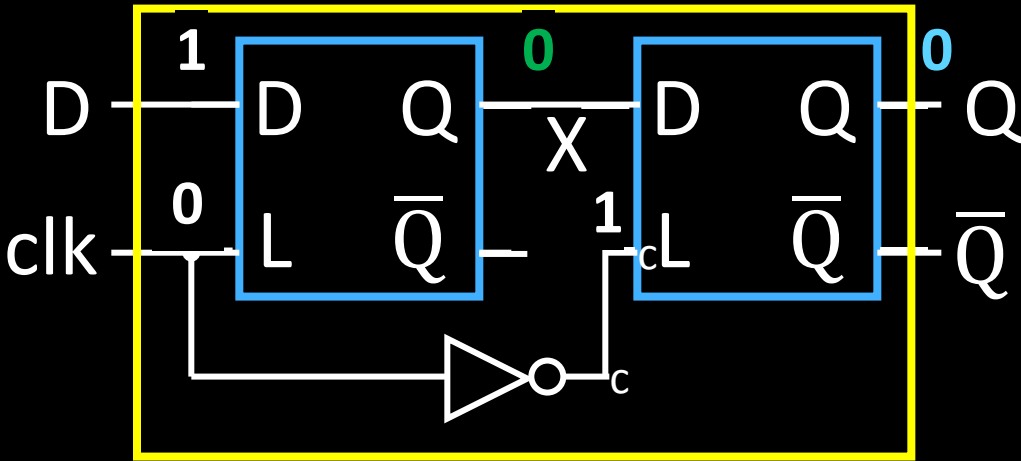


Fill in the truth table

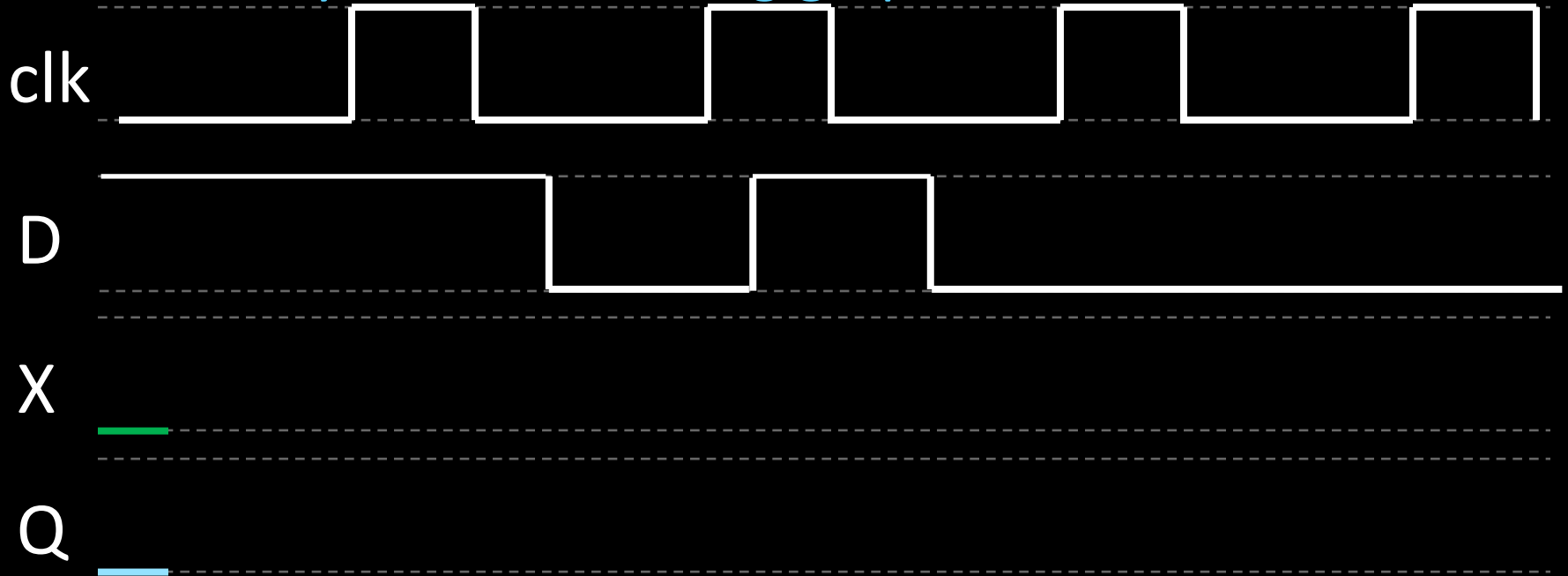


clk	D	Q	\bar{Q}
0	0		
0	1		
1	0		
1	1		

Sixth Attempt: Edge-Triggered D Flip-Flop



Activity#1: Fill in timing graph and values for X and Q



Takeaway

Set-Reset (SR) Latch can store one bit and we can change the value of the stored bit. But, SR Latch has a forbidden state.

(Unclocked) D Latch can store and change a bit like an SR Latch while avoiding a forbidden state.

An Edge-Triggered D Flip-Flop (aka Master-Slave D Flip-Flop) stores one bit. The bit can be changed in a synchronized fashion on the edge of a clock signal.

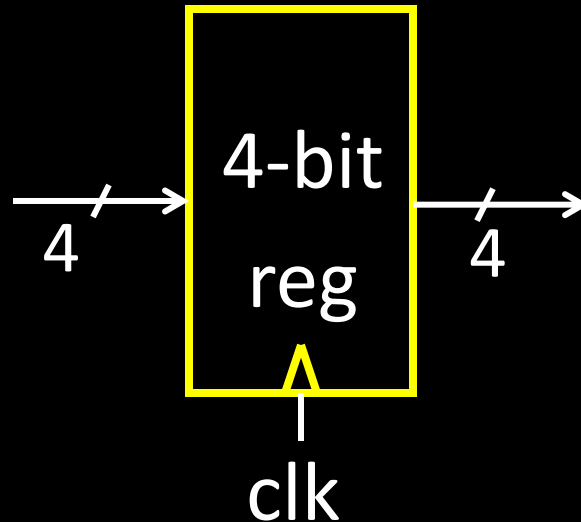
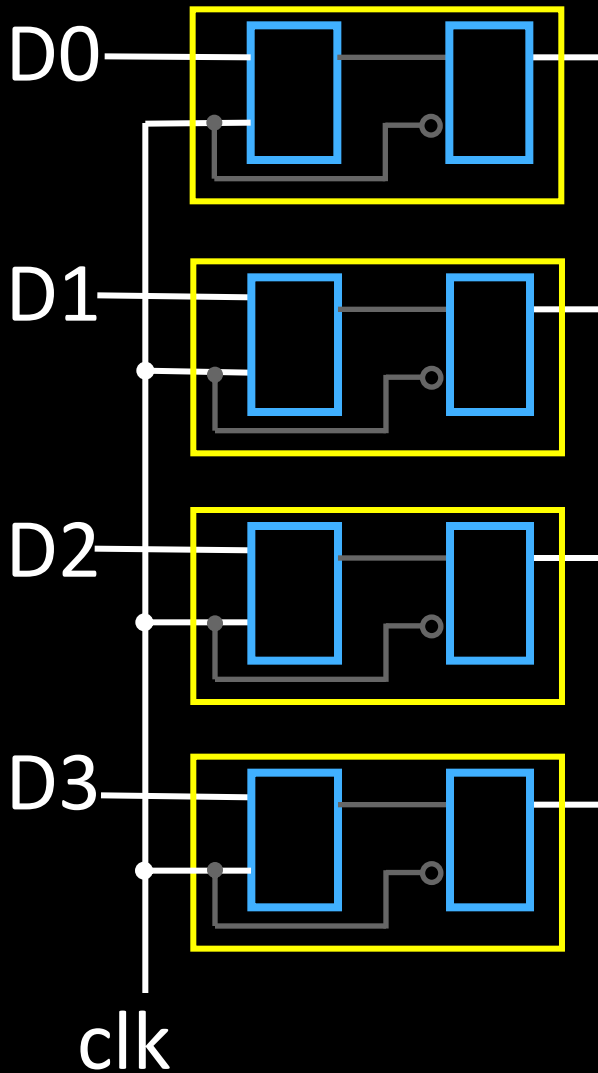
Next Goal

How do we store more than one bit, N bits?

Registers

Register

- D flip-flops in parallel
- shared clock
- extra clocked inputs: write_enable, reset, ...



