CS 2800: Discrete Structures

Spring 2016

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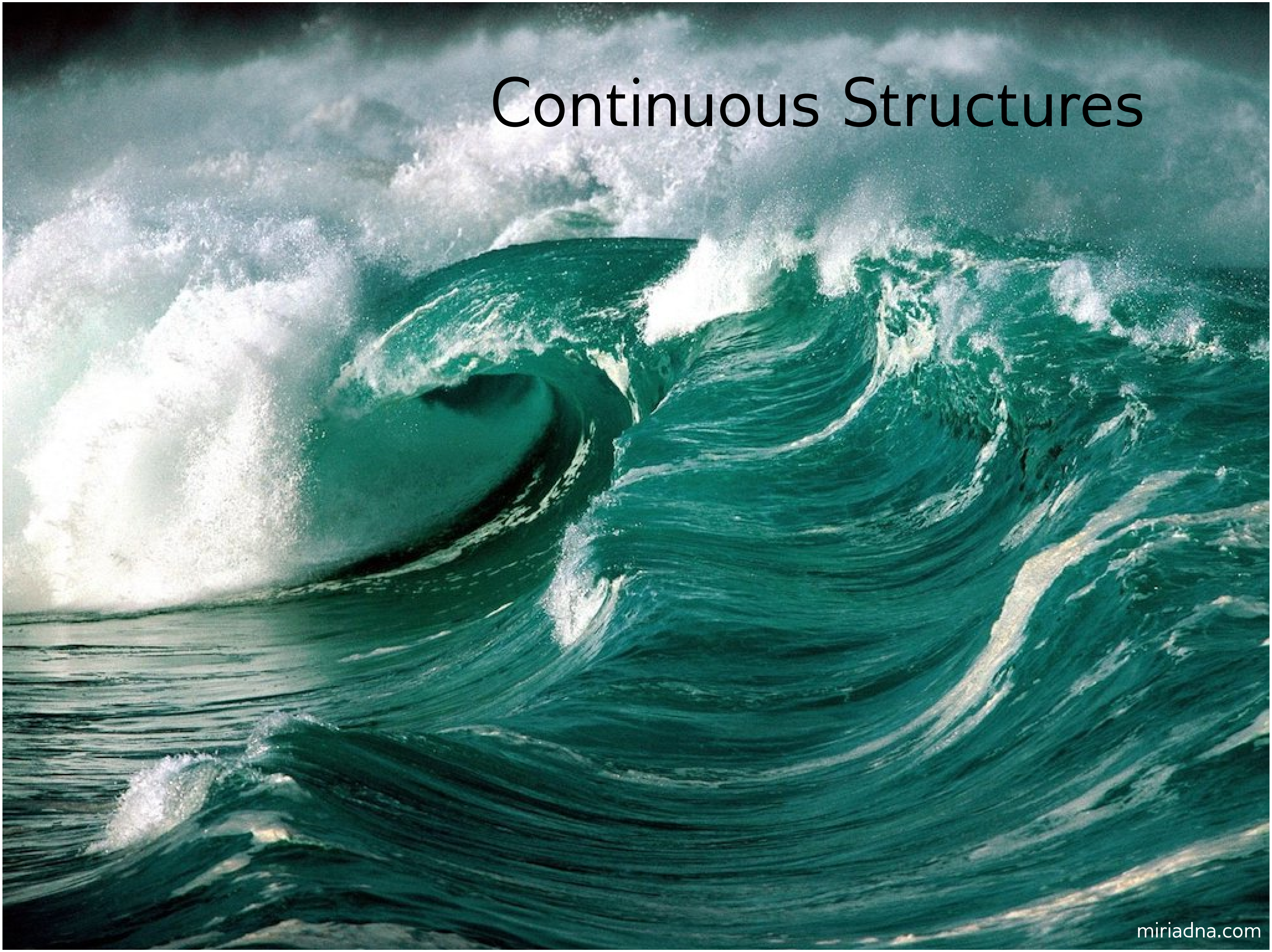
