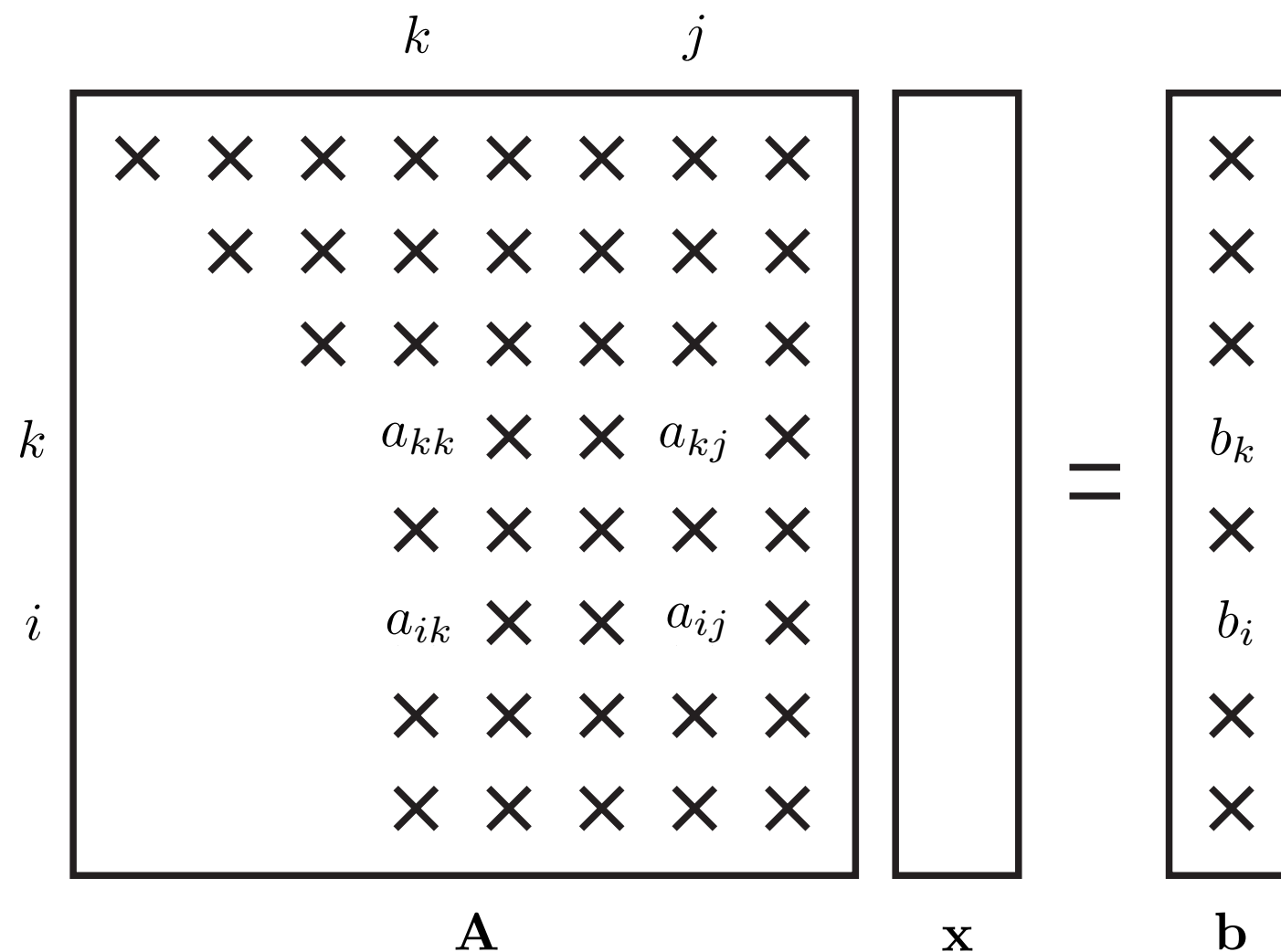


CS3220

Gaussian Elimination and LU

Steve Marschner
Spring 2010

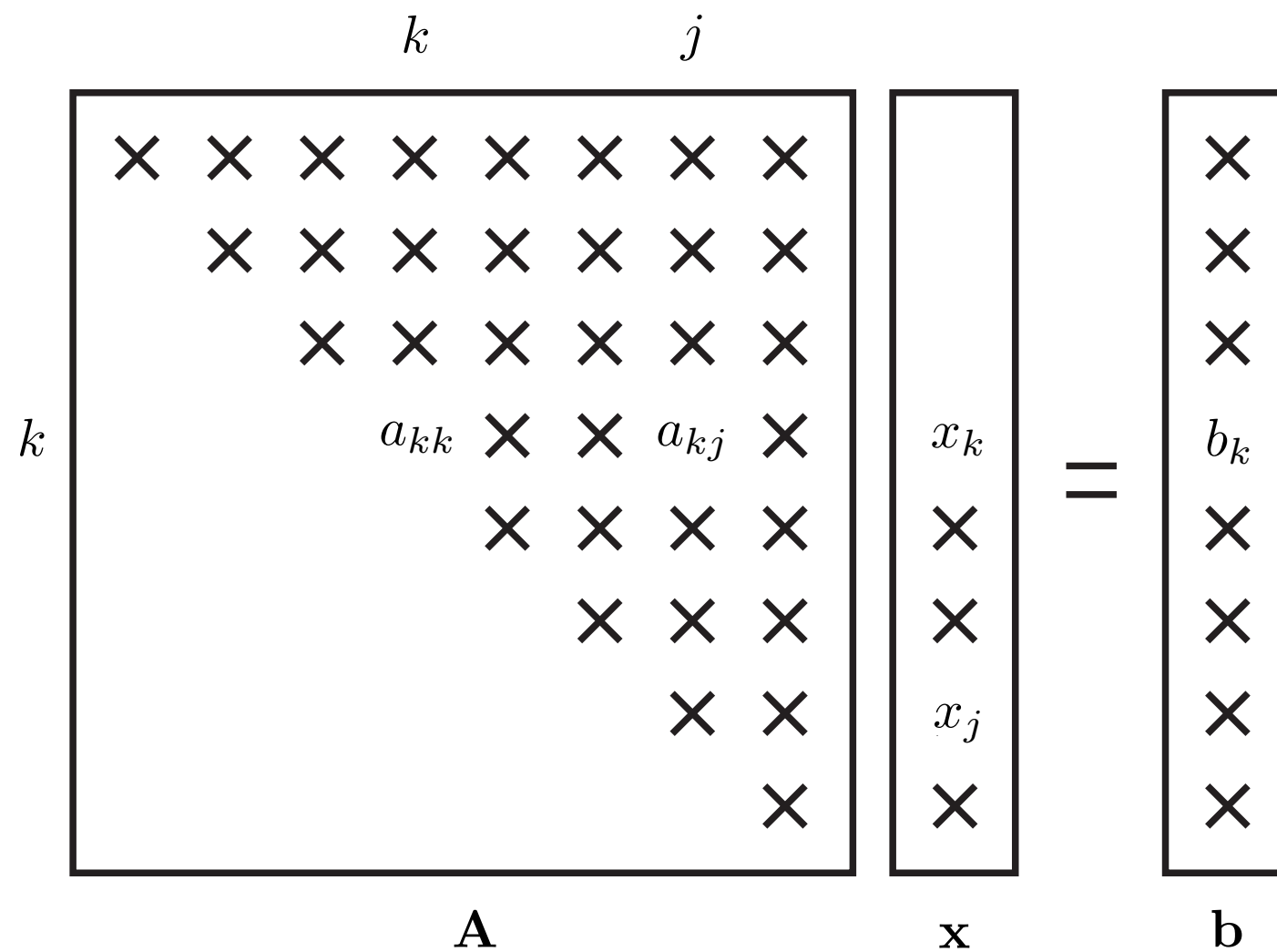
one step of the elimination phase



```

for k = 1 to n-1
  for i = k+1 to n
    r = aik / akk
    for j = k+1 to n
      aij += -r akj
    bi = bi - r bk
    aik = 0
for k = n to 1
  s = bk
  for j = (k+1) to n
    s += -akj xj
  xk = s / akk
  
