

CS 3110

Lecture 8: Closures

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Today's music: Selections from *Doctor Who* soundtracks, series 5-7

Review

Dynamic semantics:

- how expressions evaluate
- *substitution model*: substitute value for variable in let expressions, function calls, etc.
- *environment model*: maintain a data structure that binds variables to values

Today:

- semantics of function calls in environment model

Question #1

How much of PS2 have you finished?

- A. None
- B. About 25%
- C. About 50%
- D. About 75%
- E. I'm done!!!

Review: the core of OCaml

Essential sublanguage of OCaml:

```
e ::= c | (op) | x | (e1, ..., en)
      | C e
      | e1 e2
      | fun x -> e
      | let x = e1 in e2
      | match e0 with pi -> ei
```

Missing, unimportant: records, lists, options, declarations, patterns in function arguments and let bindings, **if**

Missing, important: **rec**

Review: evaluation

- Expressions evaluate to values in a dynamic environment

$$\text{env} :: e \dashrightarrow v$$

- Evaluation is meaningless if expression does not **type check**
- Values are a *syntactic subset* of expressions:

$$\begin{aligned} v ::= & c \mid (\text{op}) \mid (v_1, \dots, v_n) \\ & \mid C v \\ & \mid \text{fun } x \rightarrow e \end{aligned}$$

Review: function values

Anonymous functions **fun** **x**-> **e** are values

env :: (fun x -> e) --> (fun x -> e)

Review: let expressions

To evaluate `let x = e1 in e2` in environment `env`

Evaluate the binding expression `e1` to a value `v1` in environment `env`

$$\text{env} :: e1 \dashrightarrow v1$$

Extend the environment to bind `x` to `v1`

$$\text{env}' = \text{env} + \{x=v1\}$$

(newer bindings temporarily *shadow* older bindings)

Evaluate the body expression `e2` to a value `v2` in environment `env'`

$$\text{env}' :: e2 \dashrightarrow v2$$

Return `v2`

Function application v1.0

To evaluate $e1\ e2$ in environment env

Evaluate $e2$ to a value $v2$ in environment env

$env :: e2 \rightarrow v2$

Note: right to left order, like tuples, which matters in the presence of side effects

Evaluate $e1$ to a value $v1$ in environment env

$env :: e1 \rightarrow v1$

Note that $v1$ must be a function value $fun\ x \rightarrow e$

*because function application type checks**

Extend environment to bind formal parameter x to actual value $v2$

$env' = env + \{x=v2\}$

Evaluate body e to a value v in environment env'

$env' :: e \rightarrow v$

Return v

*Or a built-in operator (op). In which case, immediately apply (op) to $v2$ and return result.

Function application rule v1.0

If $\text{env} :: e_2 \dashrightarrow v_2$

and $\text{env} :: e_1 \dashrightarrow (\text{fun } x \rightarrow e)$

and $\text{env} + \{x=v_2\} :: e \dashrightarrow v$

then $\text{env} :: e_1 e_2 \dashrightarrow v$

Function application example

```
let f = fun x -> x in f 0 --> 0
```

1. Evaluate binding expression **fun x->x** to a value in empty environment
 - (already is a value)
2. Extend environment to bind **f** to **fun x->x**
3. Evaluate let-body expression **f 0** in environment **env = {f=fun x->x}**
 1. Evaluate argument **0** to a value
 - (already is a value)
 2. Evaluate **f** to a value
 - By variable rule, **f** evaluates to **env(f)**, i.e., **fun x->x**
 3. Extend environment to map formal parameter **x** to actual value **0**, i.e., **env' = {f=(fun x->x), x=0}**
 4. Evaluate function body **x** to value
 - By variable rule, **x** evaluates to **env'(x)**, i.e., **0**
 5. Return **0**

Function application example

```
let f = fun x -> x in f 0 --> 0
```

Another way of expressing the previous slide:

1. By function value rule,
 $\{\} :: \text{fun } x \rightarrow x \rightarrow \text{fun } x \rightarrow x$
2. By constant rule, $\{f = \text{fun } x \rightarrow x\} :: 0 \rightarrow 0$
3. By variable rule,
 $\{f = \text{fun } x \rightarrow x\} :: f \rightarrow \text{fun } x \rightarrow x$
4. By variable rule, $\{f = \text{fun } x \rightarrow x, x = 0\} :: x \rightarrow 0$
5. By function application rule with 2 and 3 and 4,
 $\{f = \text{fun } x \rightarrow x\} :: f \ 0 \rightarrow 0$
6. By **let** rule with 1 and 5,
 $\text{let } f = \text{fun } x \rightarrow x \text{ in } f \ 0 \rightarrow 0$

Hard example

```
let x = 1 in
let f = fun y -> x in
let x = 2 in
  f 0
```

What does our dynamic semantics say it evaluates to?

