CS3220
Gaussian Elimination and LU

Steve Marschner
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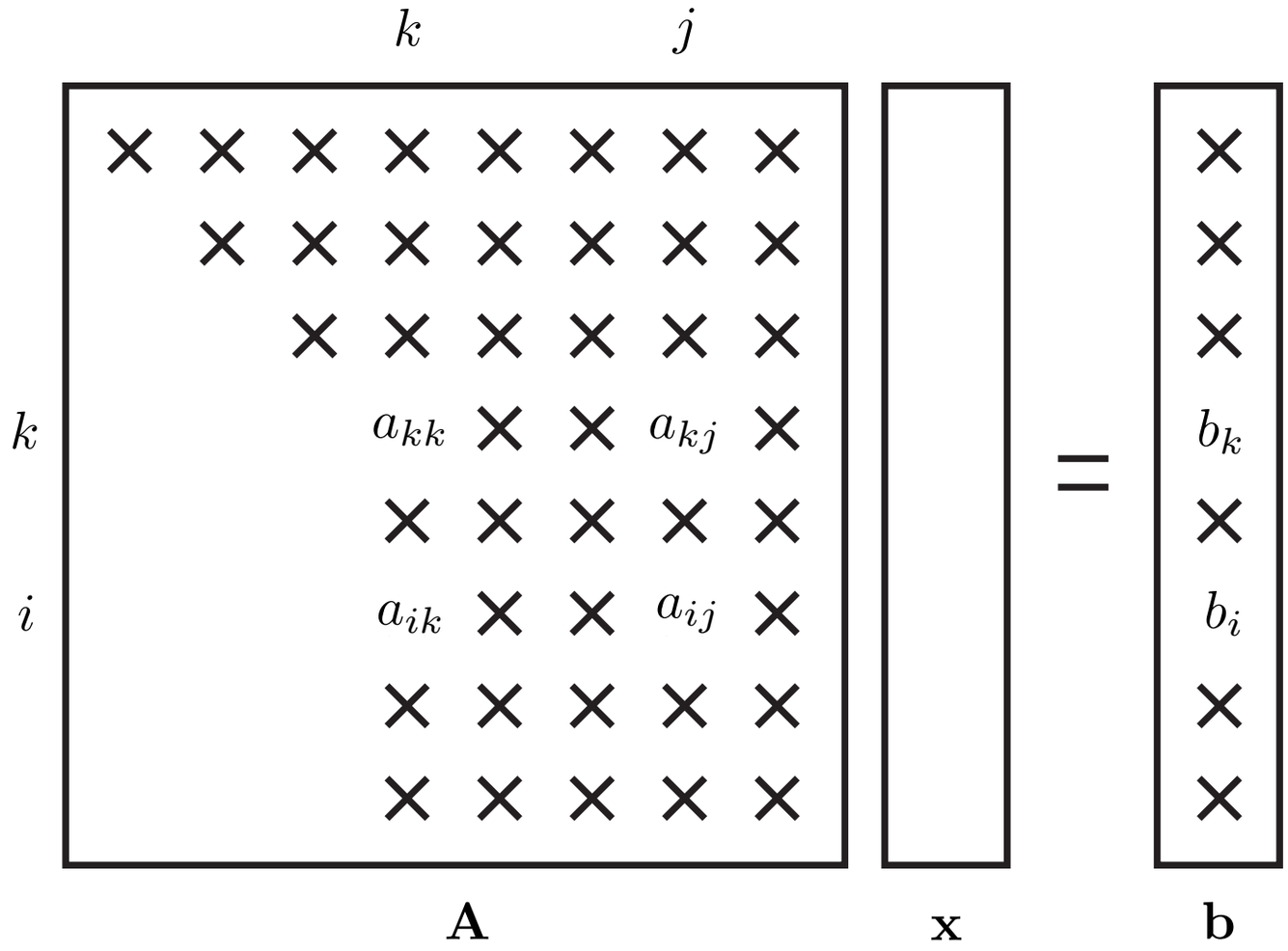
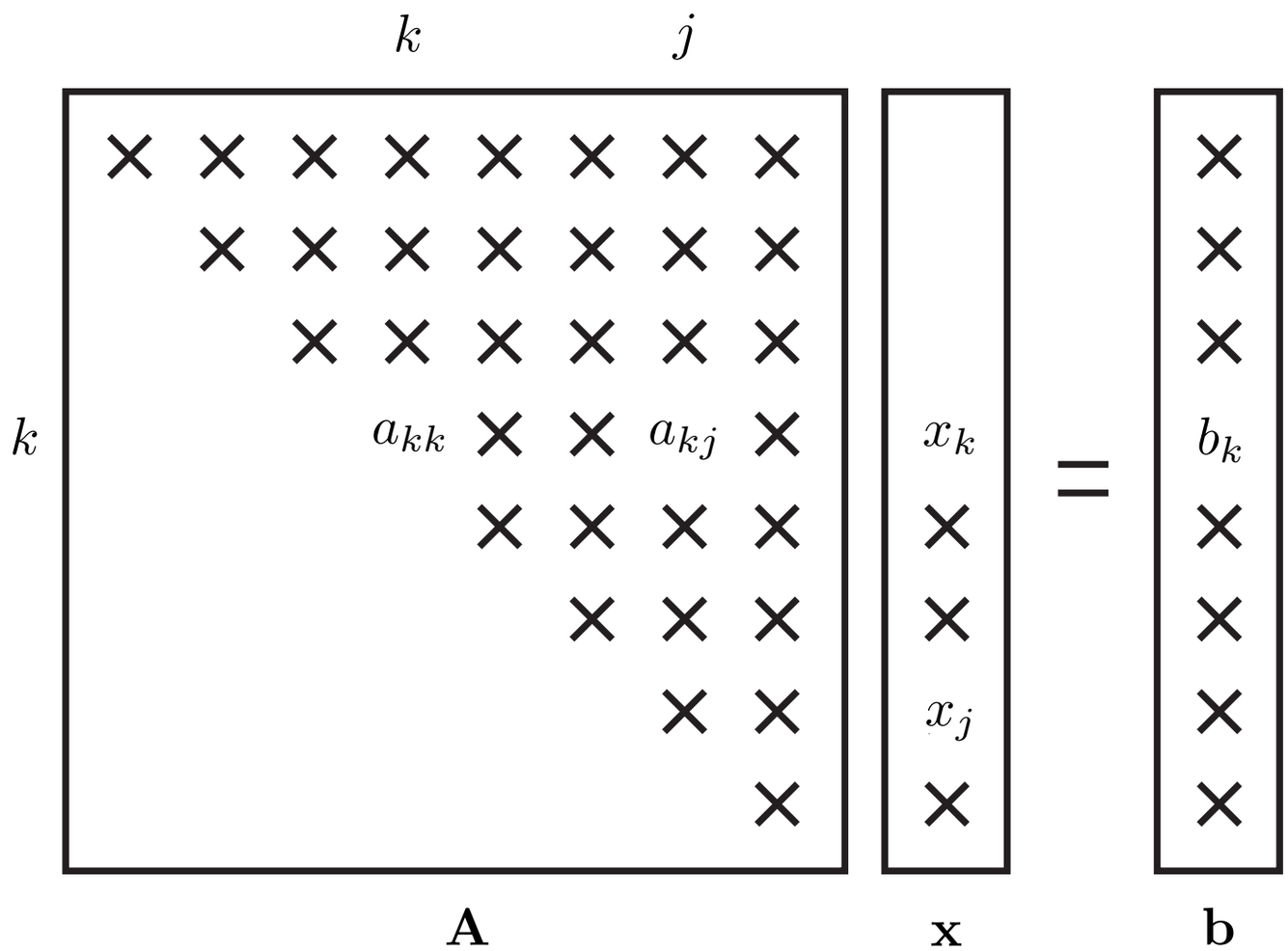
one step of the elimination phase




```
for k = 1 to n–1
   for i = k+1 to n
      r = aik /akk
      for j = k+1 to n
         aij += –r akj
      bi = bi – r bk
      aik = 0

for k = n to 1
   s = bk
   for j = (k+1) to n
      s += –akj xj
   xk = s / akk
```