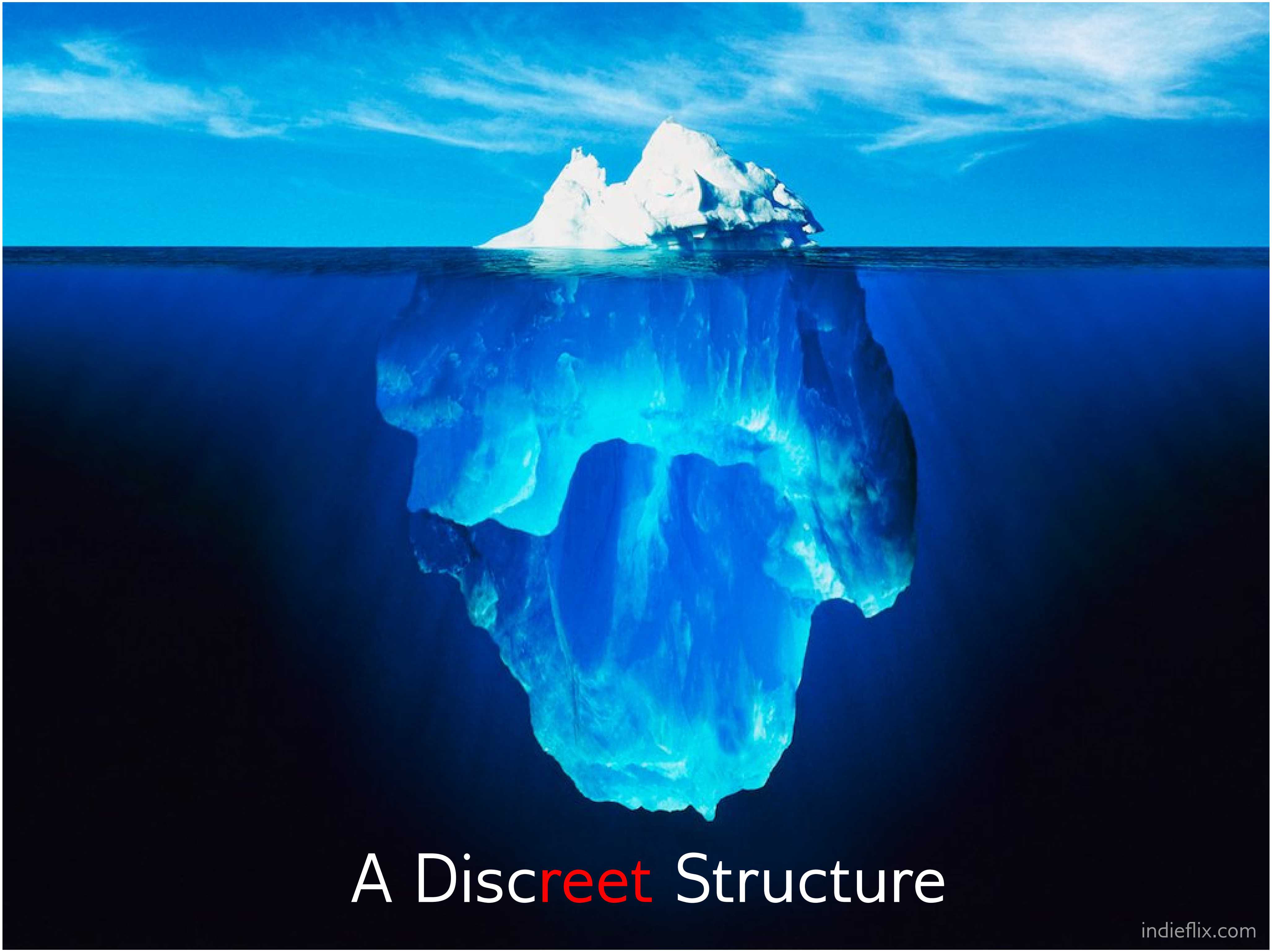
Joe Halpern

Slides largely taken from Sid Chaudhuri, with thanks.