Takeaway

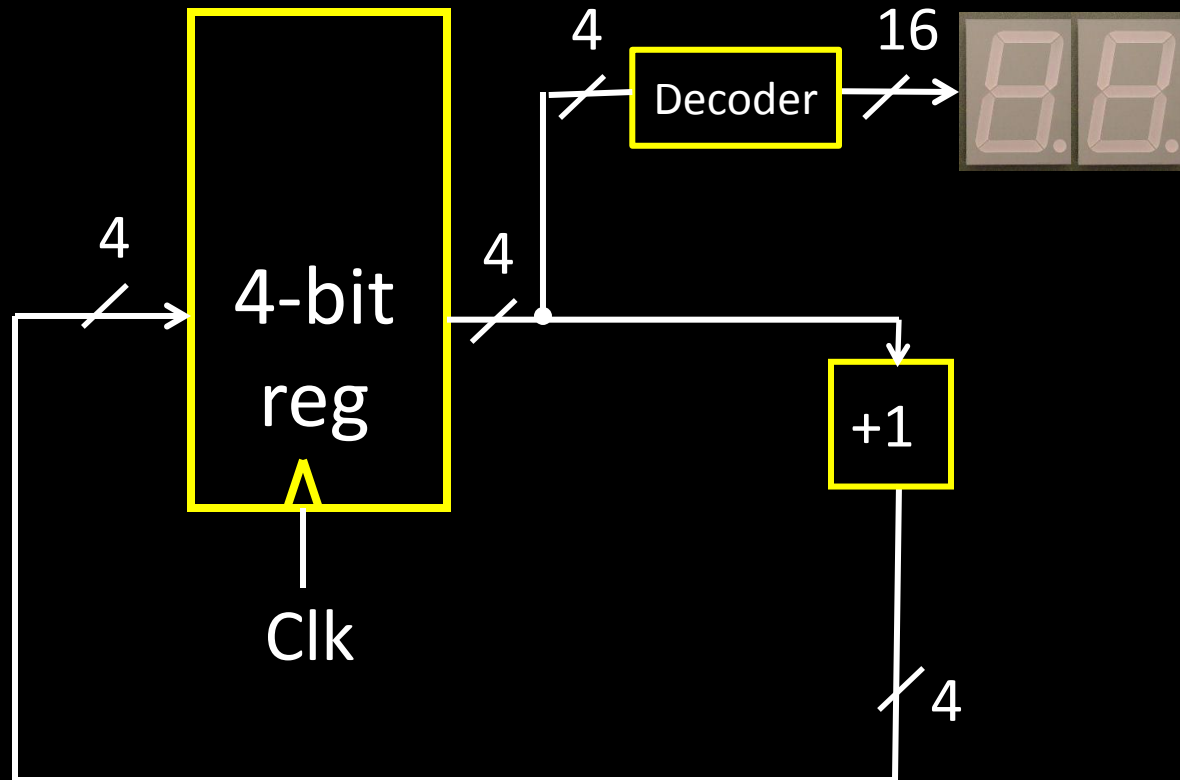
Set-Reset (SR) Latch can store one bit and we can change the value of the stored bit. But, SR Latch has a forbidden state.

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An Edge-Triggered D Flip-Flop (aka Master-Slave D Flip-Flop) stores one bit. The bit can be changed in a synchronized fashion on the edge of a clock signal.

An N -bit **register** stores N -bits. It is created with N D-Flip-Flops in parallel along with a shared clock.

An Example: What will this circuit do?



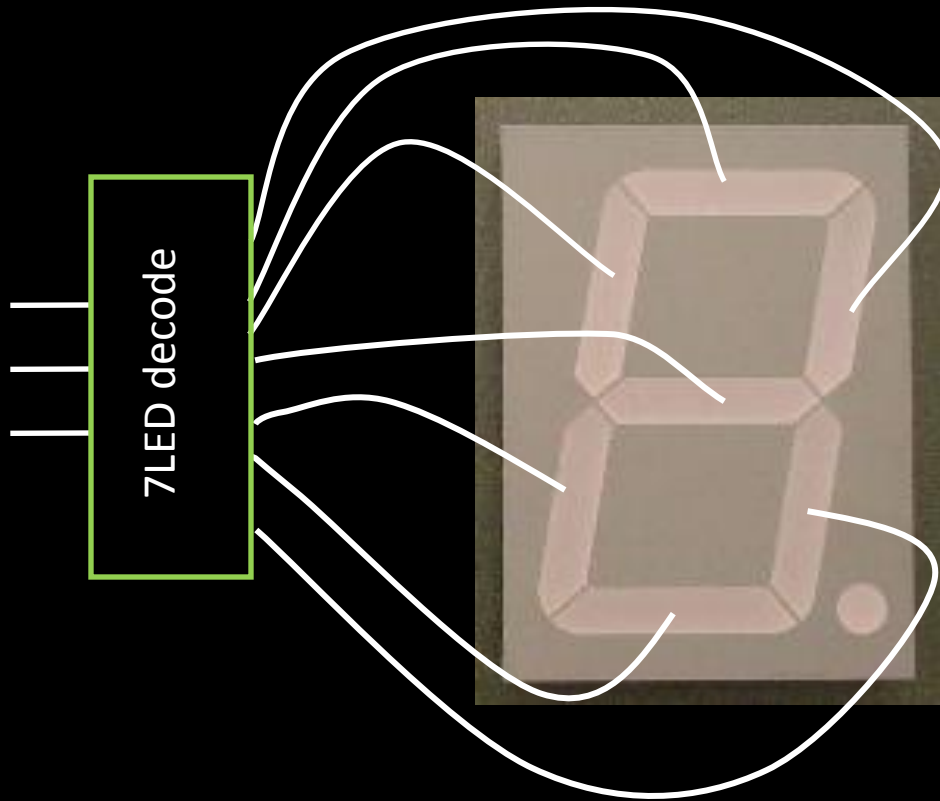
Decoder Example: 7-Segment LED

7-Segment LED

- photons emitted when electrons fall into holes



Decoder Example: 7-Segment LED Decoder



3 inputs

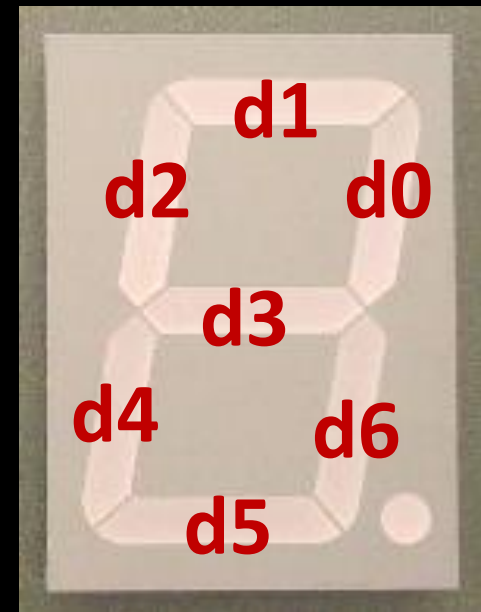
- encode 0 – 7 in binary

7 outputs

- one for each LED

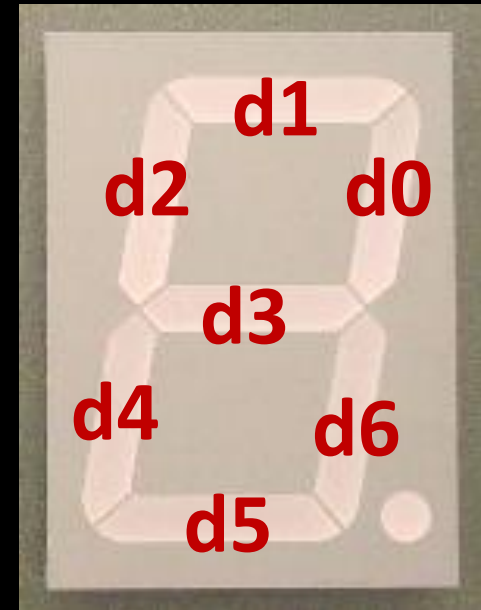
7 Segment LED Decoder Implementation

b2	b1	b0	d6	d5	d4	d3	d2	d1	d0
0	0	0							
0	0	1							
0	1	0							
0	1	1							
1	0	0							
1	0	1							
1	1	0							
1	1	1							