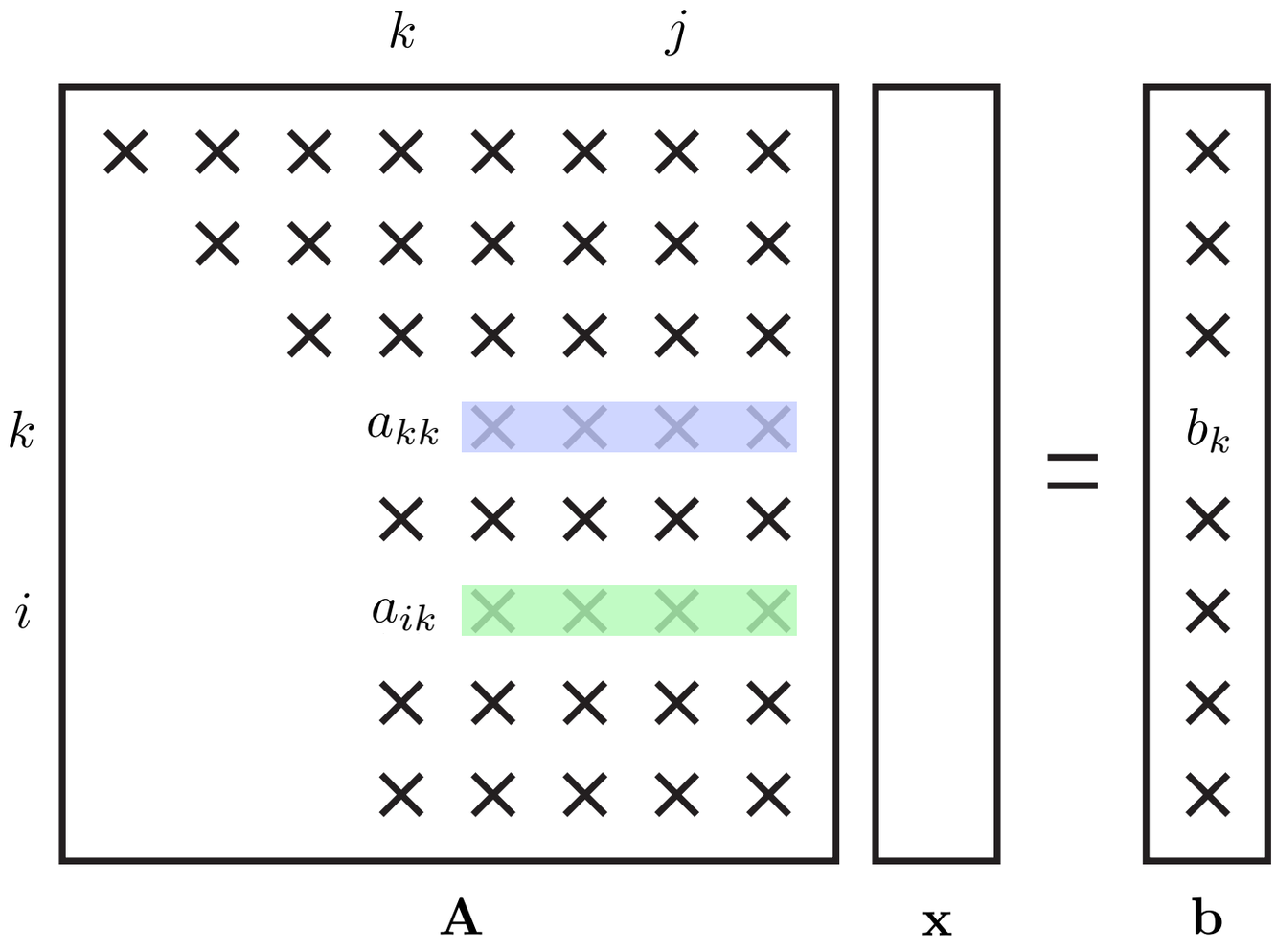
one step of the solution phase



for k = 1 **to** n–1
  for i = k+1 **to** n
    r = a_{ik} / a_{kk}
    for j = k+1 **to** n
      a_{ij} += –r a_{kj}
    b_i = b_i – r b_k
    a_{ik} = 0

for k = n **to** 1
  s = b_k
  for j = (k+1) **to** n
    s += –a_{kj} x_j
  x_k = s / a_{kk}