Discrete Structures

Continuous Structures




A Discreet Structure
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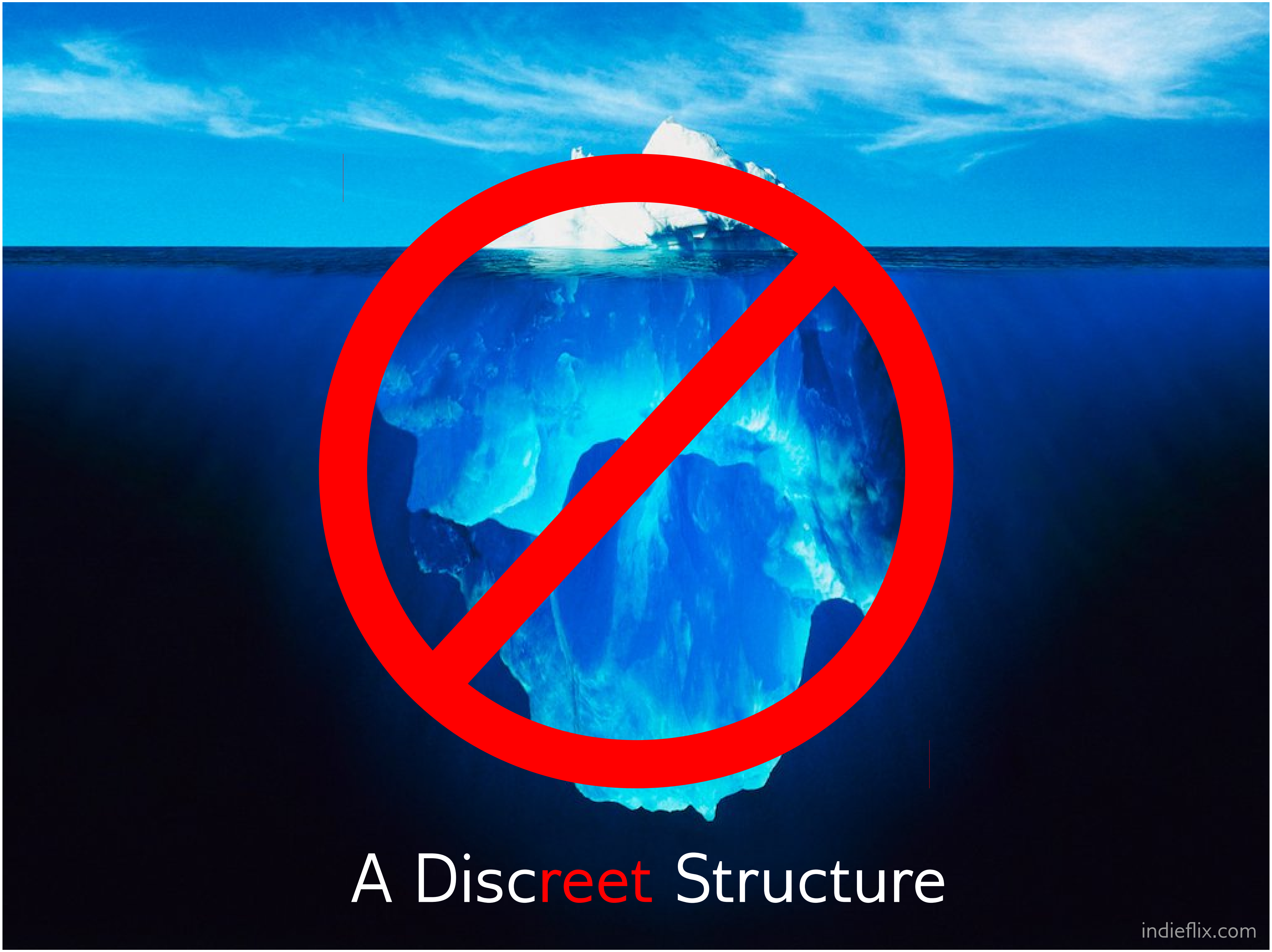
A Discreet Structure
indieflix.com

