

# CS 3110

## Lecture 11: Documenting abstractions

Prof. Clarkson

Fall 2014

Today's music: *In C* by Terry Riley

# Review

## This week:

- Programming in the large
  - Modules, signatures, functors
  - Modularity, abstraction, specification
- Today:
  - Documentation for clients: specifications
  - Documentation for implementers: abstraction functions and representation invariants

# Question #1

How much of PS3 have you finished?

- A. None
- B. About 25%
- C. About 50%
- D. About 75%
- E. I'm done!!!

# PS3 Quadtree Requirements

- We left a lot *unspecified*
  - You get to make implementation choices
  - This was deliberate!
  - (Early drafts left even more unspecified)
- Programming assignments in 3110 become increasingly less specified
- Programming assignments in 4000-level classes are even less specified
- Programming projects IRL might be entirely unspecified
- Success in programming (and in life) depends increasingly on you:
  - figuring out what's important and concentrating on that
  - making reasonable and defensible choices
  - being creative 😊

# Review: example specification

```
val sort : ('a -> 'a -> int) -> 'a list -> 'a list
```

Sort a list in increasing order according to a comparison function. The comparison function must return 0 if its arguments compare as equal, a positive integer if the first is greater, and a negative integer if the first is smaller (see `Array.sort` for a complete specification). For example, `compare` is a suitable comparison function. The resulting list is sorted in increasing order. `List.sort` is guaranteed to run in constant heap space (in addition to the size of the result list) and logarithmic stack space.

# Review: example specification

- **One-line summary of behavior:** Sort a list in increasing order according to a comparison function.
- **Precondition:** The comparison function must return 0 if its arguments compare as equal, a positive integer if the first is greater, and a negative integer if the first is smaller (see `Array.sort` for a complete specification). For example, `compare` is a suitable comparison function.
- **Postcondition:** The resulting list is sorted in increasing order.
- **Promise about behavior:** `List.sort` is guaranteed to run in constant heap space (in addition to the size of the result list) and logarithmic stack space.

# What if you had to read the implementation?

```
list.ml
let l2 = chop n1 l in
let s1 = rev_sort n1 l in
let s2 = rev_sort n2 l2 in
rev_merge_rev s1 s2 []
and rev_sort n l =
  match n, l with
  | 2, x1 :: x2 :: _ ->
    if cmp x1 x2 > 0 then [x1; x2] else [x2; x1]
  | 3, x1 :: x2 :: x3 :: _ ->
    if cmp x1 x2 > 0 then begin
      if cmp x2 x3 > 0 then [x1; x2; x3]
      else if cmp x1 x3 > 0 then [x1; x3; x2]
      else [x3; x1; x2]
    end else begin
      if cmp x1 x3 > 0 then [x2; x1; x3]
      else if cmp x2 x3 > 0 then [x2; x3; x1]
      else [x3; x2; x1]
    end
  | n, l ->
    let n1 = n asr 1 in
    let n2 = n - n1 in
    let l2 = chop n1 l in
    let s1 = sort n1 l in
    let s2 = sort n2 l2 in
    rev_merge s1 s2 []
in
let len = length l in
if len < 2 then l else sort len l
;;

let sort = stable_sort;;
let fast_sort = stable_sort;;

-:--- list.ml      58% L263 (Tuareg merlin (default))
```

# Specifications

A **specification** is a contract between an implementer of an abstraction and a client of an abstraction

- Describes behavior of abstraction
- Clarifies responsibilities
- Makes it clear who to blame

An implementation **satisfies** a specification if it provides the described behavior

Many implementations can satisfy the same specification

- Client has to assume it could be any of them
- Implementer gets to pick one



# Benefits of abstraction by specification

- **Locality:** abstraction can be understood without needing to examine implementation
  - critical in implementing large programs
  - also important in implementing smaller programs in teams
- **Modifiability:** abstraction can be reimplemented without changing implementation of other abstractions
  - update standard libraries without requiring world to rewrite code
  - performance enhancements: write the simple slow thing first, then improve bottlenecks as necessary