What does OCaml say?

What do YOU say?

Question #2

What do you think this expression should evaluate to?

```
let x = 1 in
```

```
let f = fun y -> x in
```

```
let x = 2 in
```

```
  f 0
```

A. 1

B. 2

Hard example: OCaml

What does OCaml say this evaluates to?

```
let x = 1 in
```

```
let f = fun y -> x in
```

```
let x = 2 in
```

```
  f 0
```

```
- : int = 1
```

Hard example: our semantics

What does our semantics say?

```
let x = 1 in
{x=1} let f = fun y -> x in
{x=1, f=fun y->x} let x = 2 in
  {x=2, f=fun y->x} f 0
```

$\{x=2, f=fun\ y \rightarrow x\} :: f\ 0 \dashrightarrow ?$

1. Evaluate `0` to a value, i.e., `0`
2. Evaluate `f` to a value, i.e., `fun y->x`
3. Extend environment to map parameter:
 $\{x=2, f=(fun\ y \rightarrow x), y=0\}$
4. Evaluate body `x` in that environment
5. Return `2`

2 <> 1

Why different answers?

Two different rules for variable scope:

- Rule of *dynamic scope* (our semantics)
- Rule of *lexical scope* (OCaml)

Dynamic scope

Rule of dynamic scope: The body of a function is evaluated in the current dynamic environment at the time the function is **called**, not the old dynamic environment that existed at the time the function was defined.

- Causes our semantics to use latest binding of **x**
- Thus return 2

Lexical scope

Rule of lexical scope: The body of a function is evaluated in the old dynamic environment that existed at the time the function was **defined**, not the current environment when the function is called.

- Causes OCaml to use earlier binding of **x**
- Thus return 1

Lexical scope

Rule of
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called.

- Cause
- Thus



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Scope

Rule of dynamic scope: The body of a function is evaluated in the current dynamic environment at the time the function is **called**, not the old dynamic environment that existed at the time the function was defined.

- Causes our semantics to use latest binding of **x**
- Thus return 2

Rule of lexical scope: The body of a function is evaluated in the old dynamic environment that existed at the time the function was **defined**, not the current environment when the function is called.

- Causes OCaml to use earlier binding of **x**
- Thus return 1

(In both, environment is extended to map formal parameter to actual value.)

Why would you want one vs. the other? Let's come back to that...

Implementing time travel

Q: How can functions be evaluated in old environments?

A: The language implementation keeps them around as necessary

- A function value is really a data structure that has **two parts**:
 - The **code** (obviously)
 - The **environment** that was current when the function was defined
 - Gives meaning to all the *free variables* of the function body
 - Like a “pair”
 - But you cannot access the pieces, or directly write one down in the language syntax
 - All you can do is call it
 - This data structure is called a *function closure*
- A function application:
 - evaluates the **code part** of the closure
 - in the **environment part** of the closure
 - extended to bind the function argument

Hard example revisited

```
(* 1 *) let x = 1
(* 2 *) let f = fun y -> x
(* 3 *) let x = 2
(* 4 *) let z = f 0
```

With lexical scope:

- Line 2 creates a closure and binds **f** to it:
 - Code: **fun y -> x**
 - Environment: **{x=1}**
- Line 4 calls that closure with **0** as argument
 - In function body, **y** maps to **0** and **x** maps to **1**
- So **z** is bound to **1**

Question #3

```
(* 1 *) let x = 1
(* 2 *) let f y = x + y
(* 3 *) let x = 3
(* 4 *) let y = 4
(* 5 *) let z = f (x + y)
```

What value does **z** have with lexical scope?

- A. 1
- B. 5
- C. 7
- D. 8
- E. 10

Question #3

```
(* 1 *) let x = 1
(* 2 *) let f y = x + y
(* 3 *) let x = 3
(* 4 *) let y = 4
(* 5 *) let z = f (x + y)
```

- Line 2 creates a closure and binds **f** to it:
 - Code: **fun y -> x+y**
 - Environment: **{x=1}**
- Line 5 calls that closure with **7** as argument
 - In function body, **x** maps to **1** and **y** maps to **7**
- So **z** is bound to **8**

Question #3

```
(* 1 *) let x = 1
(* 2 *) let f y = x + y
(* 3 *) let x = 3
(* 4 *) let y = 4
(* 5 *) let z = f (x + y)
```

What value does **z** have with lexical scope?