- discrete: individually separate and distinct
- discreet
  - careful and circumspect in one's speech or actions, especially in order to avoid causing offense or to gain an advantage.
  - intentionally unobtrusive.

Things we can count with the integers

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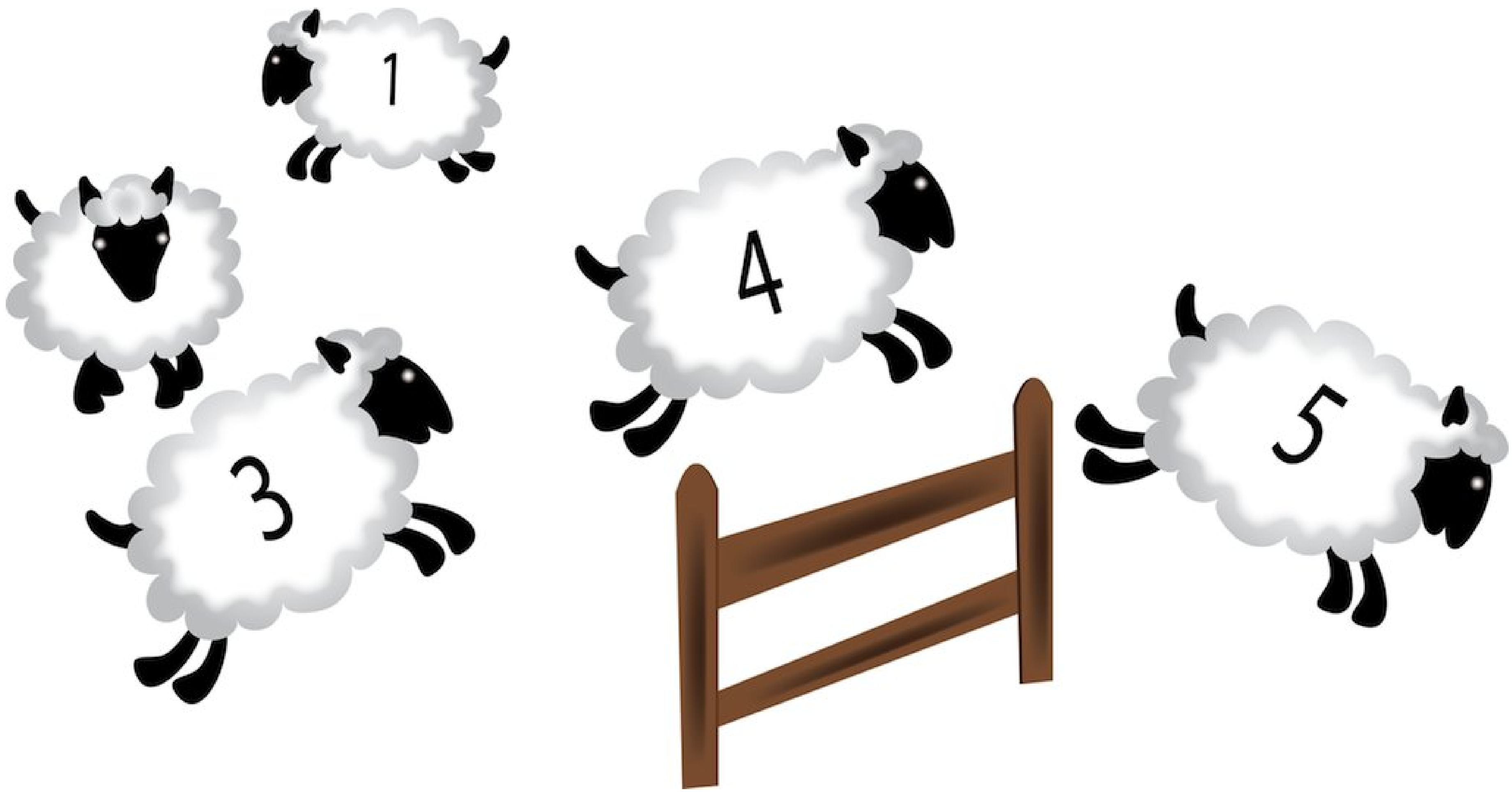
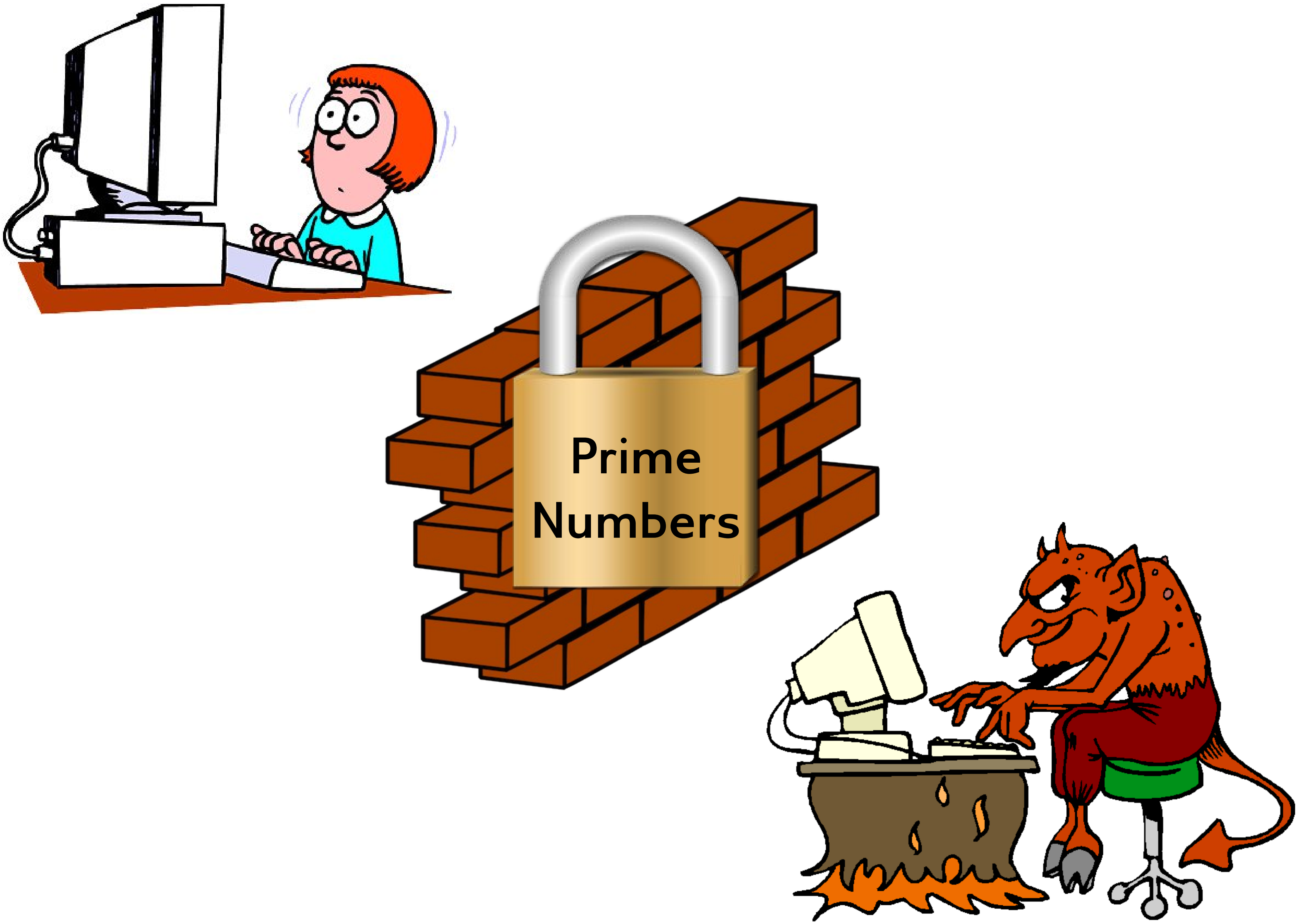
Things we can count with the integers