# Good specifications

- **Sufficiently restrictive:** rule out implementations that wouldn't be useful to clients
  - common mistakes: not stating enough in preconditions, failing to identify when exceptions will be thrown, failing to specify behavior at boundary cases
- **Sufficiently general:** do not rule out implementations that would be useful to clients
  - common mistakes: writing operational specifications instead of definitional (saying how, not what), stating too much in a postcondition
- **Sufficiently clear:** easy for clients to understand behavior
  - common mistakes: verbosity, omission of details and examples, lack of structure
  - best case: client reads spec and comes away confused
  - worst case: client read spec, thinks they understand it, but they don't hence can't use abstraction correctly

Goal is to write specifications that are restrictive AND general AND clear

# When to write specifications

- Before implementation:
  - posing and answering questions about behavior clarifies what to implement
- During implementation:
  - as soon as a design decision is made, document it in a specification
- After implementation:
  - update specification during code revisions
  - a specification becomes obsolete only when the abstraction becomes obsolete

# Audience of specification

- Clients
  - Spec informs what they must guarantee (preconditions)
  - Spec informs what they can assume (postconditions)
- Implementers
  - Spec informs what they can assume (preconditions)
  - Spec informs what they must guarantee (postconditions)

But the spec isn't enough for implementers...

# Sets without duplicates

```
module ListSetNoDup : SET = struct
  (* the list may never have duplicates *)
  type 'a set = 'a list
  let empty = []
  let mem = List.mem
  let add x l =
    if mem x l then l else x :: l
  let size = List.length
end
```

# Sets with duplicates

```
module ListSetDup : SET = struct
  (* the list may have duplicates *)
  type 'a set = 'a list
  let empty = []
  let mem = List.mem
  let add x l = x :: l
  let size l = match l with
    [] -> 0
  | h::t -> size t +
                (if mem h t then 0 else 1 )
end
```

# Compare set implementations

- Both have the same representation type
  - `'a list`
- But they interpret values of that type differently
  - `[1;1;2]` is  $\{1,2\}$  in `ListSetDup`
  - `[1;1;2]` is not meaningful in `ListSetNoDup`
  - In both, `[1;2]` and `[2;1]` are  $\{1,2\}$
- interpretation differs because they make **different assumptions** about what values of that type can be:
  - passed into operations
  - returned from operations
- e.g.,
  - `[1;1;2]` can be passed into and returned from `ListSetDup`
  - `[1;1;2]` should not be passed into or returned from `ListSetNoDup`

## Question #2

Consider this implementation of *set union* with representation type ``a list`:

```
let union l1 l2 = l1 @ l2
```

Under which assumptions about representation type will that implementation be correct?

- A. There are no duplicates in lists
- B. There could be duplicates in lists
- C. Both A and B
- D. Neither A nor B



## Question #2

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# Representation type questions

- How to interpret the representation type as the data abstraction?

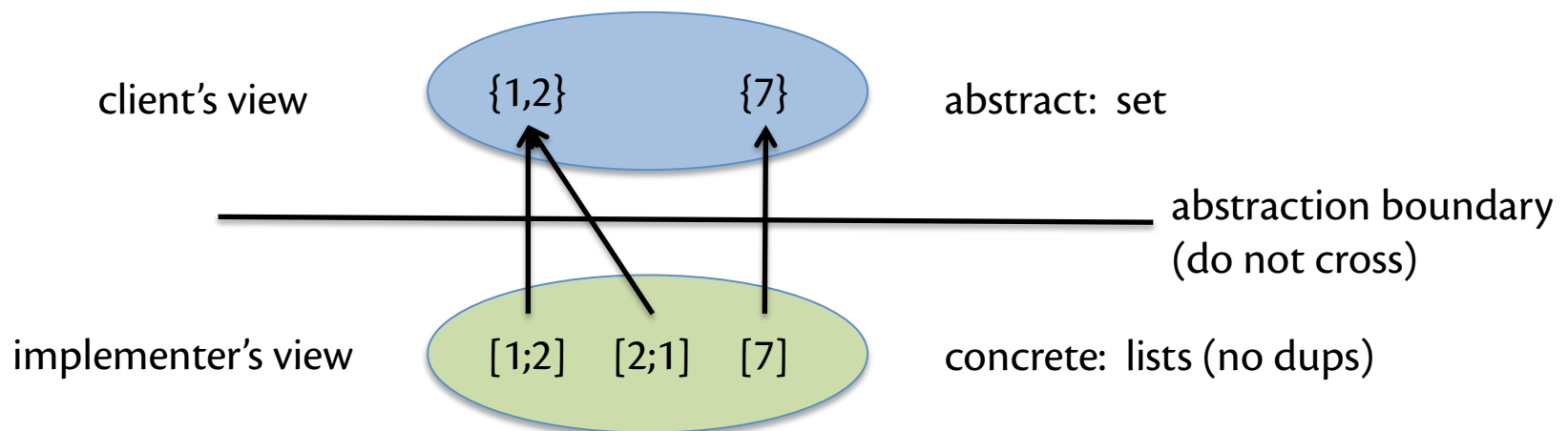
...abstraction function

- How to determine which values of representation type are meaningful?