```

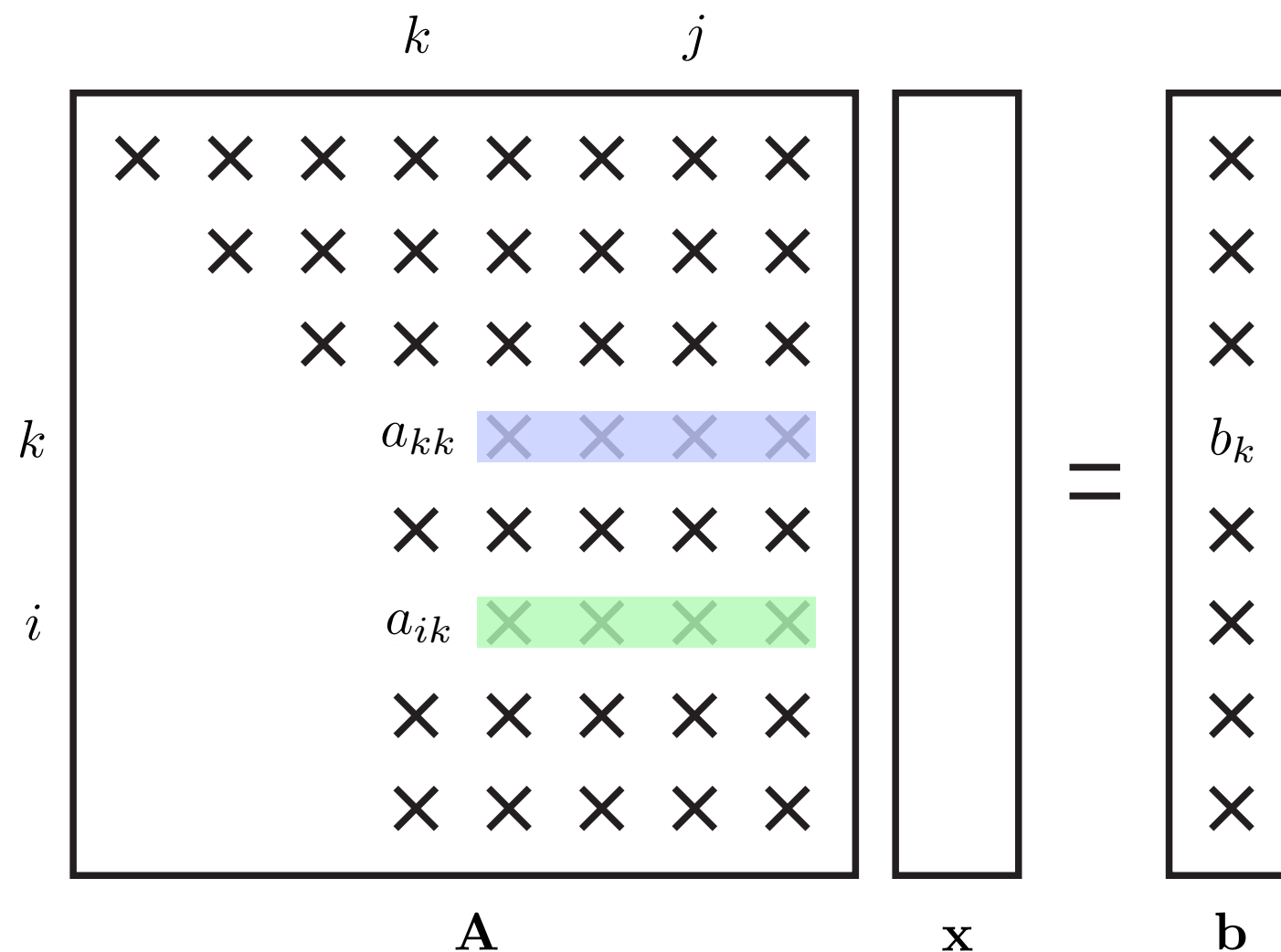
one step of the solution phase



```

for k = 1 to n-1
    for i = k+1 to n
        r = aik / akk
        for j = k+1 to n
            aij += -r akj
        bi = bi - r bk
        aik = 0
for k = n to 1
    s = bk
    for j = (k+1) to n
        s += -akj xj
    xk = s / akk
    
```