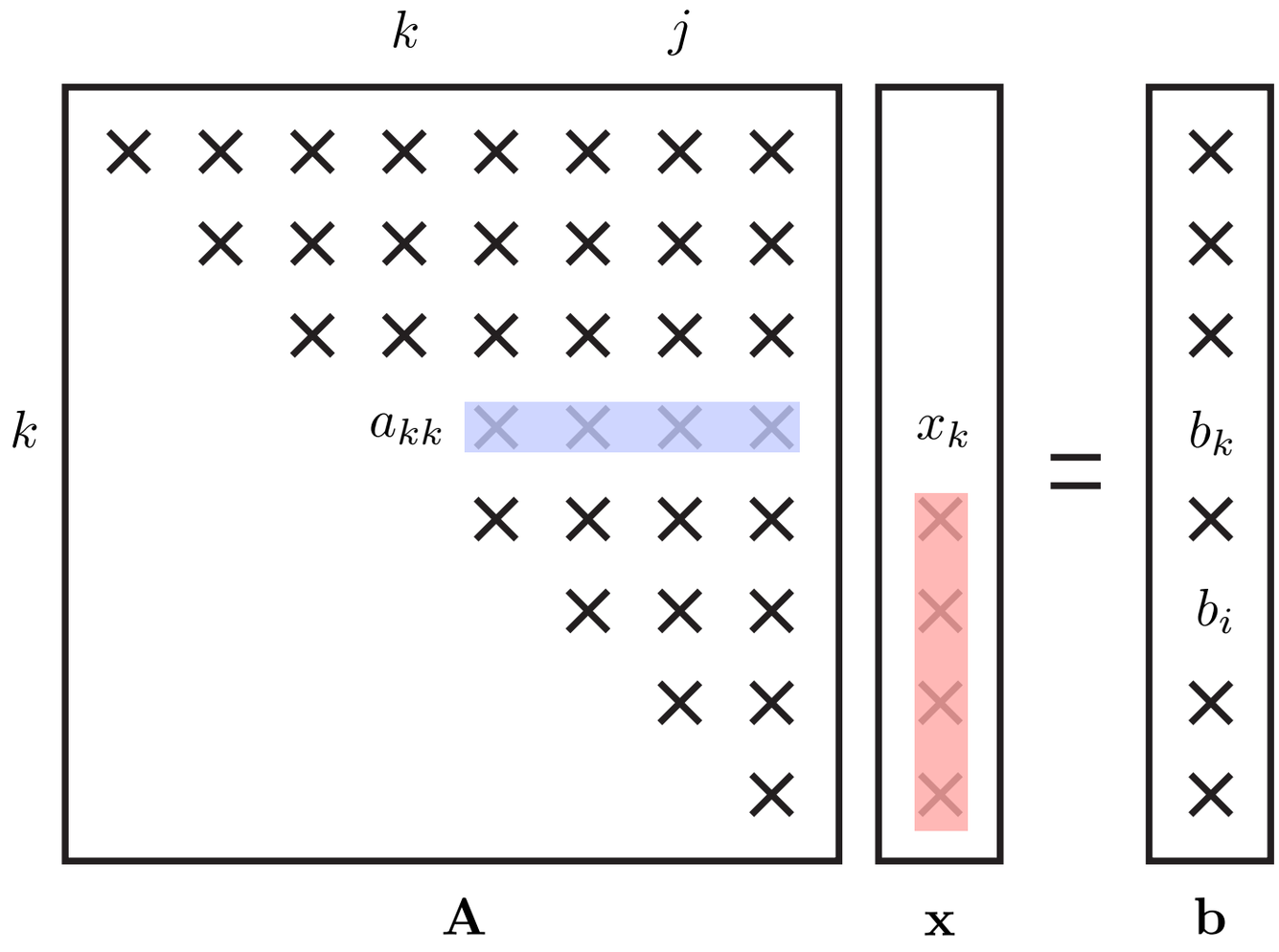
one step of the elimination phase: vector



for k = 1 **to** n–1
  for i = k+1 **to** n
    r = a_{ik} / a_{kk}
    $a_{i,k+1:n}$ += –r $a_{k,k+1:n}$
    $b_i = b_i$ – r b_k
    $a_{ik} = 0$