...representation invariant

# Abstraction function

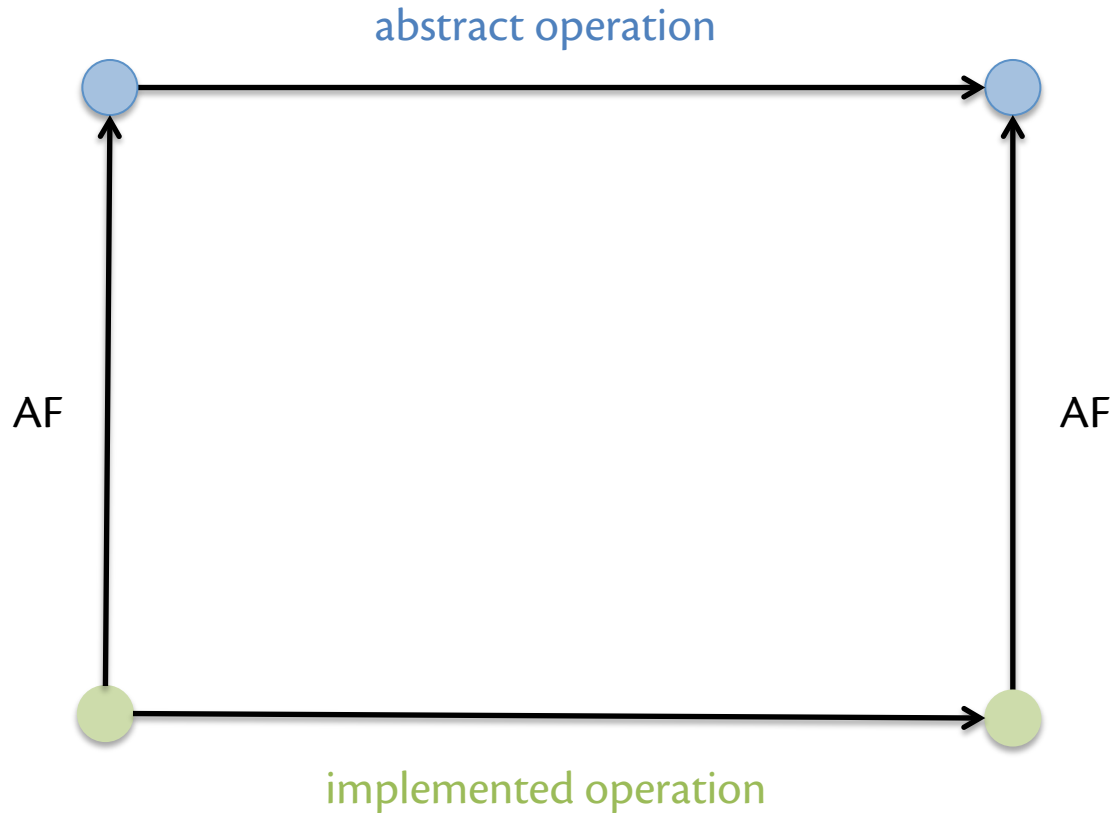
- **Abstraction function** (AF) captures designer's intent in choosing a particular representation of a data abstraction
- Not actually an OCaml function, but a mathematical function
- *Maps concrete values to abstract values*