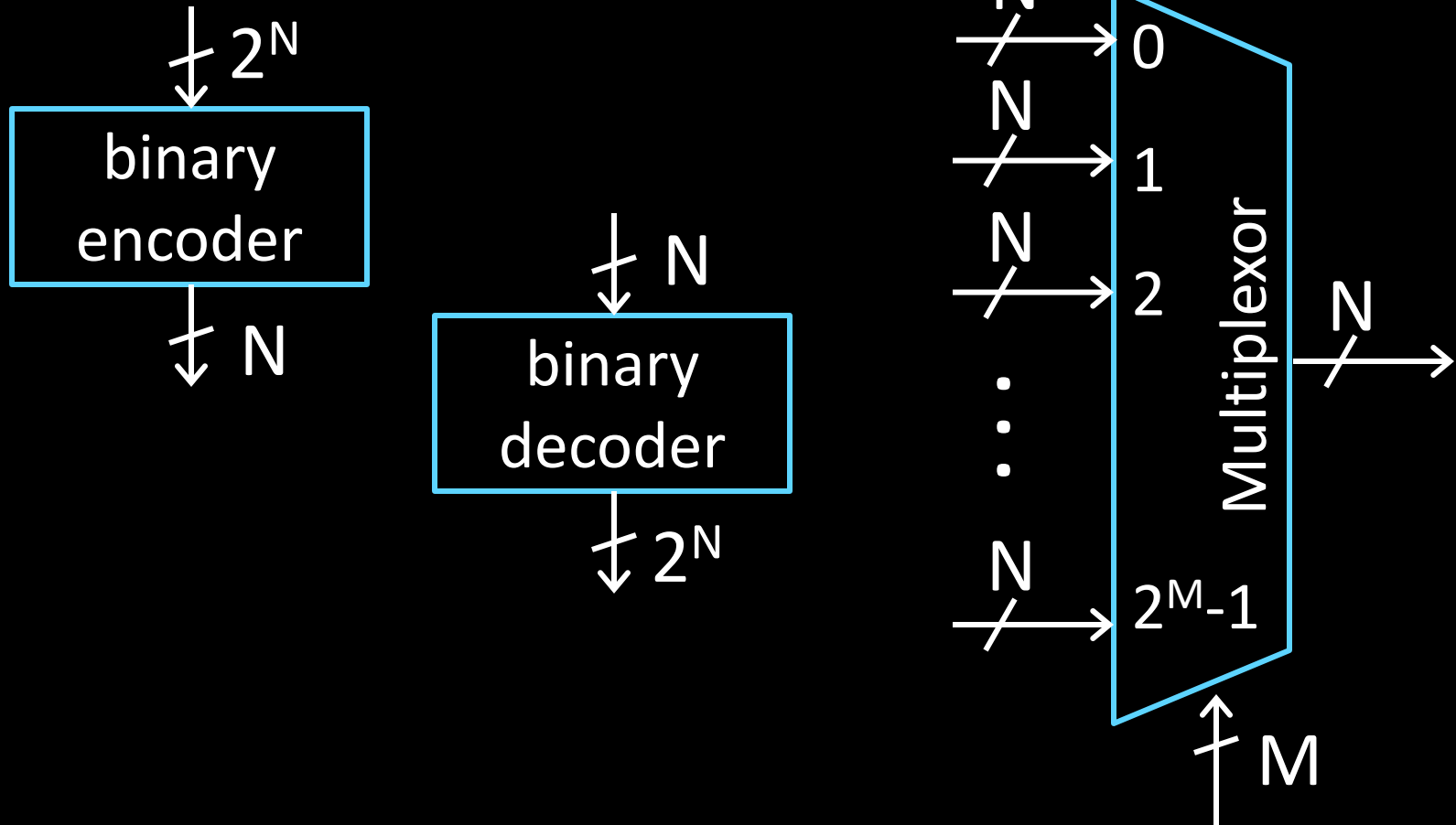


7 Segment LED Decoder Implementation

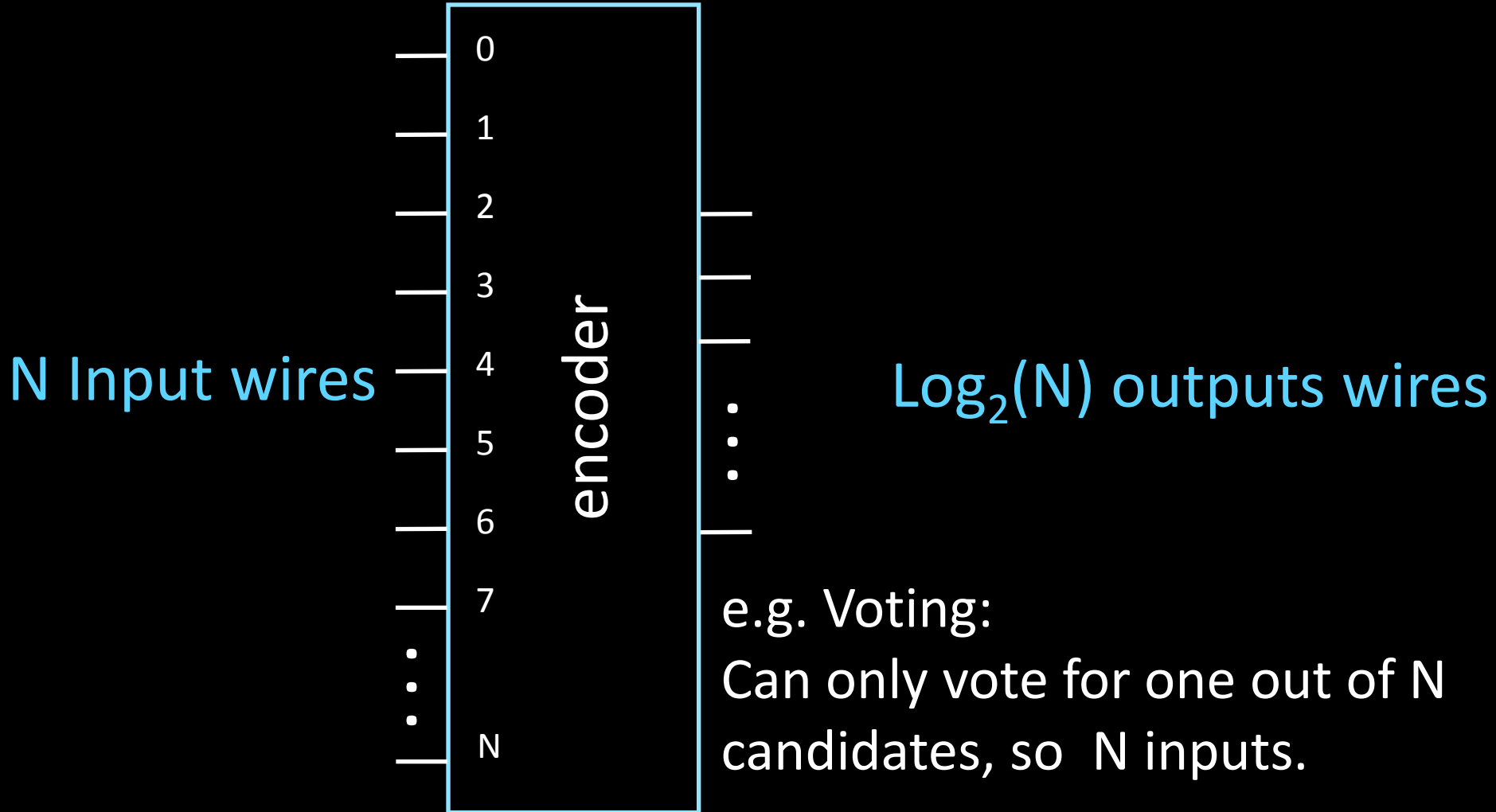
b2	b1	b0	d6	d5	d4	d3	d2	d1	d0
0	0	0	1	1	1	0	1	1	1
0	0	1	1	0	0	0	0	0	1
0	1	0	0	1	1	1	0	1	1
0	1	1	1	1	0	1	0	1	1
1	0	0	1	0	0	1	1	0	1
1	0	1	1	1	0	1	1	1	0
1	1	0	1	1	1	1	1	1	0
1	1	1	1	0	0	0	0	1	1



Basic Building Blocks We have Seen



Encoders

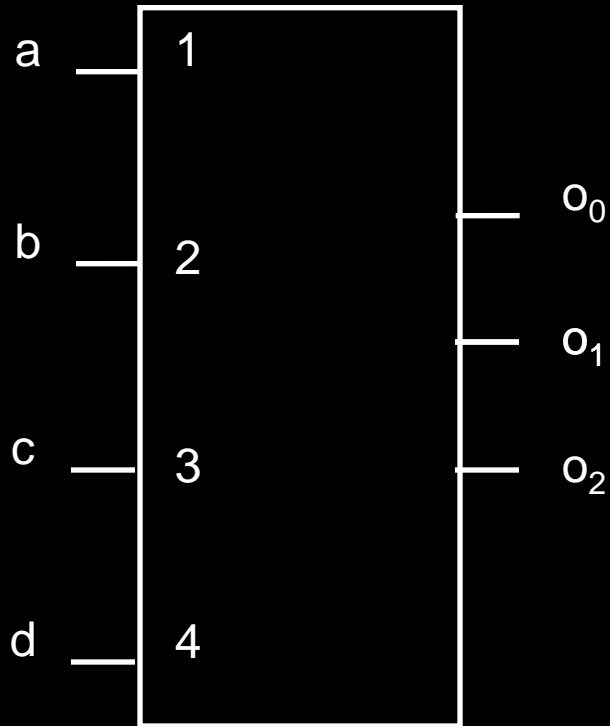


e.g. Voting:

Can only vote for one out of N candidates, so N inputs.