for k = n **to** 1
  s = b_k – $a_{k,k+1:n}$ • $x_{k+1:n}$
  x_k = s / a_{kk}

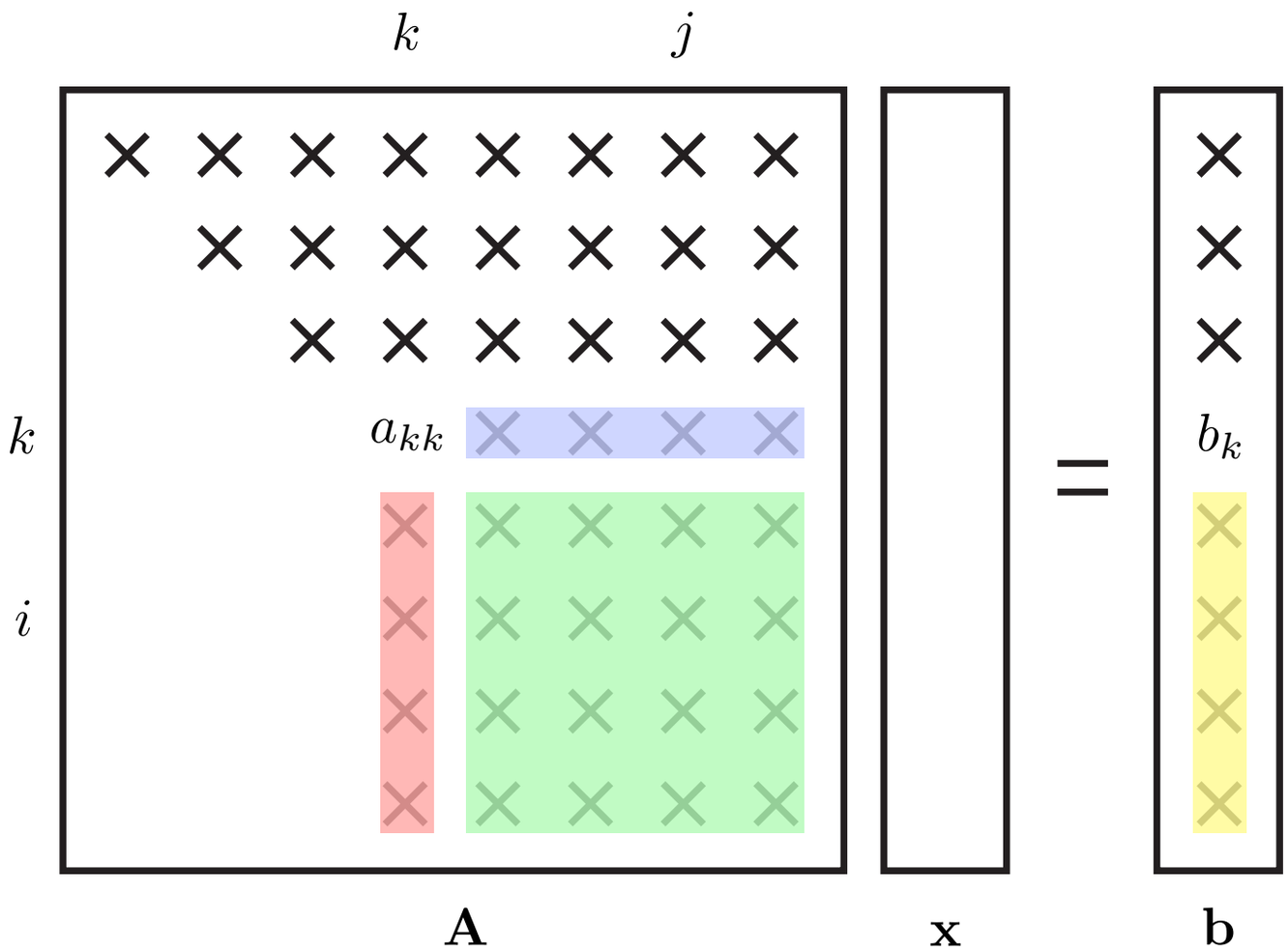
one step of the solution phase: vector



for k = 1 **to** n–1
  for i = k+1 **to** n
    r = a_{ik} / a_{kk}
    $a_{i,k+1:n}$ += –r $a_{k,k+1:n}$
    b_i = b_i – r b_k
    a_{ik} = 0