# AF properties

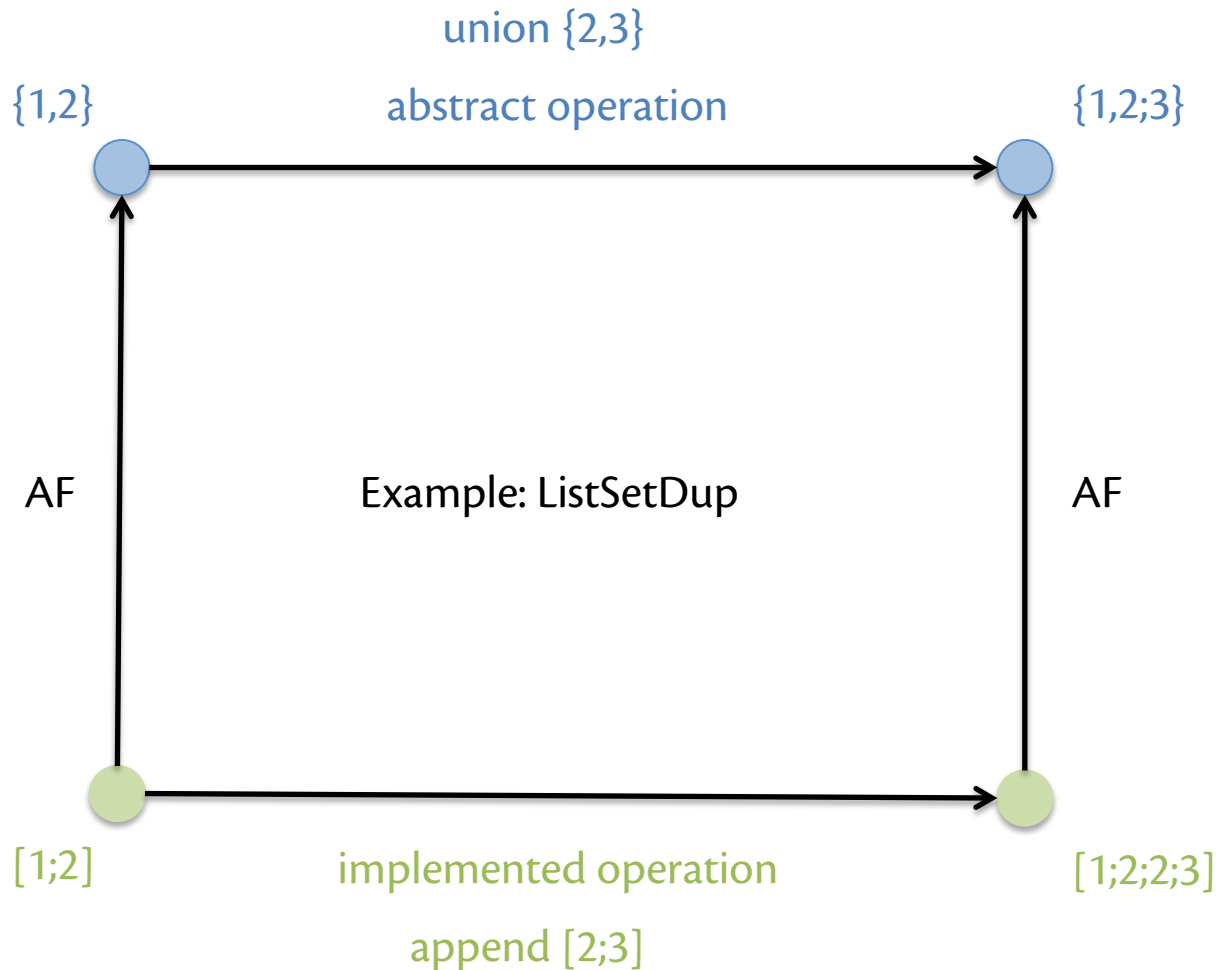
- *Many-to-one*: many values of concrete type can map to same value of abstract type
  - $[1 ; 2]$  maps to  $\{1,2\}$ , as does  $[2 ; 1]$
- *Partial*: some values of concrete type do not map to any value of abstract type
  - $[1 ; 1 ; 2]$  (in no dups) does not map to any set

# AF and operations



*commutative diagram:* both paths lead to the same place

# AF and operations



*commutative diagram: both paths lead to the same place*

# Correctness of operations

AF gives us a way to characterize correctness of operation implementation:

$$\text{op}_A(\text{AF}(c)) = \text{AF}(\text{op}_C(c))$$

“Commutative”: AF commutes with op

# Documenting AFs

```
module ListSetNoDup : SET = struct
  (* AF: the list [a1; ...; an] represents
   *   the set {a1,...,an}. [] represents
   *   the empty set. *)
  type 'a set = 'a list
  ...
end
module ListSetDup : SET = struct
  (* AF: the list [a1; ...; an] represents
   *   the smallest set containing the
   *   elements a1, ..., an. [] represents
   *   the empty set. *)
  type 'a set = 'a list
  ...
end
```



# Documenting AFs

- You might write:
  - Abstraction Function: *comment*
  - AF: *comment*
  - *comment*
- You write it FIRST
  - It's the number one decision you have to make while implementing a data abstraction
  - It dictates what fields are necessary in an object, or what values are necessary in a module
  - It gives meaning to representation
- (A large part of quadtree problem writeup on PS3 is explaining the AF! ...how to interpret regions, quadrants, etc.)