one step of the elimination phase: vector



for $k = 1$ **to** $n-1$

for $i = k+1$ **to** n

$r = a_{ik} / a_{kk}$

$a_{i,k+1:n} \leftarrow -r a_{k,k+1:n}$

$b_i \leftarrow b_i - r b_k$

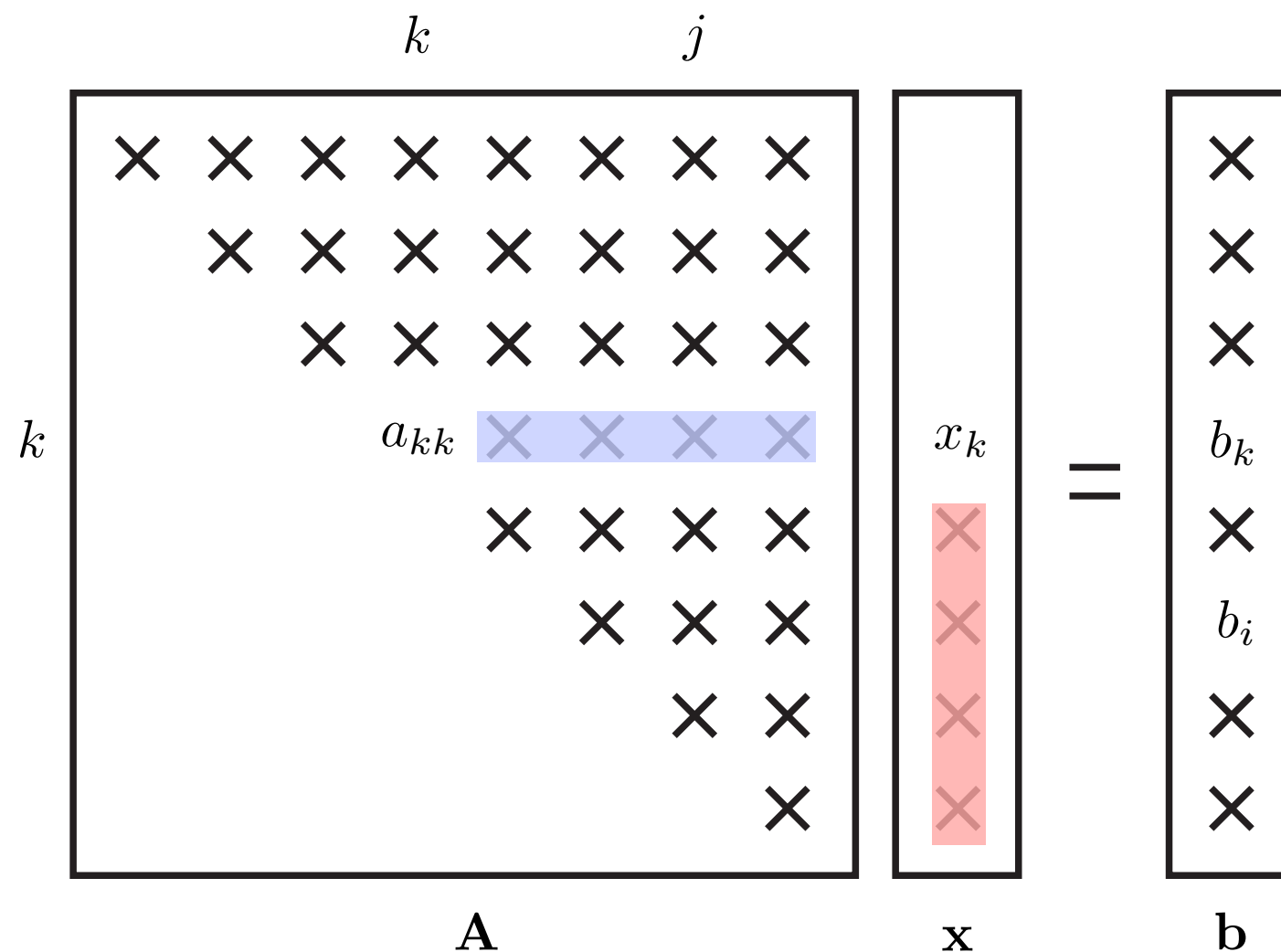
$a_{ik} = 0$

for $k = n$ **to** 1

$s = b_k - a_{k,k+1:n} \cdot x_{k+1:n}$