for k = n **to** 1
  s = b_k – $a_{k,k+1:n}$ • $x_{k+1:n}$
  x_k = s / a_{kk}

one step of the elimination phase: matrix



for k = 1 **to** n–1

B = $a_{k+1:n,k}$ • $a_{k,k+1:n}$

$a_{k+1:n,k+1:n}$ –= B / a_{kk}

$b_{k+1:n}$ += –$a_{k+1:n,k}$ / a_{kk}

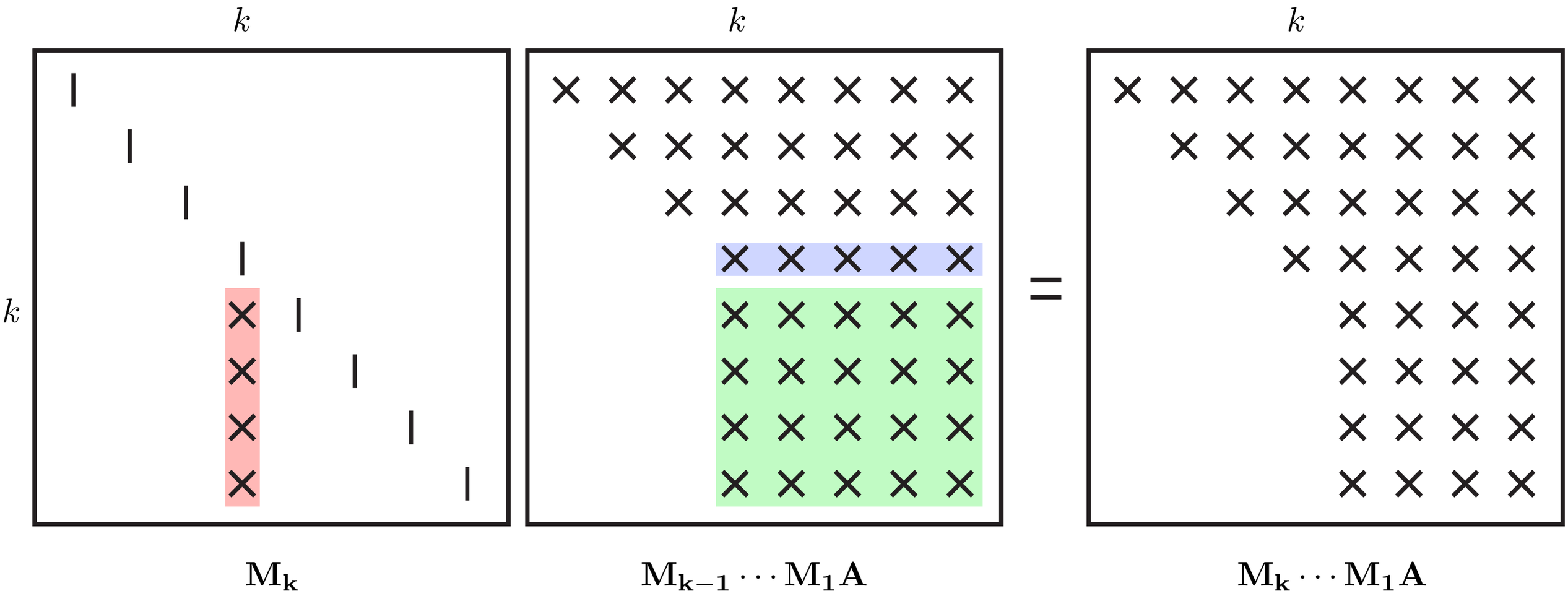
$a_{k+1:n,k}$ = 0

for k = n **to** 1

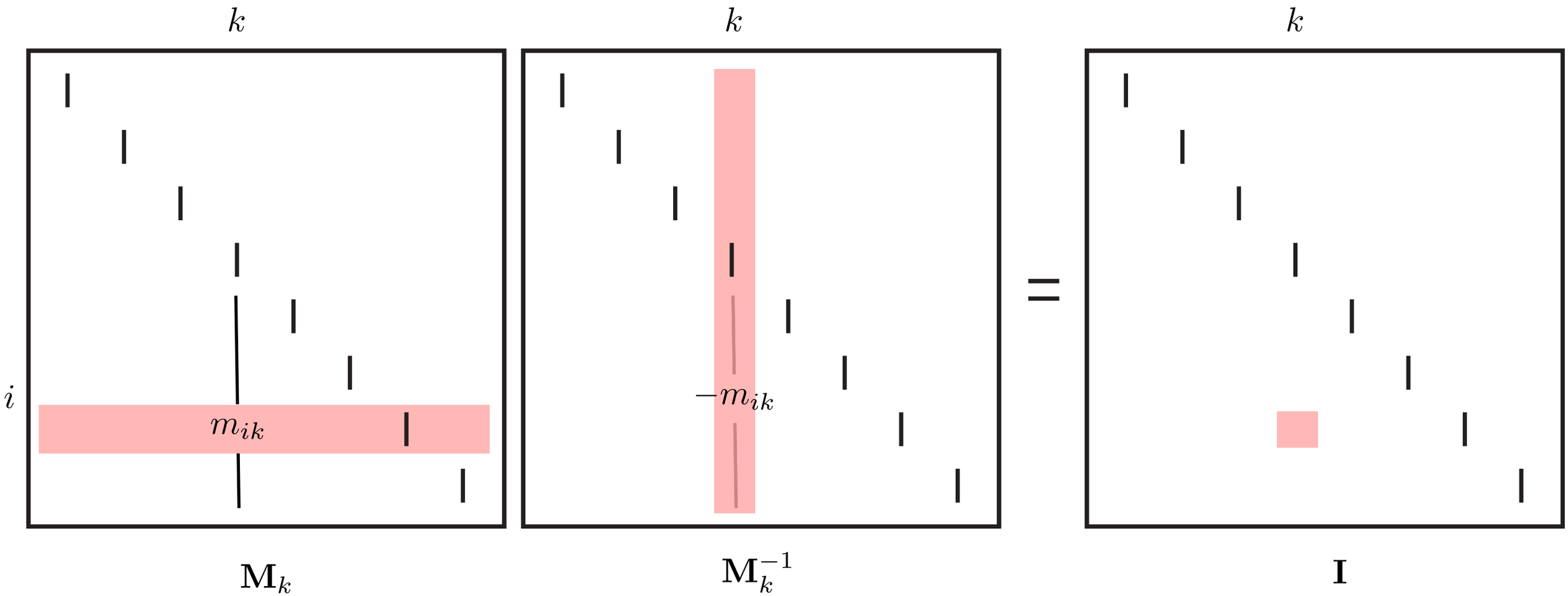
s = b_k – $a_{k,k+1:n}$ • $x_{k+1:n}$

x_k = s / a_{kk}

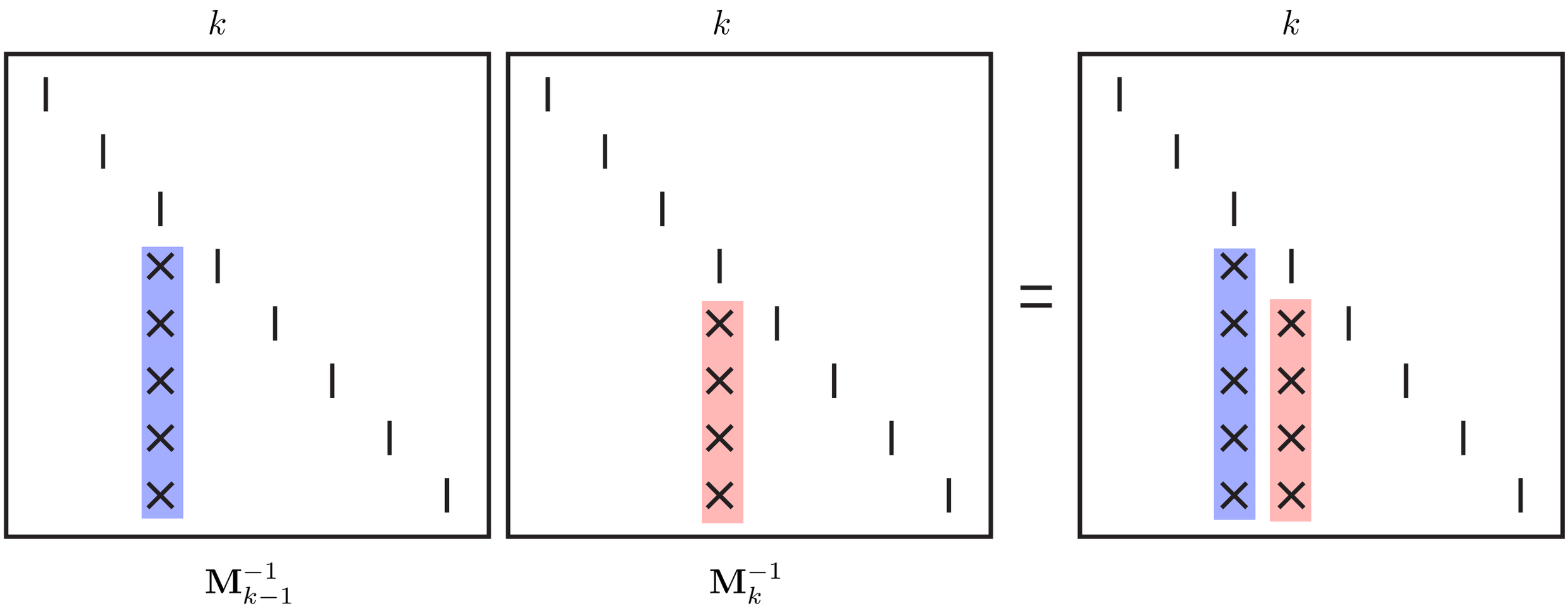
elimination step as matrix multiplication