- A. 1
- B. 5
- C. 7
- D. 8**
- E. 10

Question #4

```
(* 1 *) let x = 1
(* 2 *) let f y = x + y
(* 3 *) let x = 3
(* 4 *) let y = 4
(* 5 *) let z = f (x + y)
```

What value does **z** have with **dynamic** scope?

- A. 1
- B. 5
- C. 7
- D. 8
- E. 10

Question #4

```
(* 1 *) let x = 1
(* 2 *) let f y = x + y
(* 3 *) let x = 3
(* 4 *) let y = 4
(* 5 *) let z = f (x + y)
```

- At line 5, environment is $\{x=3, y=4\}$
- Line 5 calls f with argument 7
 - body of f is evaluated in current environment,
 - but with y bound to argument value 7
 - argument binding shadows previous binding
 - So x is 3 and y is 7 and result of call is 10
- Finally, z is bound to 10

Question #4

```
(* 1 *) let x = 1
(* 2 *) let f y = x + y
(* 3 *) let x = 3
(* 4 *) let y = 4
(* 5 *) let z = f (x + y)
```

What value does **z** have with dynamic scope?

- A. 1
- B. 5
- C. 7
- D. 8
- E. 10**

Closure notation

`<<code, environment>>`

e.g.,

`<<fun y -> x+y, {x=1}>>`

With lexical scoping, well-typed programs are guaranteed never to have any variables in the code body other than function argument and variables bound by closure environment.

Function application v2.0

To evaluate $e_1 \ e_2$ in environment env

Evaluate e_2 to a value v_2 in environment env

$env \ :: \ e_2 \ \rightarrow \ v_2$

Evaluate e_1 to a value v_1 in environment env

$env \ :: \ e_1 \ \rightarrow \ v_1$

Note that v_1 must be a function *closure* $\langle\langle \text{fun } x \rightarrow e, env' \rangle\rangle^*$

Extend *closure* environment to bind formal parameter x to actual value v_2

$env'' \ = \ env' \ + \ \{x=v_2\}$

Evaluate body e to a value v in environment env''

$env'' \ :: \ e \ \rightarrow \ v$

Return v

*Or a built-in operator (op). In which case, immediately apply (op) to v_2 and return result.

Function application rule v2.0

If $\text{env} :: e2 \dashrightarrow v2$

and $\text{env} :: e1 \dashrightarrow$

$\langle\langle \text{fun } x \rightarrow e, \text{env}' \rangle\rangle$

and $\text{env}' + \{x=v2\} :: e \dashrightarrow v$

then $\text{env} :: e1 e2 \dashrightarrow v$

Function values v2.0

Anonymous functions **fun x -> e** are **closures**

`env :: (fun x -> e) -->`

`<<fun x -> e, env>>`

Lexical vs. dynamic scope

- Consensus after decades of programming language design is that **lexical scope is the right choice**
- Dynamic scope is convenient in some situations
 - Some languages use it as the norm (e.g., Emacs LISP, LaTeX)
 - Some languages have special ways to do it (e.g., Perl, Racket)
 - But most languages just don't have it
- Exception handling resembles dynamic scope:
 - **raise e** transfers control to the “most recent” exception handler
 - like how dynamic scope uses “most recent” binding of variable

Why lexical scope?

1. Programmer can freely change names of local variables

```
(* 1 *) let x = 1
(* 2 *) let f y =
          let x = y + 1 in
          fun z -> x+y+z
(* 3 *) let x = 3
(* 4 *) let w = (f 4) 6
```

Lexical scope: evaluates to 15

Dynamic scope: evaluates to 13

```
(* 1 *) let x = 0
(* 2 *) let f y =
          let q = y + 1 in
          fun z -> q+y+z
(* 3 *) let x = 3
(* 4 *) let w = (f 4) 6
```

Lexical scope: evaluates to 15

Dynamic scope: run-time error

- (f 4) --> fun z -> q+y+z
- (fun z -> q+y+z) 6 --> q+y+6
- at line 4, env. doesn't bind q

Why lexical scope?