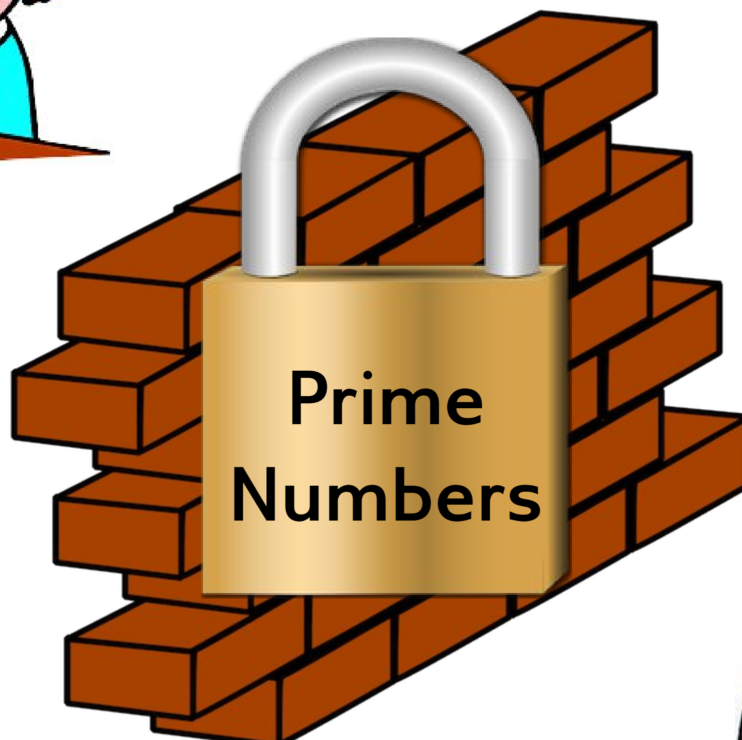
Prime Numbers

A number with exactly two divisors:
1 and **itself**

2, 3, 5, 7, 11, 13, 17...



eldhughes.com, pleasureinlearning.com


Prime
Numbers

How many prime numbers exist?

How many prime numbers exist?

1,000?

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1,000?

1,000,000?

How many prime numbers exist?

1,000?

1,000,000?

An infinite number?

How many prime numbers exist?

1,000?

1,000,000?

An infinite number

Euclid's Proof of Infinitude of Primes

(~300BC)

Euclid's Proof of Infinitude of Primes

- Suppose there is a finite number of primes

Euclid's Proof of Infinitude of Primes

- Suppose there is a finite number of primes
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Euclid's Proof of Infinitude of Primes

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- But no number from 2 to p divides n
- So n has a **prime divisor greater than p**

Contradiction!!!