# Question #3

Which of the following are part of the AF for quadtrees?

- A. A quadtree is a data structure that provides a sparse representation of 2D space.
- B. The first quadtree in a Node is the north-eastern quadrant (I) of the region, ..., and the fourth is the south-eastern quadrant (IV).
- C. To find an object near point  $(x, y)$ , a quadtree is traversed starting from the root, walking down the appropriate sequence of child nodes that contain the point until a leaf node is reached.
- D. A Leaf is never separated if doing so would cause the size of the region to become too small.

# Question #3

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- D. A Leaf is never separated if doing so would cause the size of the region to become too small.

# Implementing AFs

- Mostly you don't
  - Would need to have an OCaml type for abstract values
  - If you had that type, you'd already be done...
- But sometimes you do
  - `string_of_X`
  - `toString()`
  - `Matrix.show`
  - ...all really useful for debugging
    - example: crypto library

# Representation invariant

- Recall: AF may be partial
  - [1;1;2] is not a valid ListSetNoDup
  - You might have decided some tuples don't represent valid quadtrees in your implementation
- **Representation invariant** characterizes which concrete values are *valid* and which are *invalid*
  - “Rep invariant” or RI for short
  - Valid concrete values will be mapped by AF to abstract values
  - Invalid concrete value will not be mapped by AF to abstract values
    - CANNOT meaningfully apply AF to values that don't satisfy RI
  - Many Piazza questions about quadtrees on PS3 are about RI
    - ...which we mostly let you define 😊

# Representation invariant

## Operations of data abstraction...

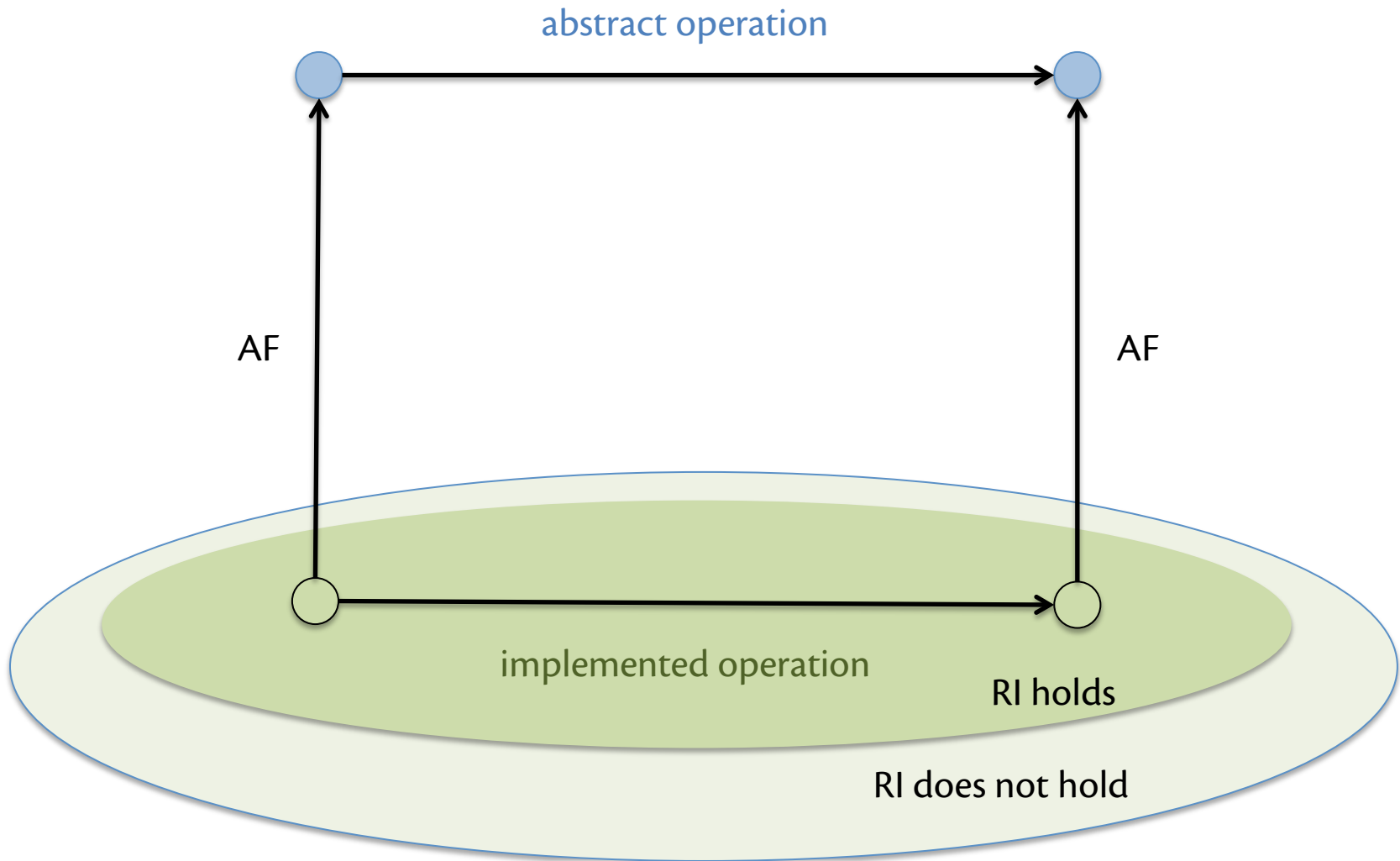
- Assume that any inputs from client satisfy RI
  - e.g., `ListSetNoDup` operations assume that values passed in contain no duplicates
- Internally might produce intermediate values that violate RI
  - e.g., `ListSetNoDup` could temporarily be processing a list with duplicates
- Must produce output to client that satisfy RI
  - e.g., `ListSetNoDup` operations produce values that contain no duplicates
- Hence RI is a **fact whose truth is *invariant*** except for exactly the period of time when data abstraction operates on values