$x_k = s / a_{kk}$

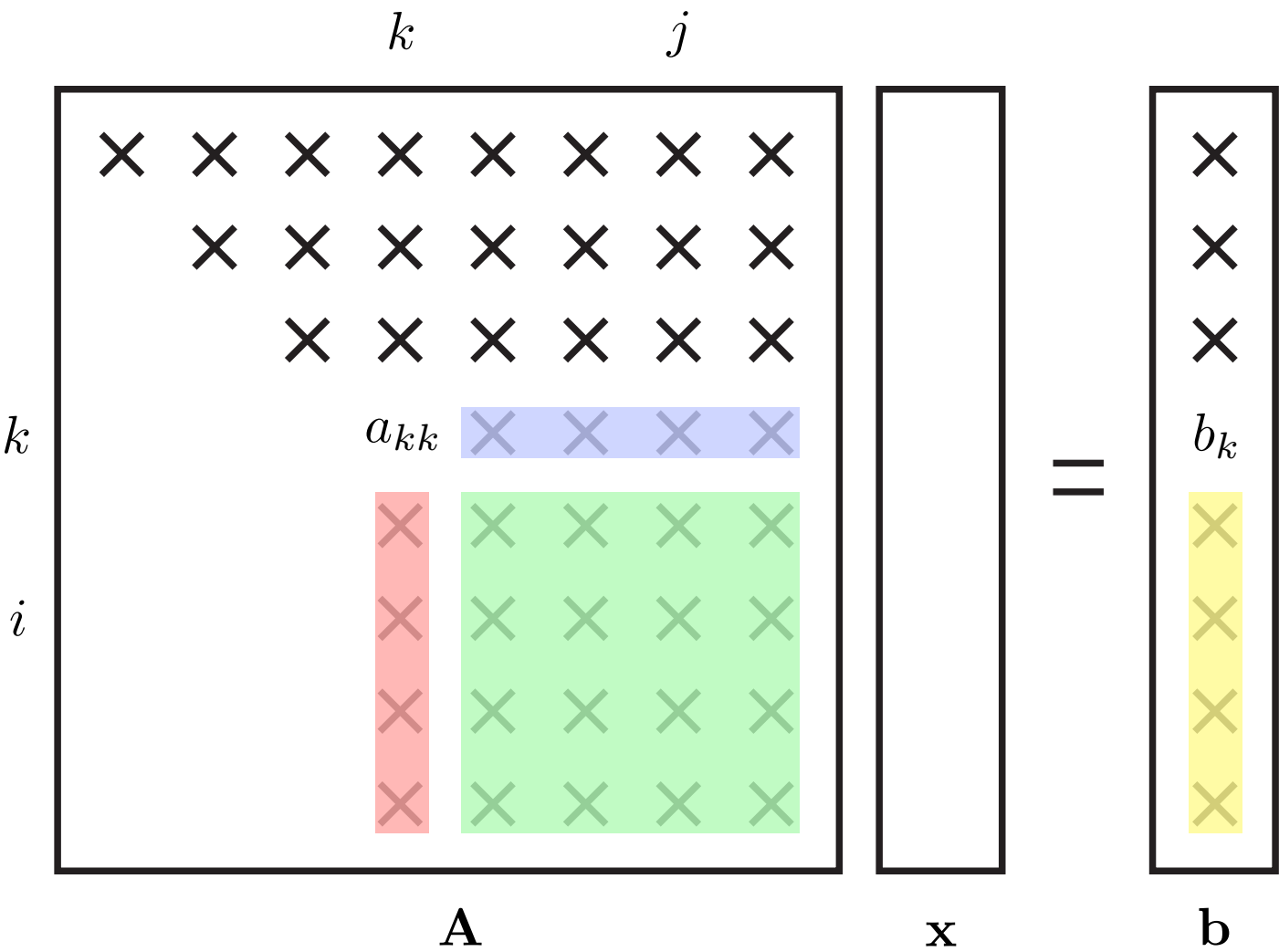
one step of the solution phase: vector



```

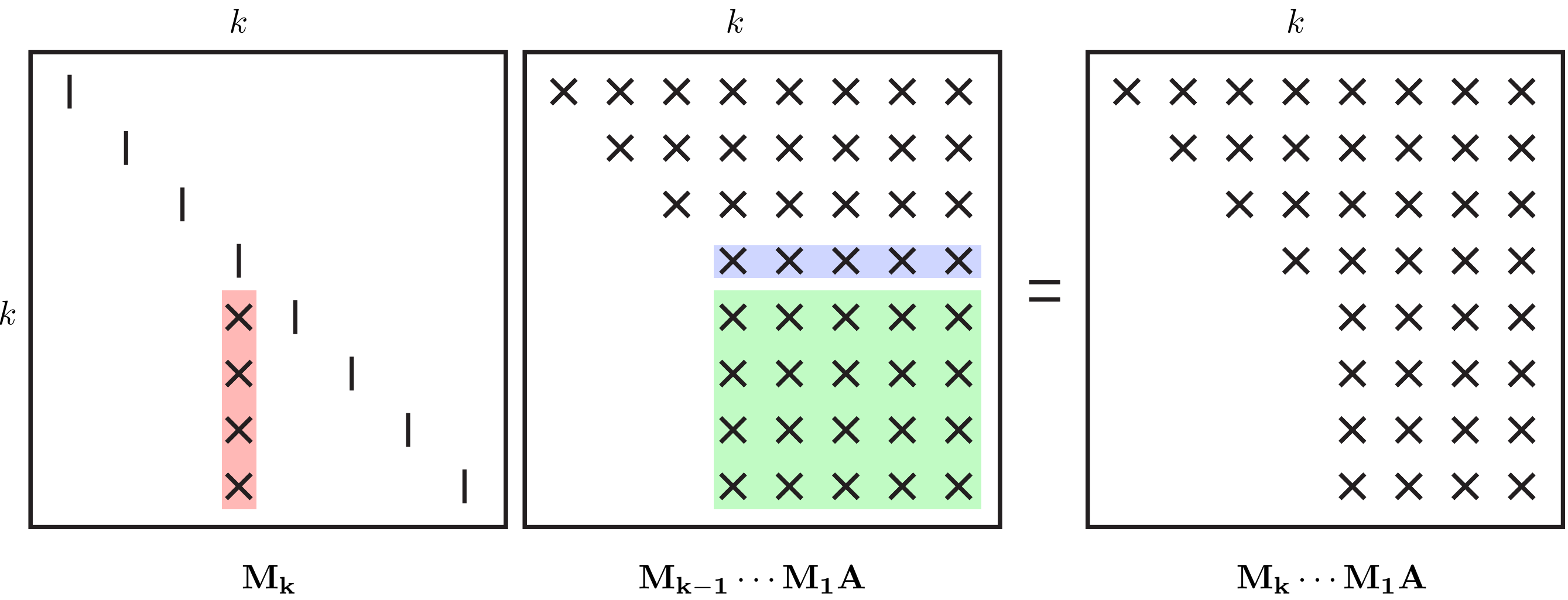
for k = 1 to n-1
  for i = k+1 to n
    r = aik / akk
    ai,k+1:n += -r ak,k+1:n
    bi = bi - r bk
    aik = 0
for k = n to 1
  s = bk - ak,k+1:n • xk+1:n
  xk = s / akk
  