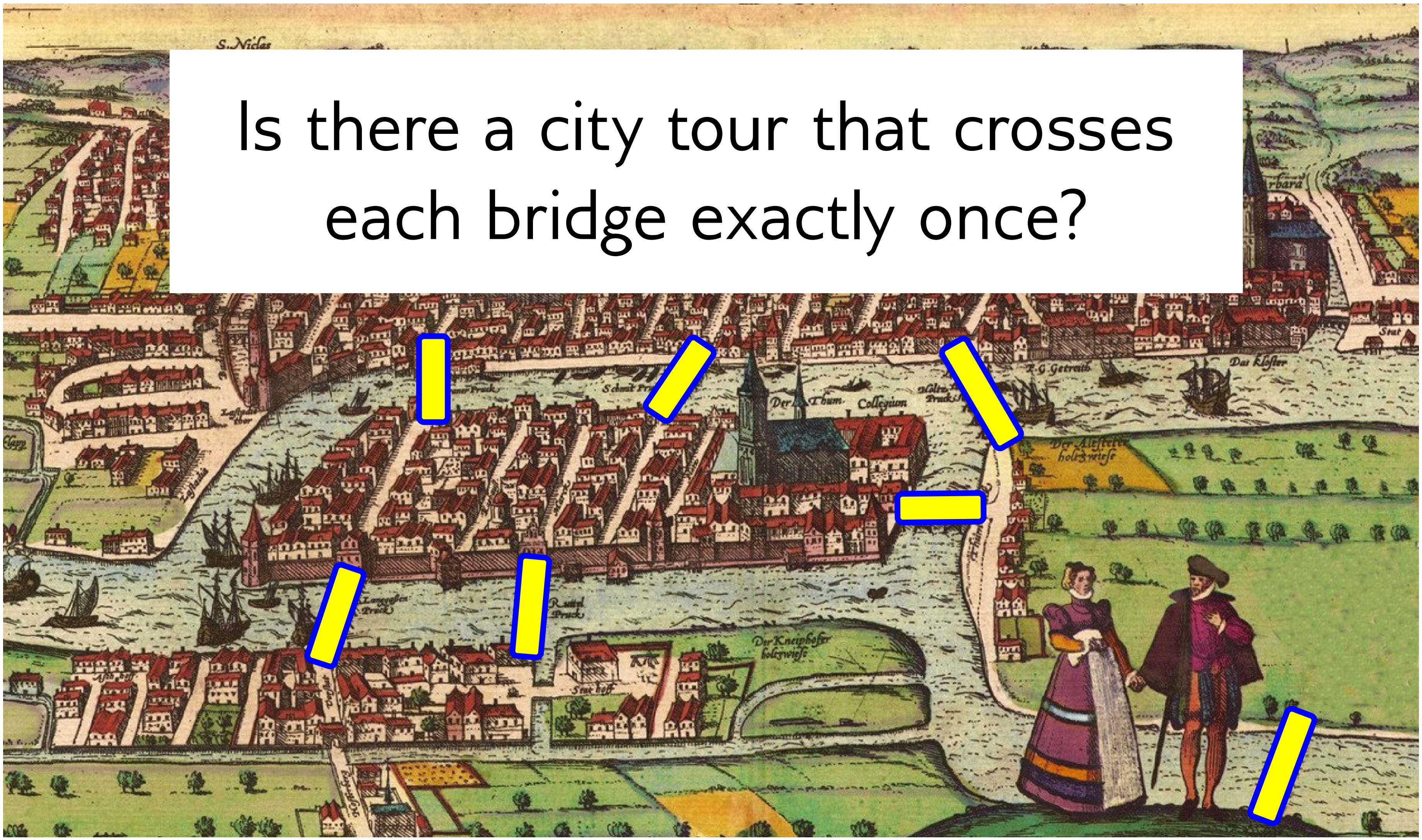
Discrete Structures

- Number theory
- Proof systems
- Sets, functions, relations
- Counting and probability

Bridges of Königsberg

Braun & Hogenberg, "Civitates Orbis Terrarum", Cologne 1585. Photoshopped to clean up right side and add 7th bridge.