But can encode vote efficiently with binary encoding.

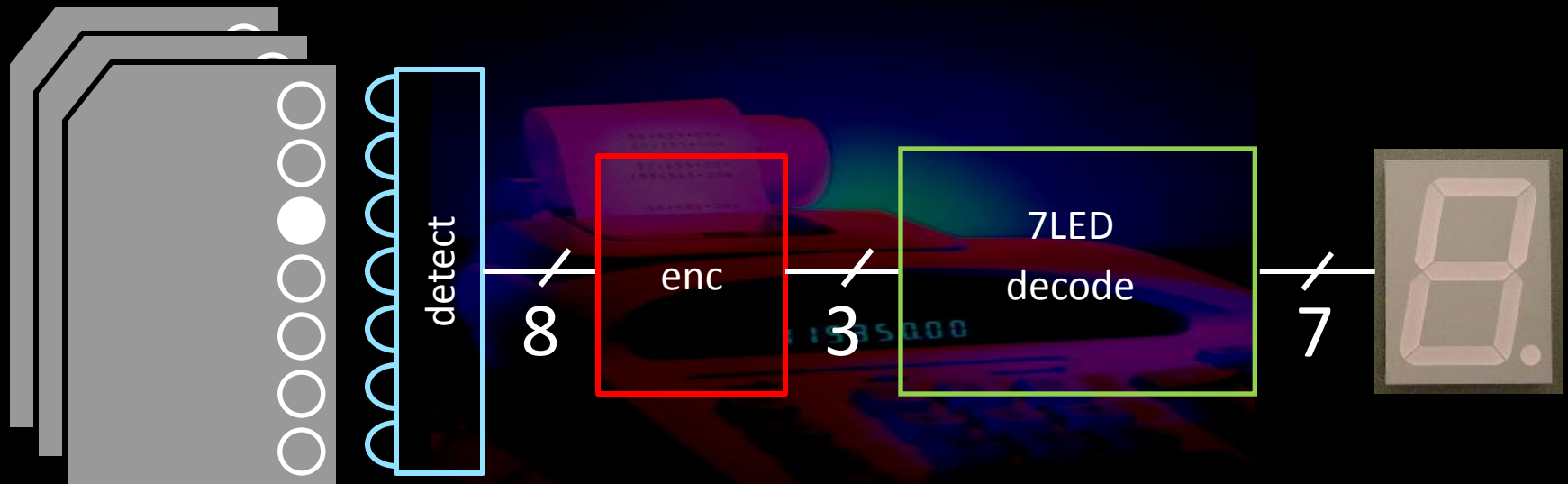
Example Encoder Truth Table



A 3-bit
encoder
with 4 inputs
for simplicity

a	b	c	d				
0	0	0	0				
1	0	0	0				
0	1	0	0				
0	0	1	0				
0	0	0	1				

Basic Building Blocks Example: Voting



Ballots

The 3410 optical scan
vote reader
machine

Recap

We can now build interesting devices with sensors

- Using combinatorial logic

We can also store data values (aka Sequential Logic)

- In state-holding elements
- Coupled with clocks

Administrivia

Make sure to go to **your** Lab Section this week

Completed Lab1 due **before** winter break, Friday, Feb 14th

Note, a Design Document is due when you submit Lab1 final circuit

Work **alone**

Homework1 is out

Due a week before prelim1, Monday, February 24th

Work on problems incrementally, as we cover them in lecture

Office Hours for help

Work **alone**

Work alone, **BUT** use your resources

- Lab Section, Piazza.com, Office Hours
- Class notes, book, Sections, CSUGLab

Administrivia

Check online syllabus/schedule

- <http://www.cs.cornell.edu/Courses/CS3410/2014sp/schedule.html>

Slides and Reading for lectures

Office Hours

Homework and Programming Assignments

Prelims (in evenings):

- Tuesday, March 4th
- Thursday, May 1th

Schedule is subject to change

Collaboration, Late, Re-grading Policies

“Black Board” Collaboration Policy

- Can discuss approach together on a “black board”
- Leave and write up solution independently
- Do not copy solutions

Late Policy

- Each person has a total of **four** “slip days”
- Max of **two** slip days for any individual assignment
- Slip days deducted first for *any* late assignment, cannot selectively apply slip days
- For projects, slip days are deducted from all partners
- 25% deducted per day late after slip days are exhausted

Regrade policy

- Submit written request to lead TA,
and lead TA will pick a different grader
- Submit another written request,
lead TA will regrade directly
- Submit yet another written request for professor to regrade.

Goals for Today

State

- How do we store *one* bit?
- Attempts at storing (and changing) one bit
 - Set-Reset Latch
 - D Latch
 - D Flip-Flops
 - Master-Slave Flip-Flops
- Register: storing more than one bit, N-bits

Basic Building Blocks

- Decoders and Encoders

Finite State Machines (FSM)

- How do we design logic circuits with state?
- Types of FSMs: Mealy and Moore Machines
- Examples: Serial Adder and a Digital Door Lock

Finite State Machines

Next Goal

How do we design logic circuits with state?