```

one step of the elimination phase: matrix

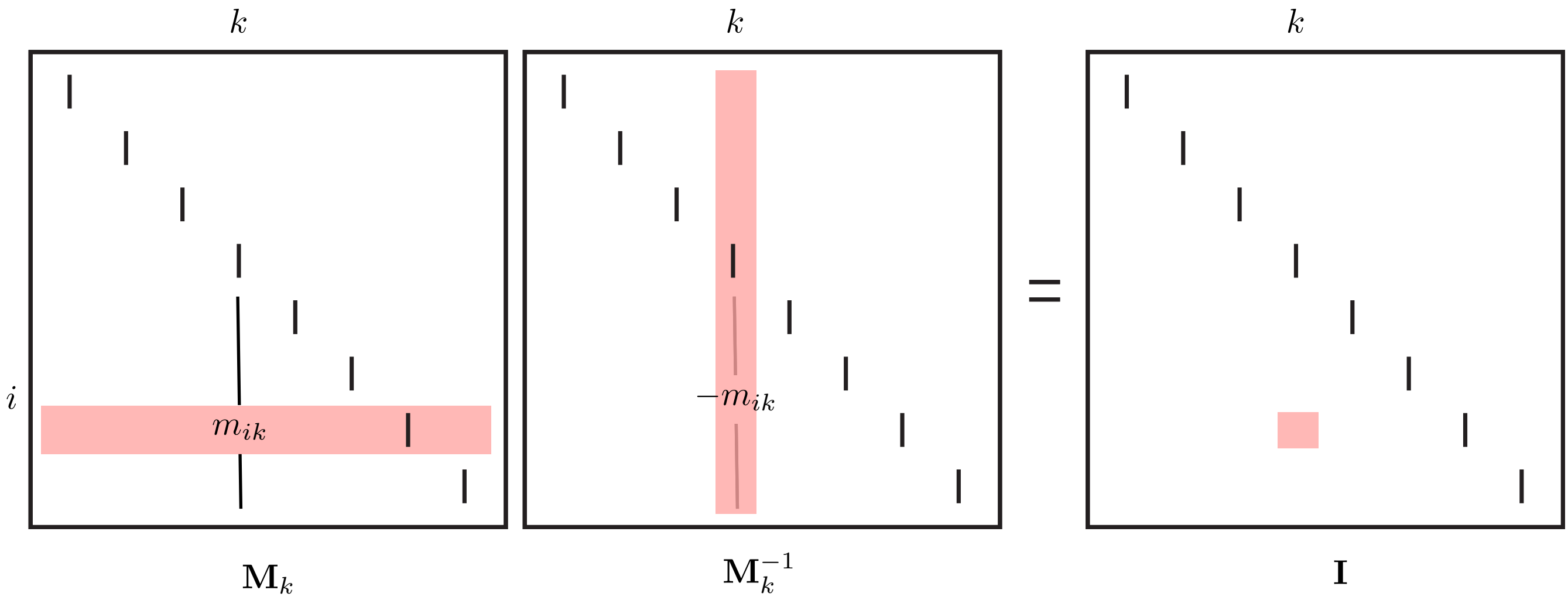


```
for k = 1 to n-1
    B = ak+1:n,k • ak,k+1:n
    ak+1:n,k+1:n -= B / akk
    bk+1:n += -ak+1:n,k / akk
    ak+1:n,k = 0
for k = n to 1
    s = bk - ak,k+1:n • xk+1:n
    xk = s / akk
```