inverting elimination matrices



multiplying the multipliers

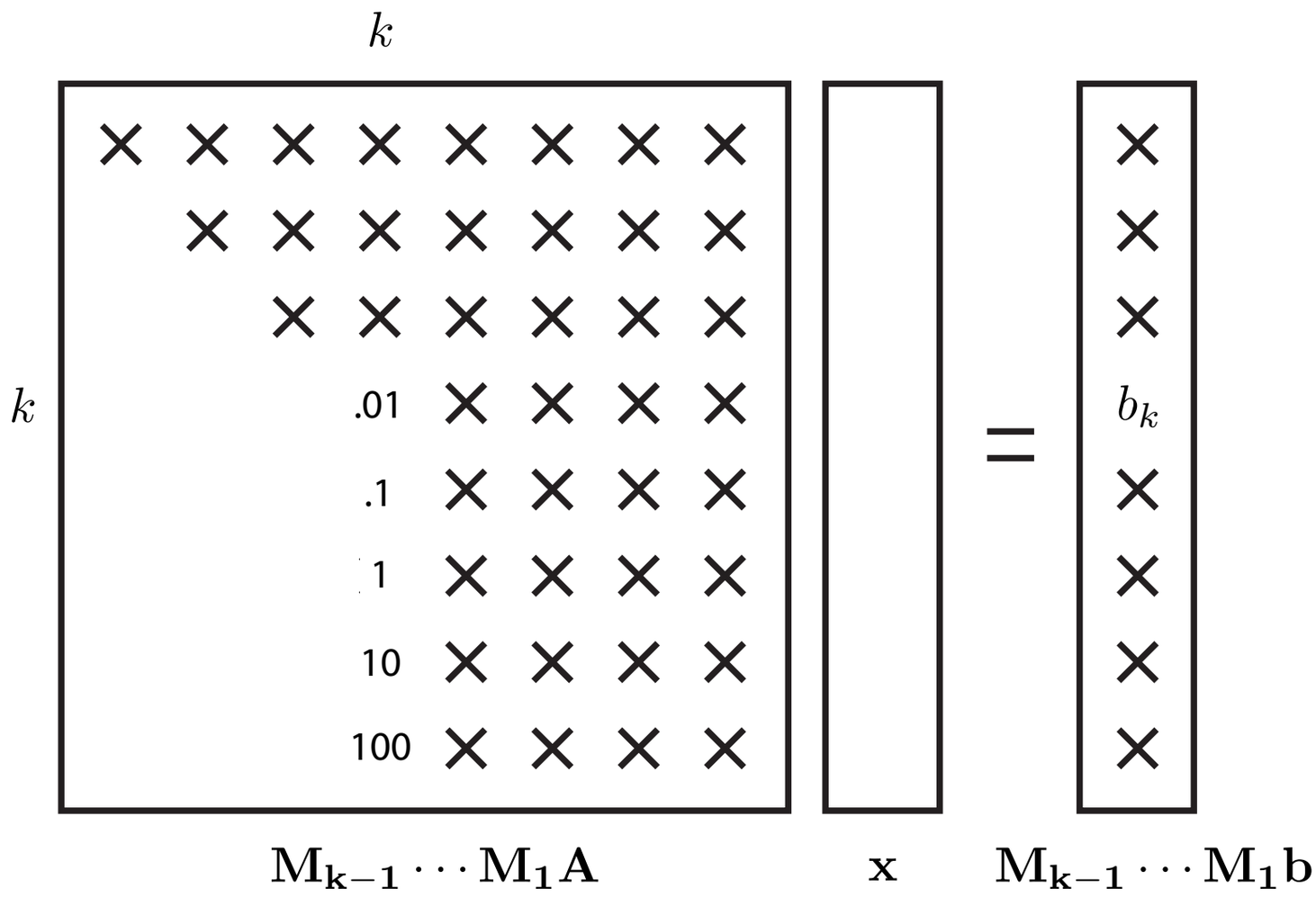


matrix full of multipliers

$$L = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ | & | & \ddots & & \\ \mathbf{m}_1 & \mathbf{m}_2 & 1 & & \\ | & | & | & 1 & \\ & & \mathbf{m}_{n-2} & \mathbf{m}_{n-1} & 1 \end{bmatrix}$$

pivoting

multipliers will be 10, 100, 1000, 10000—not good!