2. Type checker can prevent run-time errors

```
(* 1 *) let x = 1
(* 2 *) let f y =
          let x = y + 1 in
          fun z -> x+y+z
(* 3 *) let x = 3
(* 4 *) let w = (f 4) 6
```

Lexical scope: evaluates to 15

Dynamic scope: evaluates to 13

```
(* 1 *) let x = 0
(* 2 *) let f y =
          let x = y + 1 in
          fun z -> x+y+z
(* 3 *) let x = "hi"
(* 4 *) let w = (f 4) 6
```

Lexical scope: evaluates to 15

Dynamic scope: run-time error

- (f 4) --> fun z -> x+y+z
- (fun z -> x+y+z) 6 --> x+y+6
- at line 4, env. binds x to string; can't add string to int

Progress

```
e ::= c | (op) | x | (e1, ..., en)
    | C e
    | e1 e2
    | fun x -> e
    | let x = e1 in e2
    | match e0 with pi -> ei
```

But we still don't have **let rec**

Recursive functions

```
e ::= c | (op) | x | (e1, ..., en)
    | C e
    | e1 e2
    | fun x -> e
    | let x = e1 in e2
    | match e0 with pi -> ei
    | let rec f x = e1 in e2
```

Let rec expressions

To evaluate `let rec f x = e1 in e2` in environment `env`

don't evaluate the binding expression e1

Extend the environment to bind `f` to a *recursive closure*

`env' = env +`

`{ f = <<f, fun x -> e1, env >> }`

Evaluate the body expression `e2` to a value `v2` in environment `env'`

`env' :: e2 --> v2`

Return `v2`

Function application v3.0

To evaluate e_1 e_2 in environment env

Evaluate e_2 to a value v_2 in environment env

$env :: e_2 \rightarrow v_2$

Evaluate e_1 to a value v_1 in environment env

$env :: e_1 \rightarrow v_1$

Note that v_1 must be a *recursive* closure $c_1 = \langle\langle f, \text{fun } x \rightarrow e, env' \rangle\rangle$
or a closure $\langle\langle \text{fun } x \rightarrow e, env' \rangle\rangle$

Extend closure environment to bind formal parameter x to actual value v_2 and
(if present) function name f to the closure

$env'' = env' + \{x=v_2, f=c_1\}$

That's where the recursion happens: name is bound to "itself" inside call

Evaluate body e to a value v in environment env''

$env'' :: e \rightarrow v$

Return v

Closures in OCaml

```
clarkson@chardonnay ~/share/ocaml-4.02.0/  
bytecomp  
$ grep Kclosure *.ml  
bytegen.ml:          (Kclosure(lbl, List.length  
fv) :: cont)  
bytegen.ml:          (Kclosurerec(lbls,  
List.length fv) ::  
emitcode.ml:  | Kclosure(lbl, n) -> out  
opCLOSURE; out_int n; out_label lbl  
emitcode.ml:  | Kclosurerec(lbls, n) ->  
instruct.ml:  | Kclosure of label * int  
instruct.ml:  | Kclosurerec of label list * int  
printinstr.ml: | Kclosure(lbl, n) ->  
printinstr.ml: | Kclosurerec(lbls, n) ->
```


Closures in OCaml

- *Closure conversion* is an important phase of compiling many functional languages
- Expands on ideas we've seen here
 - Many optimizations possible
 - Especially, better handling of recursive functions

Closures in Java

- Nested classes can simulate closures
 - Used everywhere for Swing GUI!
<http://docs.oracle.com/javase/tutorial/uiswing/events/generalrules.html#innerClasses>
 - You've done it yourself already in 2110
- Java 8 adds higher-order functions and closures
- Can even think of OCaml closures as resembling Java objects:
 - closure has a single method, the code part, that can be invoked
 - closure has many fields, the environment part, that can be accessed

Closures in C

- In C, a *function pointer* is just a code pointer, period. No environment.
- To simulate closures, a common **idiom**:
Define function pointers to take an extra, explicit environment argument
 - But without generics, no good choice for type of list elements or the environment
 - Use `void*` and various type casts...
- From Linux kernel:
<http://lxr.free-electrons.com/source/include/linux/kthread.h#L13>

Please hold still for 1 more minute

WRAP-UP FOR TODAY

Upcoming events

- **PS2 is due tonight at 11:59 pm**
- Clarkson permanent(?) office hours:
Tuesday & Thursday 3-4 pm

This is closure.

THIS IS 3110