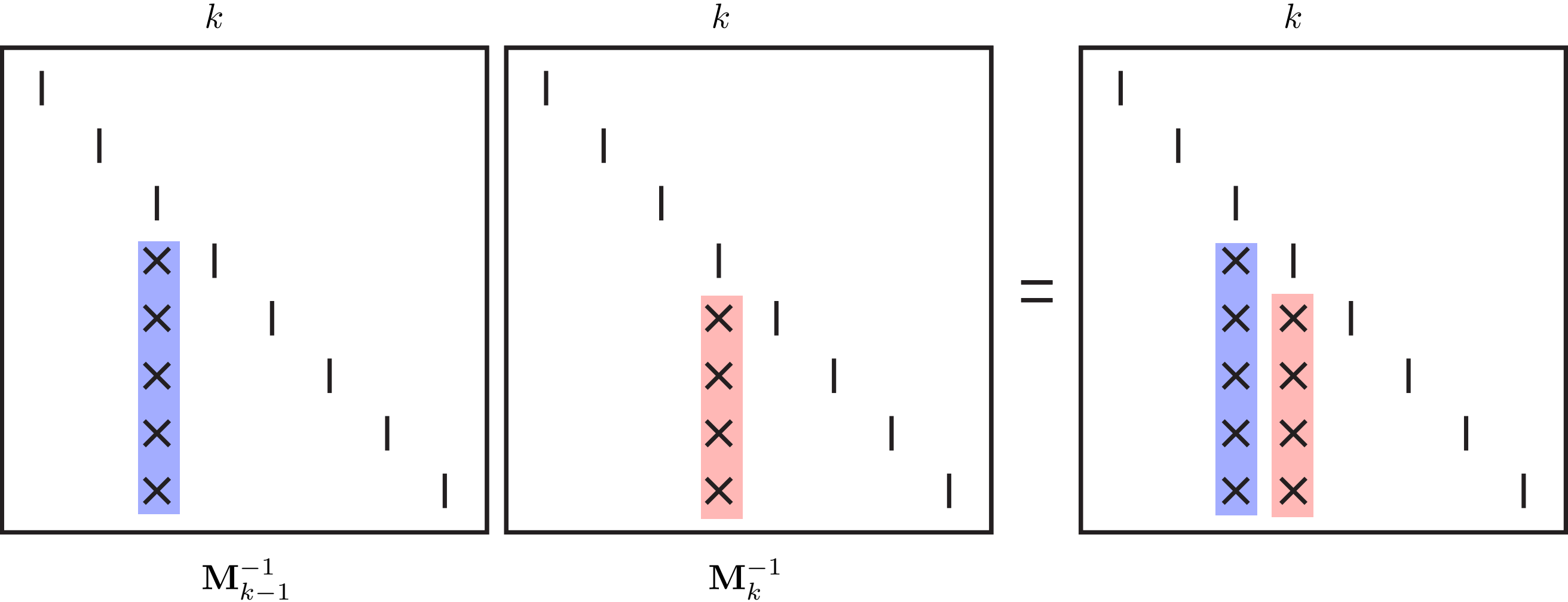
elimination step as matrix multiplication



inverting elimination matrices



multiplying the multipliers



matrix full of multipliers

$$L = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & | & | & \ddots & & \\ \mathbf{m}_1 & \mathbf{m}_2 & & & 1 & \\ & | & & & | & \\ & & & & \mathbf{m}_{n-2} & 1 \\ & & & & & \mathbf{m}_{n-1} \\ & & & & & & 1 \end{bmatrix}$$

pivoting

multipliers will be 10, 100, 1000, 10000—not good!

k

k

$M_{k-1} \cdots M_1 A$

x

$M_{k-1} \cdots M_1 b$

b_k

pivoting

multipliers will be 10, 100, 1000, 10000—not good!

k

k	×	×	×	×	×	×	×	×					
		×	×	×	×	×	×		×				
			×	×	×	×	×			×			
		.01	×	×	×	×	=				b_k		
		.1	×	×	×	×						×	
		1	×	×	×	×							×
		10	×	×	×	×							
	100	×	×	×	×	×							

$M_{k-1} \cdots M_1 A \quad x \quad M_{k-1} \cdots M_1 b$

solution:
swap these rows

pivot vector

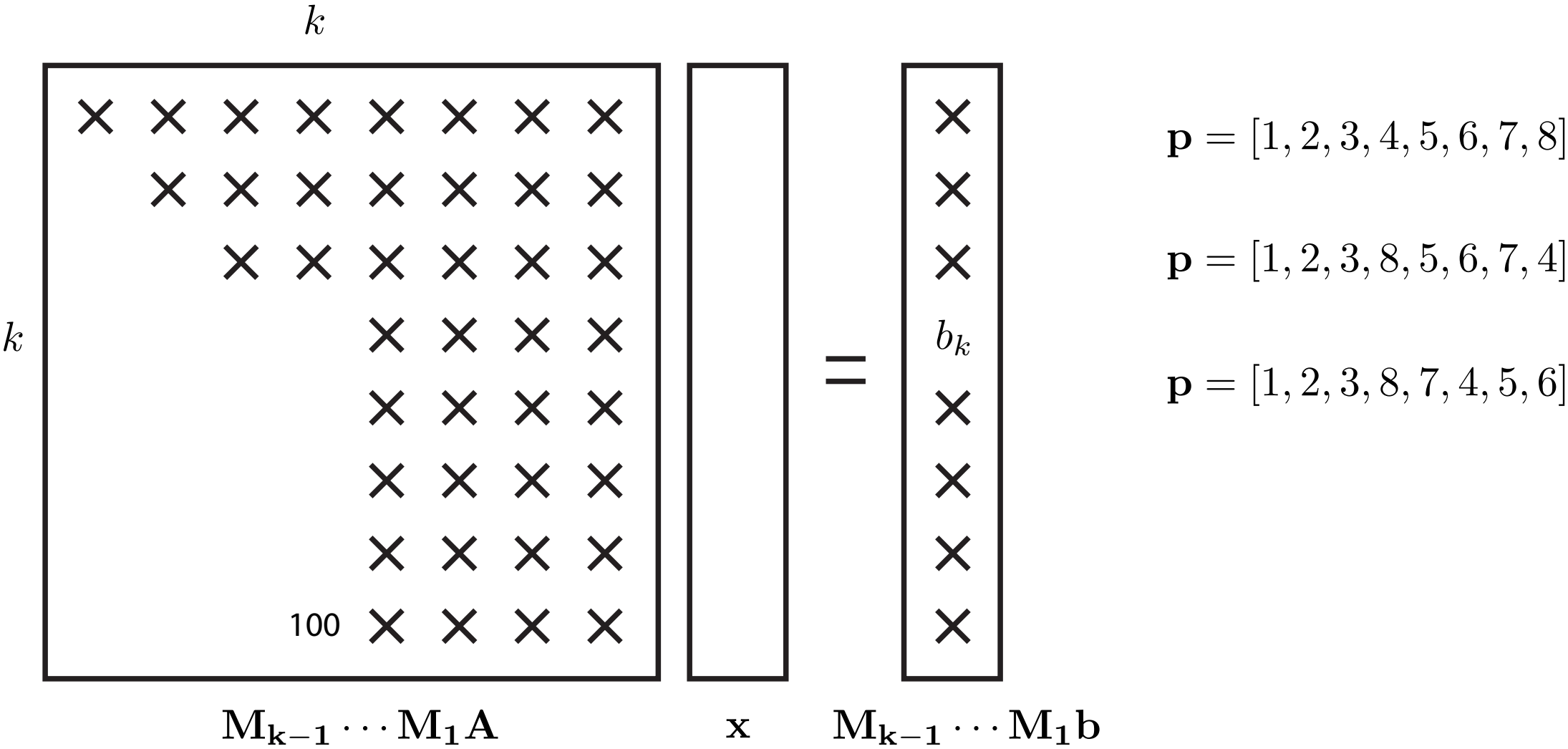
don't actually swap rows—just renumber them.