Finite State Machines

An electronic machine which has

- external inputs
- externally visible outputs
- internal state

Output and next state depend on

- inputs
- current state

Abstract Model of FSM

Machine is

$$M = (S, I, O, \delta)$$

S : Finite set of states

I : Finite set of inputs

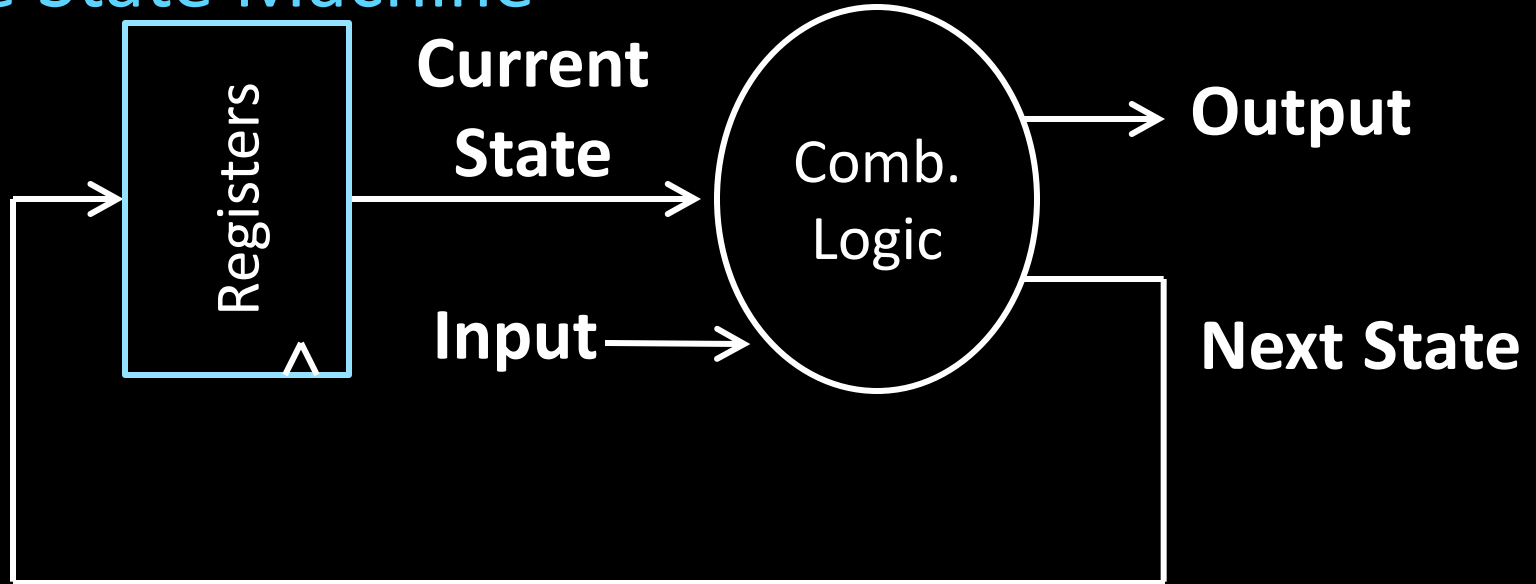
O : Finite set of outputs

δ : State transition function

Next state depends on present input *and* present state

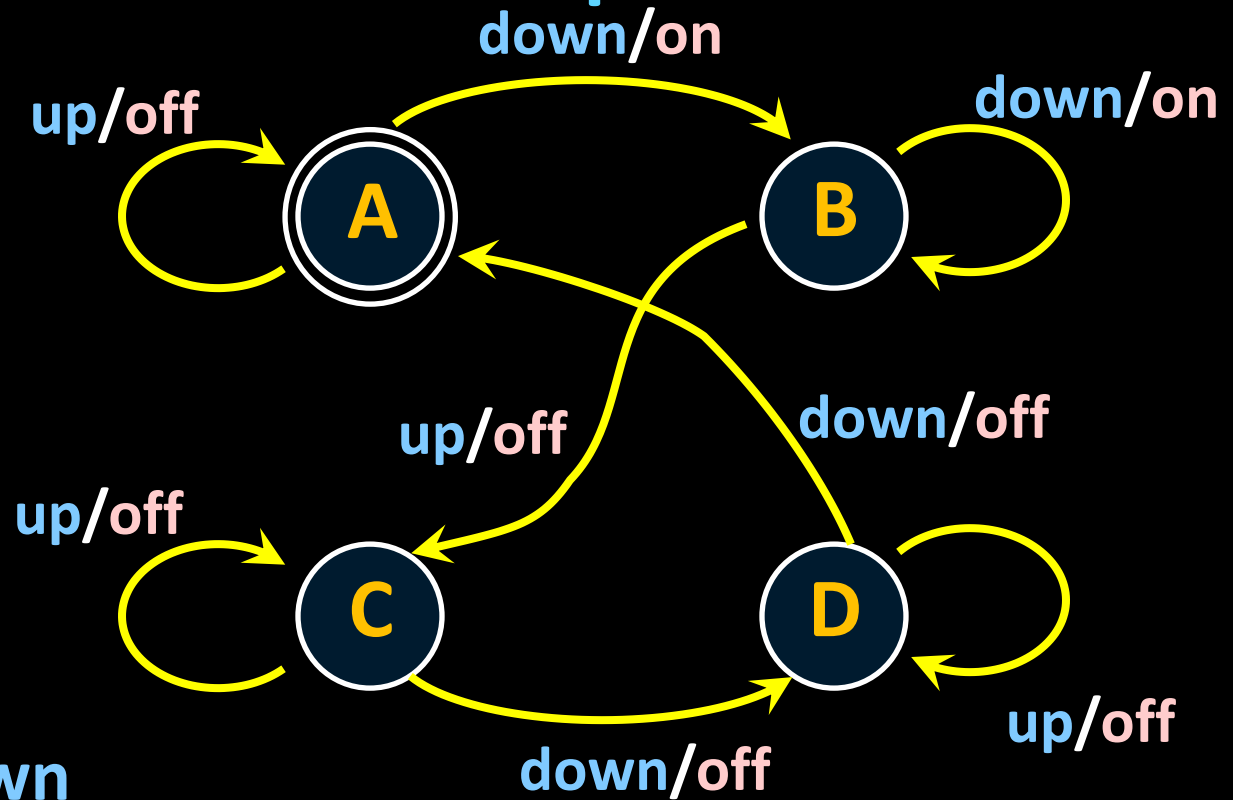
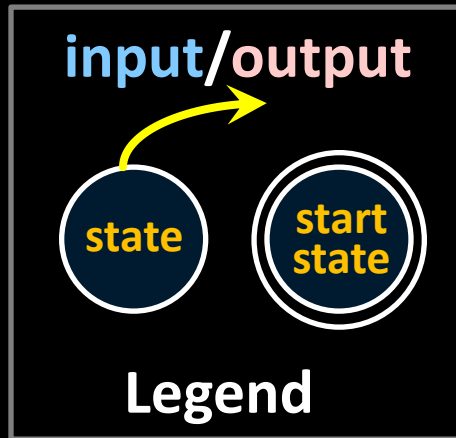
Automata Model

Finite State Machine



- inputs from external world
- outputs to external world
- internal state
- combinational logic

FSM Example

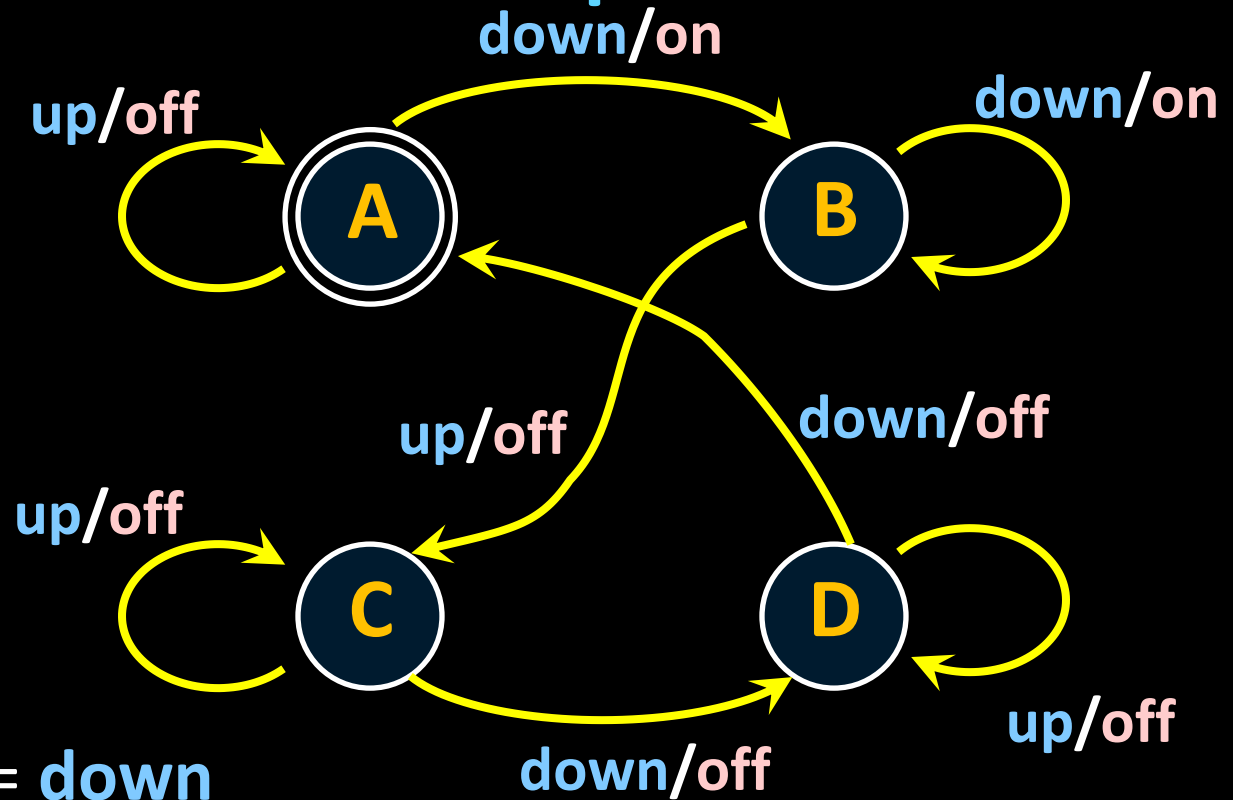
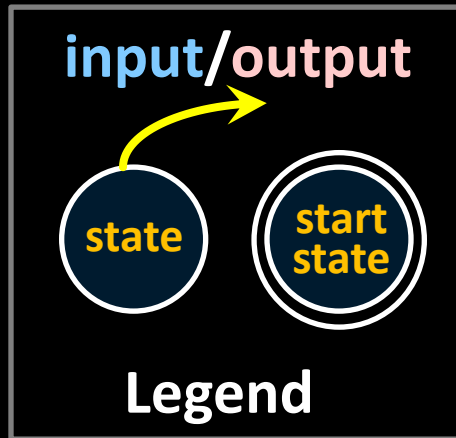


