CS 2802: Homework 5

September 27, 2020

Handed out Sept. 28, due Oct. 5

- Read Chapter 9 (except for 9.10, which we won't cover, other than Fermat's Little Theorem, at the end)
- Do the following problems:
  - 9.3
  - 9.7
  - 9.8(a)
  - 9.11(a), (b), (e) [Hint: Theorem 9.2.2 is *really* useful here.] Note that 9.11(c) was proved in class as Corollary 3, and 9.11(d) follows from Corollary 4.
  - 9.30(i), (ii), (iii) (If the statement is true, provide a short proof; if it's false, give a counterexample.)
  - 9.47
  - Additional Problem 1: Recall the inductive definition of transitive closure of R given in class:
    - Suppose that R is a relation on S. Let $R_0 = R$.
    - Let $R_{n+1} = R_n \cup \{(s,t) : \exists u \in S((s,u),\, (u,t) \in R_n)\}$.
    - Let $R' = \cup_{n=0}^{\infty} R_n$.

    Prove that R' is the transitive closure of R.
  - Additional problem 2: Prove that if $a|b$ and $b|c$, then $a|c$ (so divisibility is transitive).
  - Additional problem 4: Suppose you can show that (a) $P(0)$ holds and (b) for all n, if $P(n)$ holds, then so does (i) $P(n+3)$ and (ii) $P(2n+1)$. What can you conclude? Formally, you have to find the smallest set that contains 0 and if it contains n, it also contains $n+3$ and $2n+1$. (Hint: Try some numbers and use a little number theory.) Prove whatever you claim carefully!
  - Challenge problem (don't hand it in): The sum of two positive integers is 2310. Can their product be divisible by 2310? (Hint: modular arithmetic is useful here. Also, there's nothing particularly special about 2310. Think about 35 or 77 first to get your intuitions in gear.)

For recitation: 9.11(a), 9.30, Additional problem 3