$$\begin{array}{c}
 k \\
 \begin{array}{|c|} \hline \begin{array}{cccccccc}
 \times & \times & \times & \times & \times & \times & \times & \times \\
 & \times & \times & \times & \times & \times & \times & \times \\
 & & \times & \times & \times & \times & \times & \times \\
 & & & .01 & \times & \times & \times & \times \\
 & & & .1 & \times & \times & \times & \times \\
 & & & 1 & \times & \times & \times & \times \\
 & & & 10 & \times & \times & \times & \times \\
 & & & 100 & \times & \times & \times & \times
 \end{array} \\ \hline
 \end{array}
 \end{array}
 \begin{array}{|c|} \hline \mathbf{x} \\ \hline
 \end{array}
 =
 \begin{array}{|c|} \hline \begin{array}{c}
 \times \\
 \times \\
 \times \\
 b_k \\
 \times \\
 \times \\
 \times \\
 \times \\
 \times
 \end{array} \\ \hline
 \end{array}
 \begin{array}{l}
 \mathbf{p} = [1, 2, 3, 4, 5, 6, 7, 8] \\
 \mathbf{p} = [1, 2, 3, 8, 5, 6, 7, 4] \\
 \mathbf{p} = [1, 2, 3, 8, 7, 4, 5, 6]
 \end{array}$$

$\mathbf{M}_{k-1} \cdots \mathbf{M}_1 \mathbf{A}$
 $\mathbf{x}$
 $\mathbf{M}_{k-1} \cdots \mathbf{M}_1 \mathbf{b}$

pivot vector

don't actually swap rows—just renumber them.



pivot vector

don't actually swap rows—just renumber them.

$$\begin{array}{c}
 k \\
 \boxed{
 \begin{array}{cccccccc}
 \times & \times & \times & \times & \times & \times & \times & \times \\
 & \times & \times & \times & \times & \times & \times & \times \\
 & & \times & \times & \times & \times & \times & \times \\
 & & & & & \times & \times & \times \\
 & & & & & & \times & \times \\
 & & & & & & & \times \\
 & & & & \times & \times & \times & \times \\
 & & \times & \times & \times & \times & \times &
 \end{array}
 } \\
 k \\
 \mathbf{M}_{k-1} \cdots \mathbf{M}_1 \mathbf{A}
 \end{array}
 \quad
 \begin{array}{c}
 \boxed{} \\
 \mathbf{x}
 \end{array}
 =
 \begin{array}{c}
 \boxed{
 \begin{array}{c}
 \times \\
 \times \\
 \times \\
 b_k \\
 \times \\
 \times \\
 \times \\
 \times \\
 \times
 \end{array}
 } \\
 \mathbf{M}_{k-1} \cdots \mathbf{M}_1 \mathbf{b}
 \end{array}
 \quad
 \begin{array}{l}
 \mathbf{p} = [1, 2, 3, 4, 5, 6, 7, 8] \\
 \mathbf{p} = [1, 2, 3, 8, 5, 6, 7, 4] \\
 \mathbf{p} = [1, 2, 3, 8, 7, 4, 5, 6]
 \end{array}$$