Input: **up** or **down**

Output: **on** or **off**

States: **A**, **B**, **C**, or **D**

FSM Example

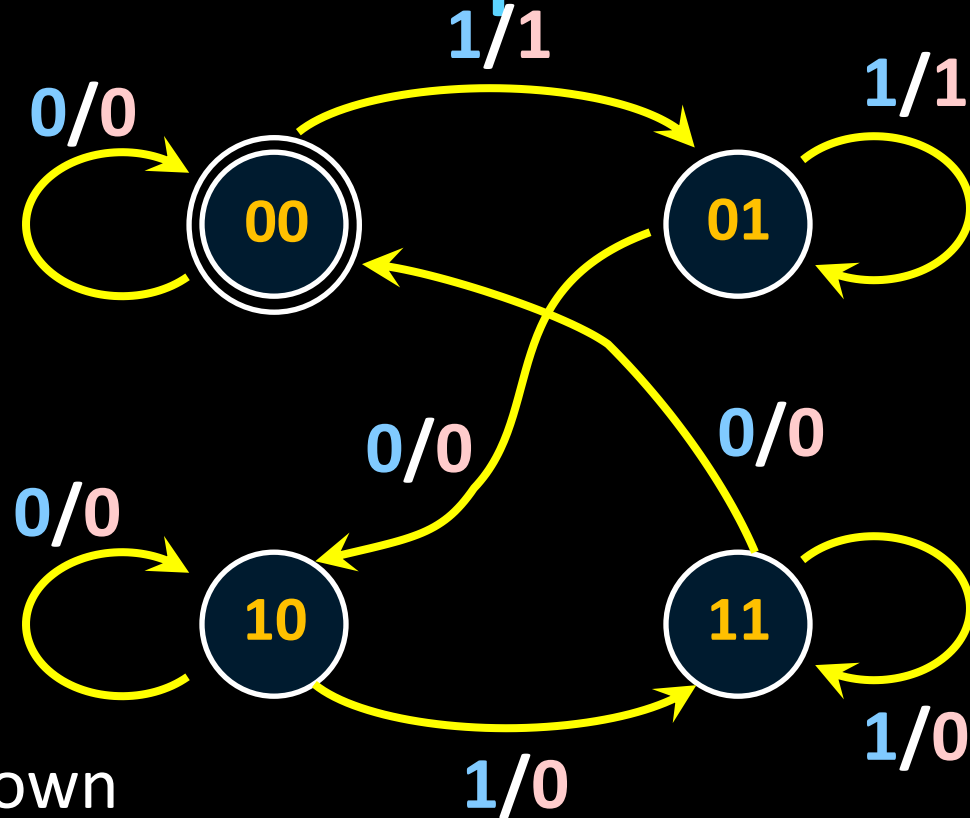
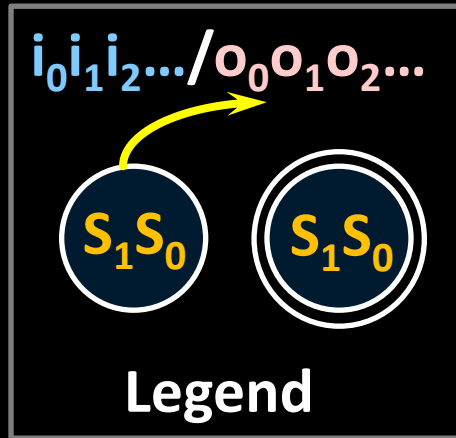


