



Proof of  $\vdash \neg(P \wedge Q) \rightarrow (\neg P \vee \neg Q)$ :

$$\frac{\frac{\overline{\dots \vdash P \vee \neg P} \text{ excl mid} \quad \frac{\text{(see below)}}{\dots, P \vdash \neg P \vee \neg Q} \vee\text{elim} \quad \frac{\overline{\dots, \neg P \vdash \neg P} \text{ assum}}{\dots, \neg P \vdash \neg P \vee \neg Q} \vee\text{intro}}{\dots, \neg P \vdash \neg P \vee \neg Q} \vee\text{elim}}{\vdash \neg(P \wedge Q) \rightarrow (\neg P \vee \neg Q)} \rightarrow \text{intro}$$

$$\frac{\overline{\dots \vdash Q \vee \neg Q} \text{ excl mid} \quad \frac{\text{(see below)}}{\dots, Q \vdash \neg P \vee \neg Q} \text{ absurd} \quad \frac{\overline{\dots, \neg Q \vdash \neg Q} \text{ assum}}{\dots, \neg Q \vdash \neg P \vee \neg Q} \vee\text{intro}}{\dots, P \vdash \neg P \vee \neg Q} \vee\text{elim}$$

$$\frac{\frac{\overline{\dots, P \vdash P} \text{ assum} \quad \overline{\dots, Q \vdash Q} \text{ assum}}{\dots \vdash P \wedge Q} \wedge\text{intro} \quad \frac{\overline{\dots, \neg(P \wedge Q) \vdash \neg(P \wedge Q)} \text{ assum}}{\dots, Q \vdash \neg P \vee \neg Q} \text{ absurd}}{\dots, Q \vdash \neg P \vee \neg Q} \text{ absurd}$$