pivoting

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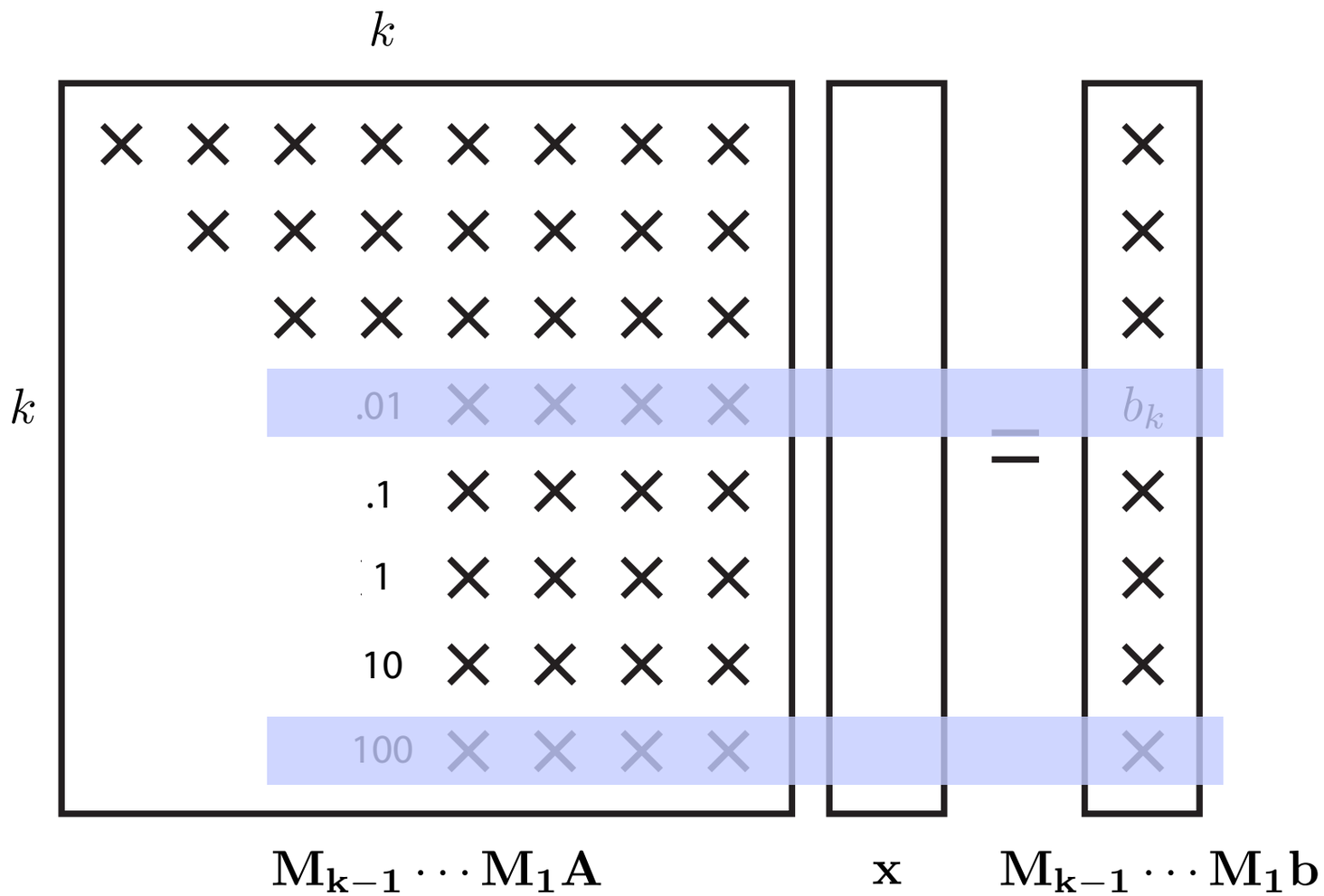
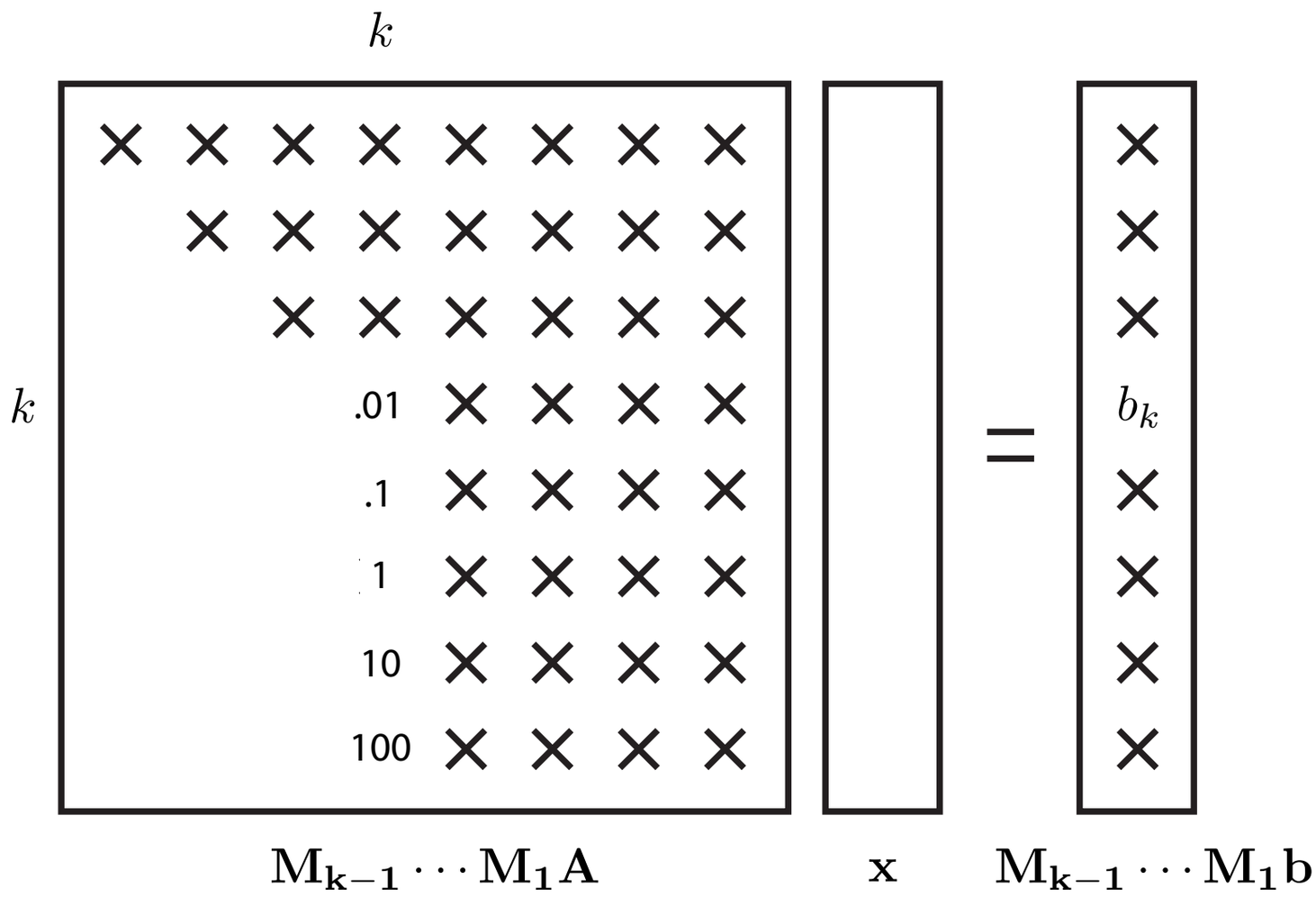


solution:
swap these rows

pivot vector

don’t actually swap rows—just renumber them.