# Representation invariant

Imagine a different person implements each operation of a data abstraction

- The RI is what they agree on
- Needs to express all the relevant constraints so the people don't have to talk to one another

# RI with commutative diagram





# Documenting RI

```
module ListSetNoDup : SET = struct
  (* AF: the list [a1; ...; an] represents
   *   the set {a1, ..., an}. [] represents
   *   the empty set. *)
  (* RI: the list contains no duplicates *)
  type 'a set = 'a list
end

module ListSetDup : SET = struct
  (* AF: the list [a1; ...; an] represents
   *   the smallest set containing the
   *   elements a1, ..., an. [] represents
   *   the empty set. *)
  type 'a set = 'a list
end
```

# Implementing the RI

- Great habit to cultivate
- Implement it EARLY, before any operations are implemented
- Common **idiom**: if RI fails then raise exception, otherwise return concrete value

```
let repOK (x: 'a list) : 'a list =  
  if has_dups x then raise "RI failure"  
  else x
```

- When debugging, check repOK on every input to an operation and on every output
- Effectively happened on PS2:
  - Matrices could be invalid
  - Operations were supposed to raise MatrixFailure
  - So your solutions were essentially checking a repOK!

# Checking the RI

```
module ListSetNoDup : SET = struct
  (* AF: ... *)
  (* RI: ... *)
  type 'a set = 'a list
  let repOK = ...
  let empty = repOK []
  let mem x l = List.mem x (repOK l)
  let add x l =
    if mem x (repOK l) then (repOK l)
    else repOK(x :: l)
  let size l = List.length (repOK l)
end
```

Funny story...this saved the CS 3110 tournament one year

# Checking the RI

- Can be expensive!
- For production code:
  - only check “cheap parts” of RI
  - change repOK to identity function, let compiler optimize call away
    - but keep the real repOK around in a comment!
  - use language feature for condition compilation (or various assertion libraries)

Please hold still for 1 more minute

**WRAP-UP FOR TODAY**

# Prelim 1

- Everything from day 1 through tomorrow's recitation (inclusive) is covered by Prelim 1
- Will post a sample prelim on Piazza
- Will have review session in recitation day before prelim
- Will cancel lecture day of prelim

# Upcoming events

- PS3 due Thursday

*This is invariant.*

**THIS IS 3110**