Input: = **up** or = **down**

Output: = **on** or = **off**

States: = **A**, = **B**, = **C**, or = **D**

FSM Example



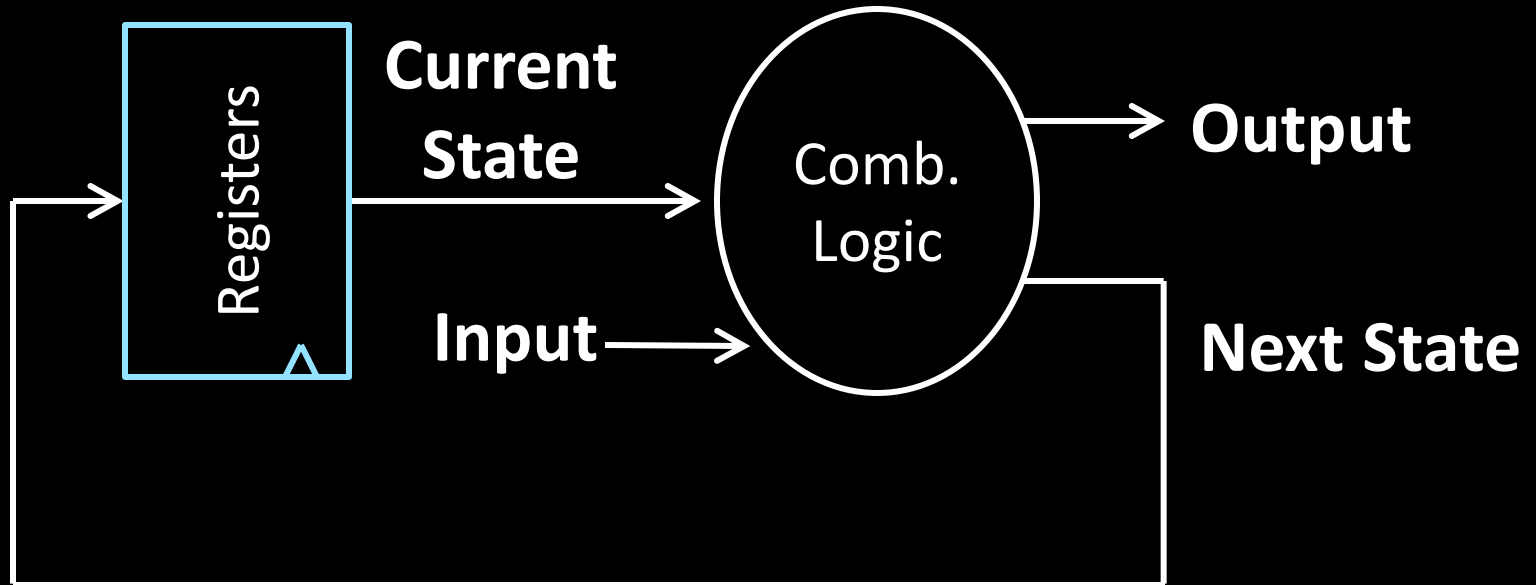
Input: 0=up or 1=down

Output: 1=on or 1=off

States: 00=A, 01=B, 10=C, or 11=D

Mealy Machine

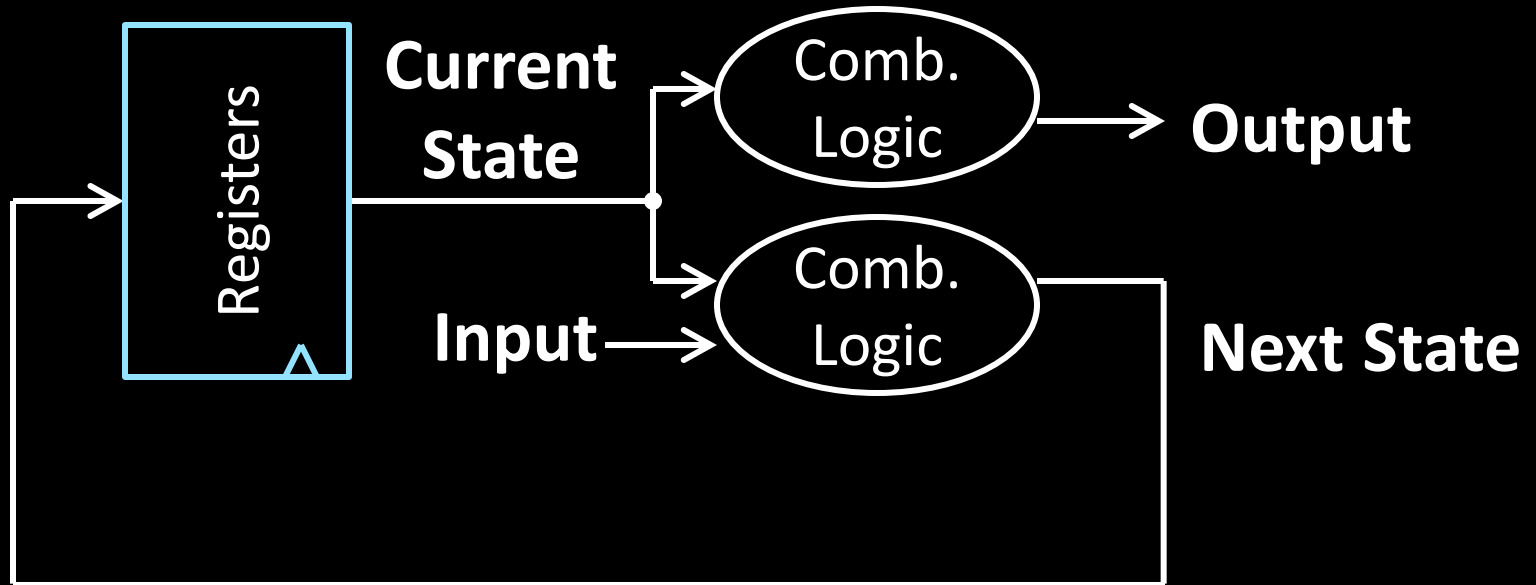
General Case: Mealy Machine



Outputs and next state depend on both current state and input

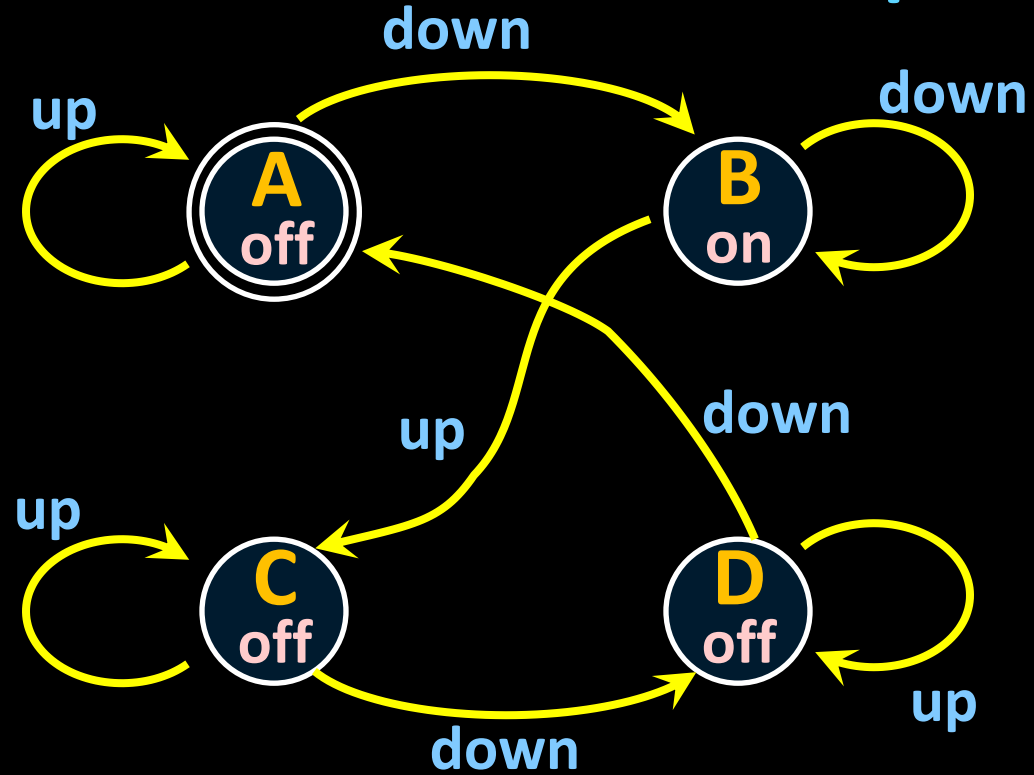
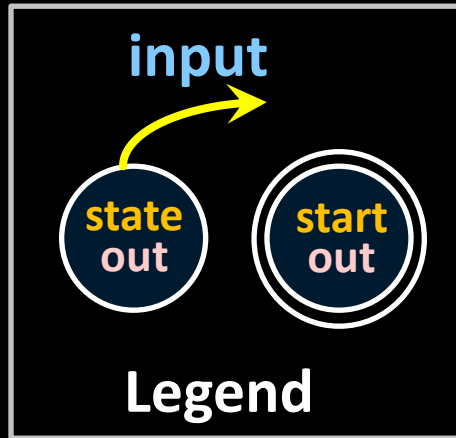
Moore Machine

Special Case: Moore Machine



Outputs depend only on current state

Moore Machine FSM Example

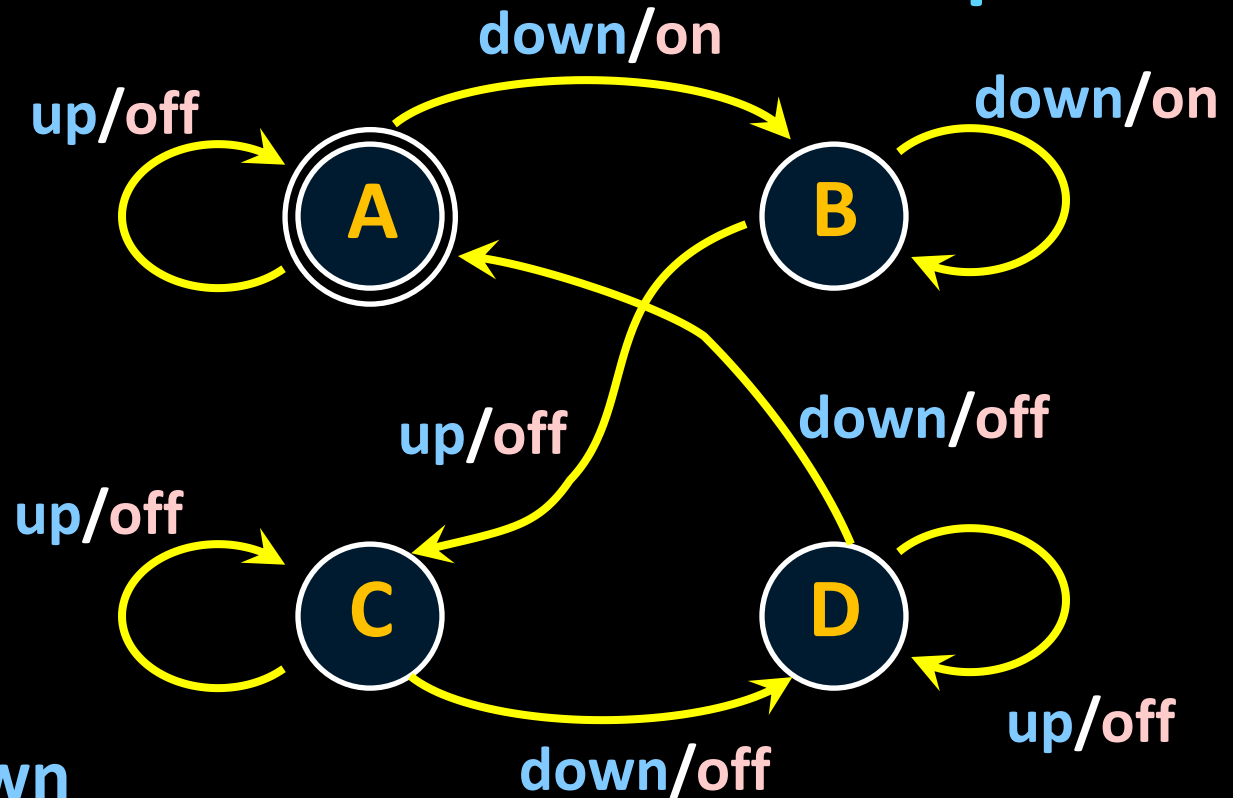
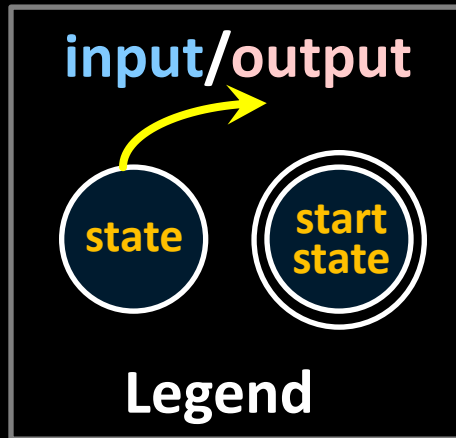