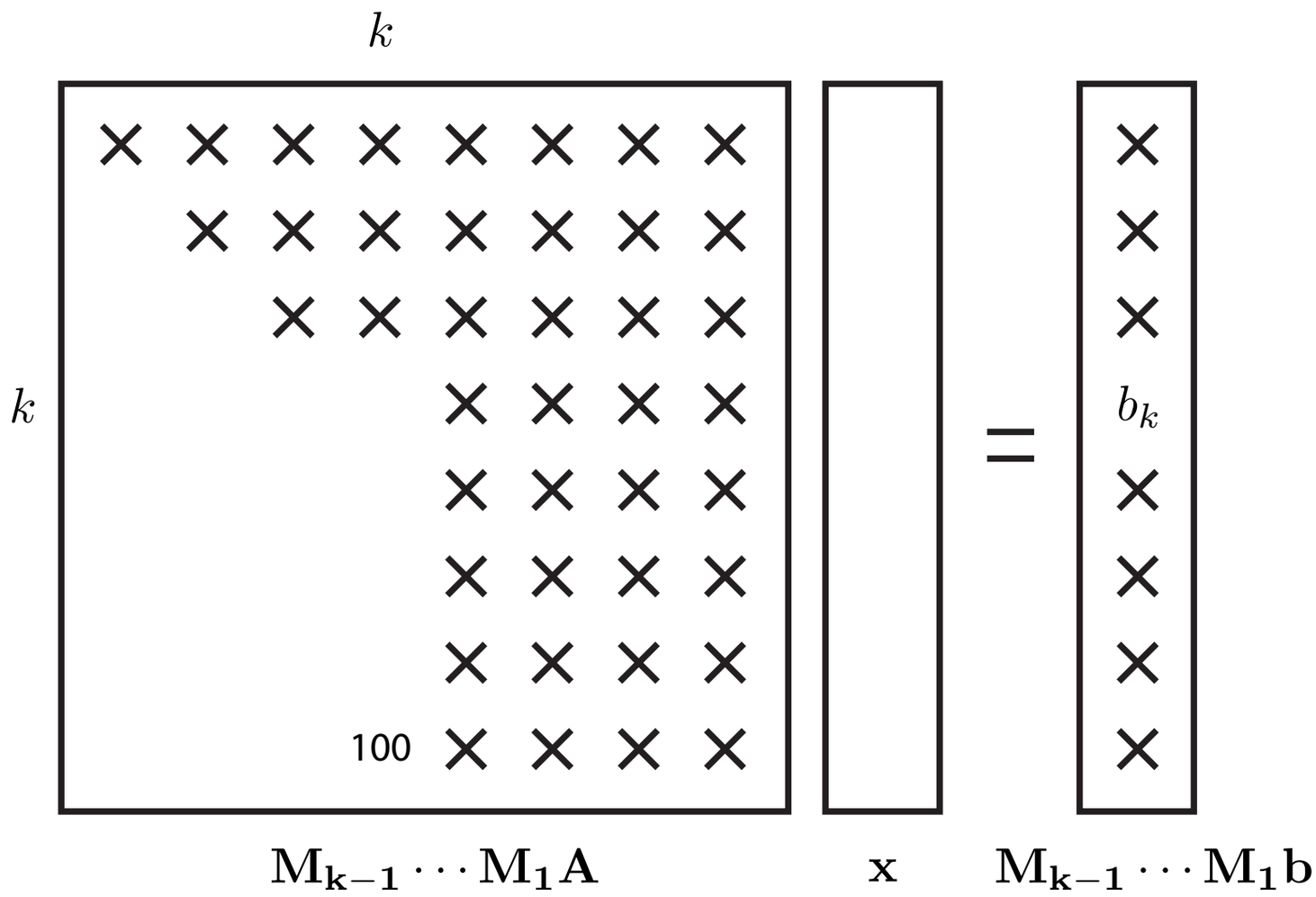
$\mathbf{p} = [1, 2, 3, 4, 5, 6, 7, 8]$

$\mathbf{p} = [1, 2, 3, 8, 5, 6, 7, 4]$

$\mathbf{p} = [1, 2, 3, 8, 7, 4, 5, 6]$

pivot vector

don’t actually swap rows—just renumber them.



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pivot vector

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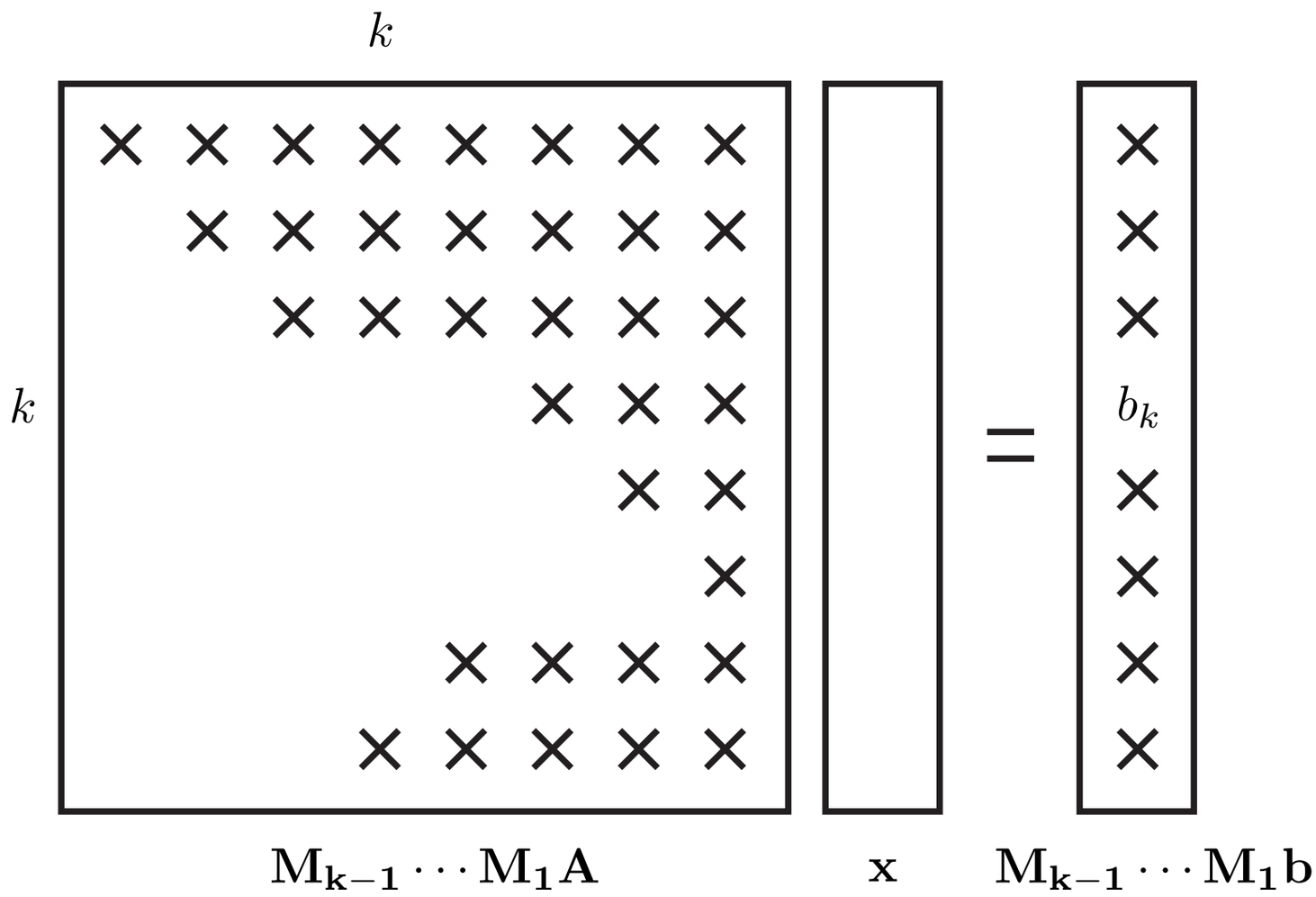

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