Input: **up** or **down**

Output: **on** or **off**

States: **A**, **B**, **C**, or **D**

Mealy Machine FSM Example



Input: **up** or **down**

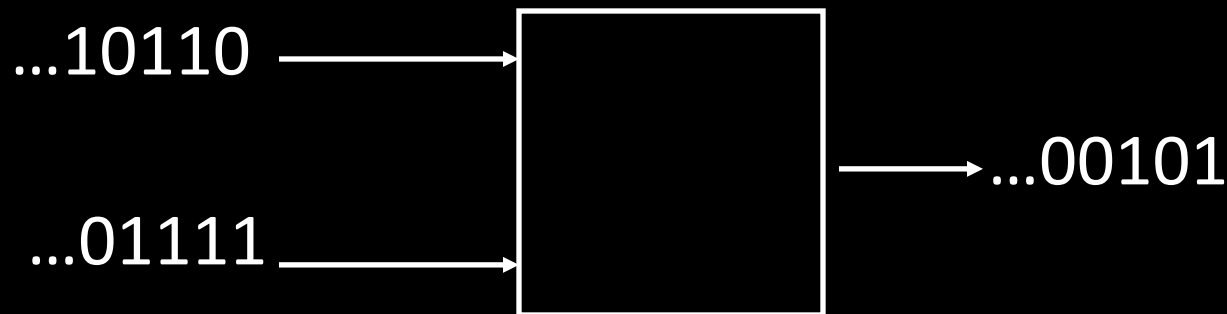
Output: **on** or **off**

States: **A**, **B**, **C**, or **D**

Activity#2: Create a Logic Circuit for a Serial Adder

Add two infinite input bit streams

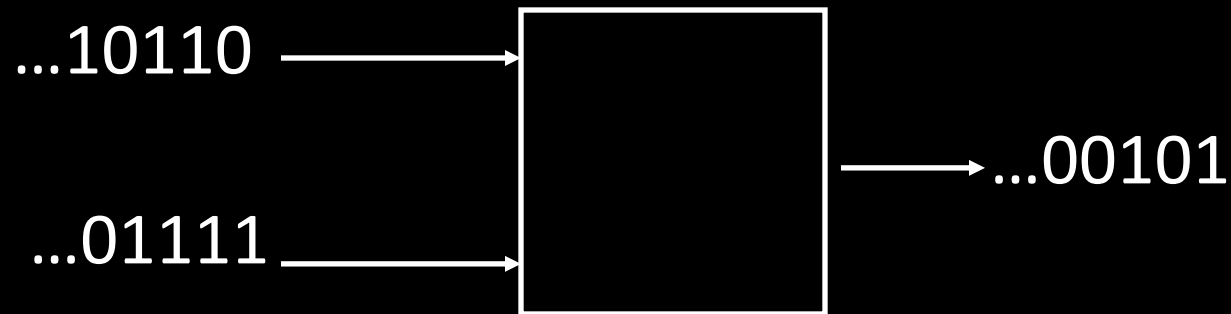
- streams are sent with least-significant-bit (lsb) first
- How many states are needed to represent FSM?
- Draw and Fill in FSM diagram



Strategy:

- (1) Draw a state diagram (e.g. Mealy Machine)
- (2) Write output and next-state tables
- (3) Encode states, inputs, and outputs as bits
- (4) Determine logic equations for next state and outputs

FSM: State Diagram

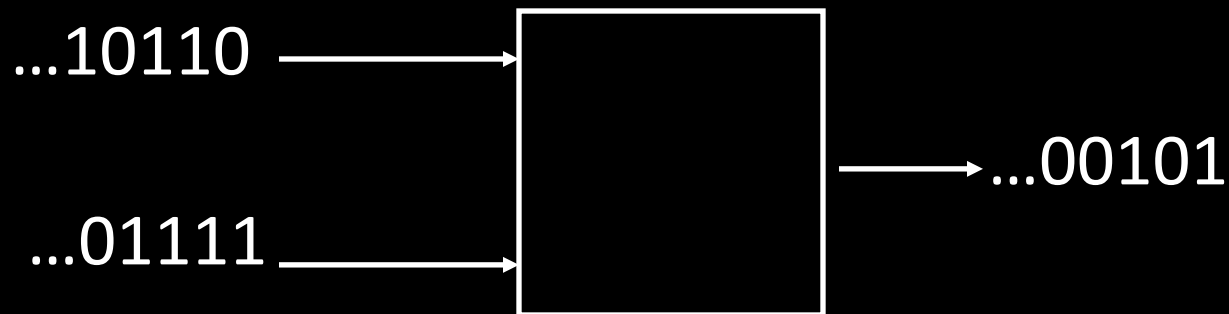
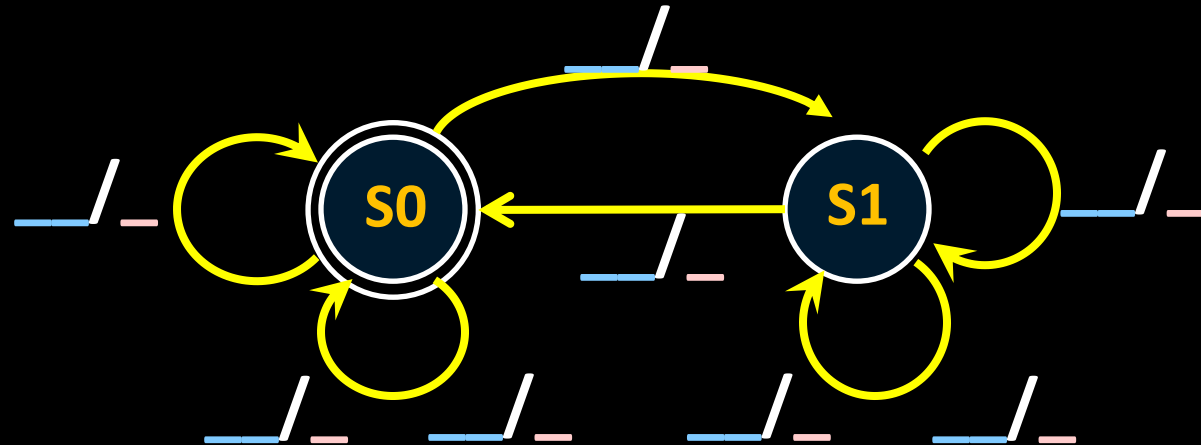


 states:

Inputs: ??? and ???

Output: ???

FSM: State Diagram

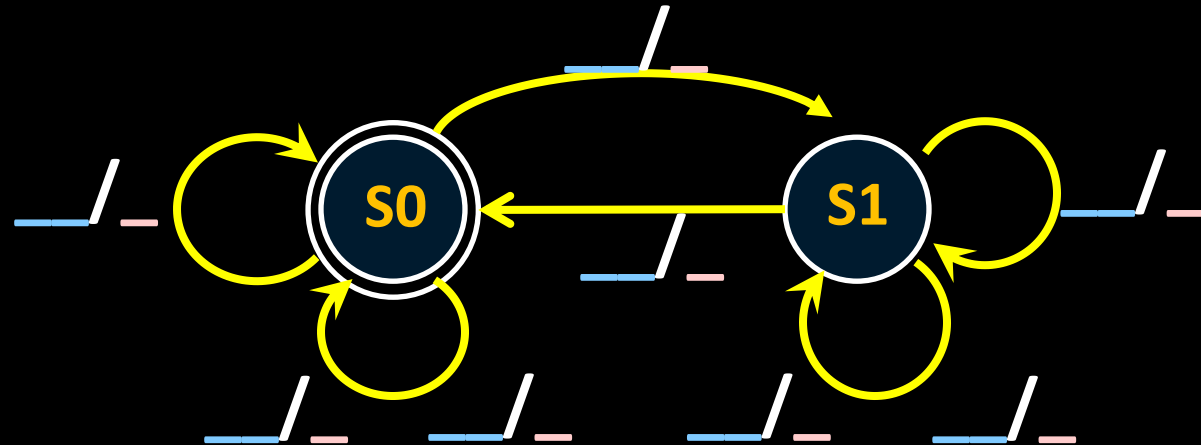


___ states:

Inputs: ??? and ???

Output: ???

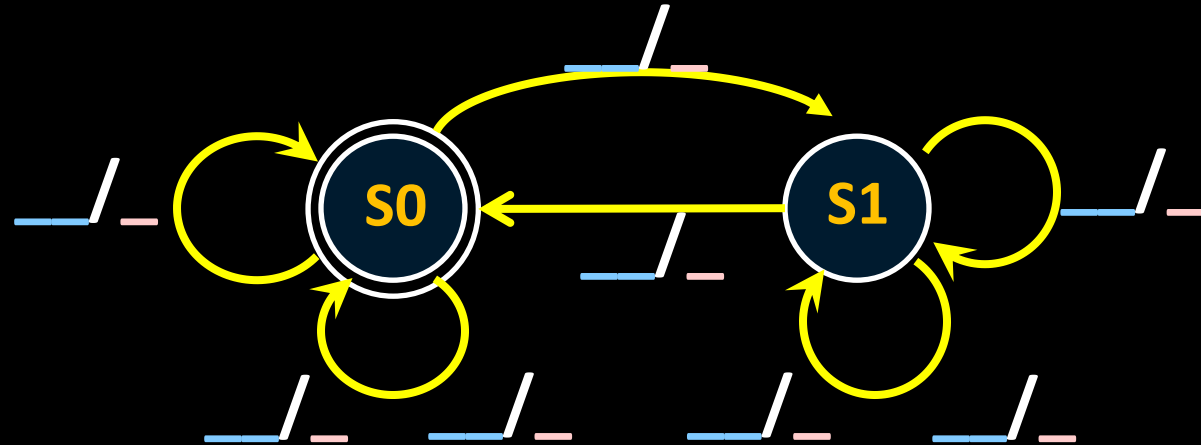
FSM: State Diagram



??	??	Current state	?	Next state

(2) Write down all input and state combinations

FSM: State Diagram



??	??	Current state	?	Next state

(3) Encode states, inputs, and outputs as bits

FSM: State Diagram

??	??	Current state	?	Next state

(4) Determine logic equations for next state and outputs

Summary

We can now build interesting devices with sensors

- Using combinational logic

We can also store data values

- Stateful circuit elements (D Flip Flops, Registers, ...)
- Clock to synchronize state changes
- State Machines or Ad-Hoc Circuits