Bridges of Königsberg



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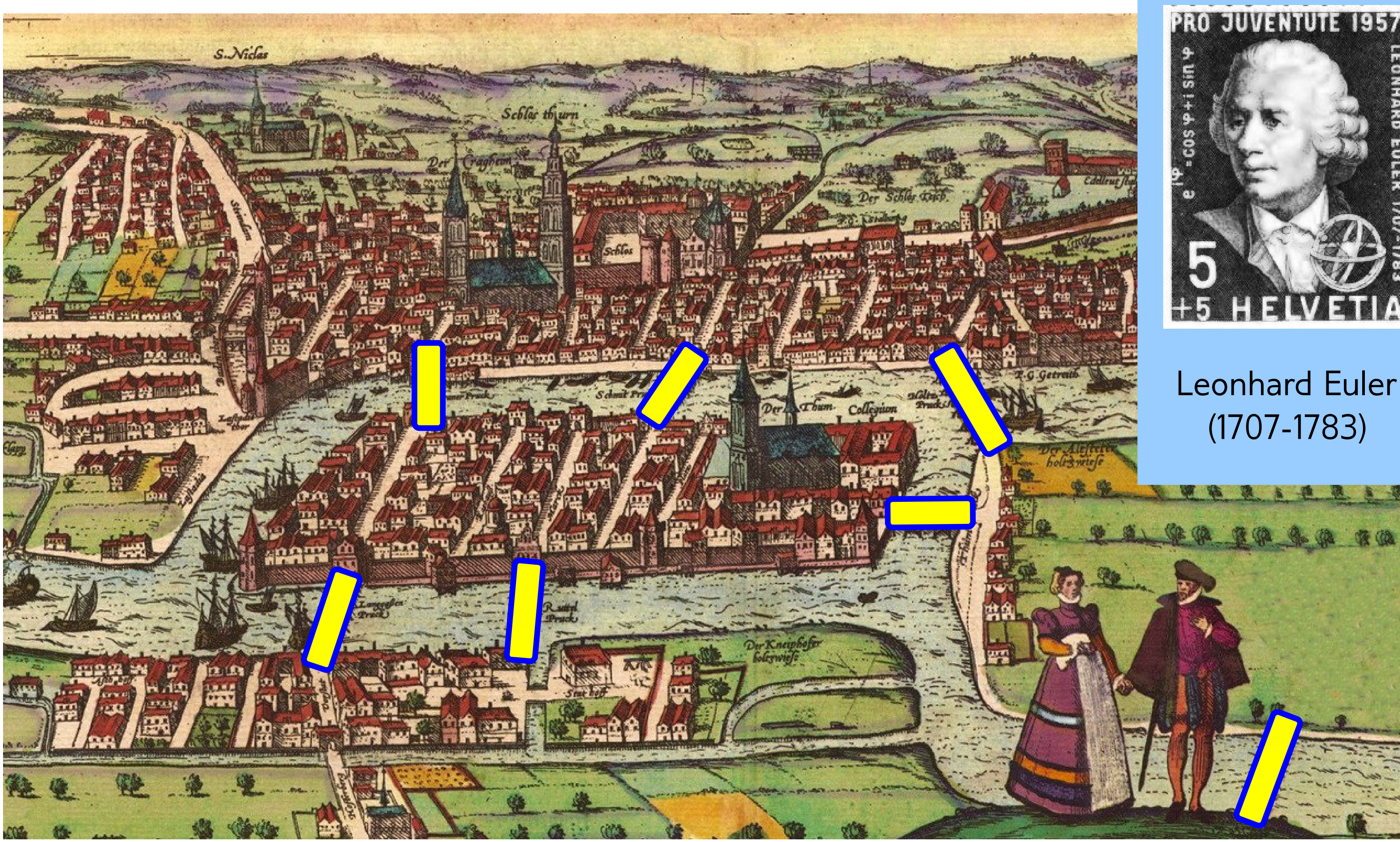
Bridges of Königsberg



Leonhard Euler
(1707-1783)

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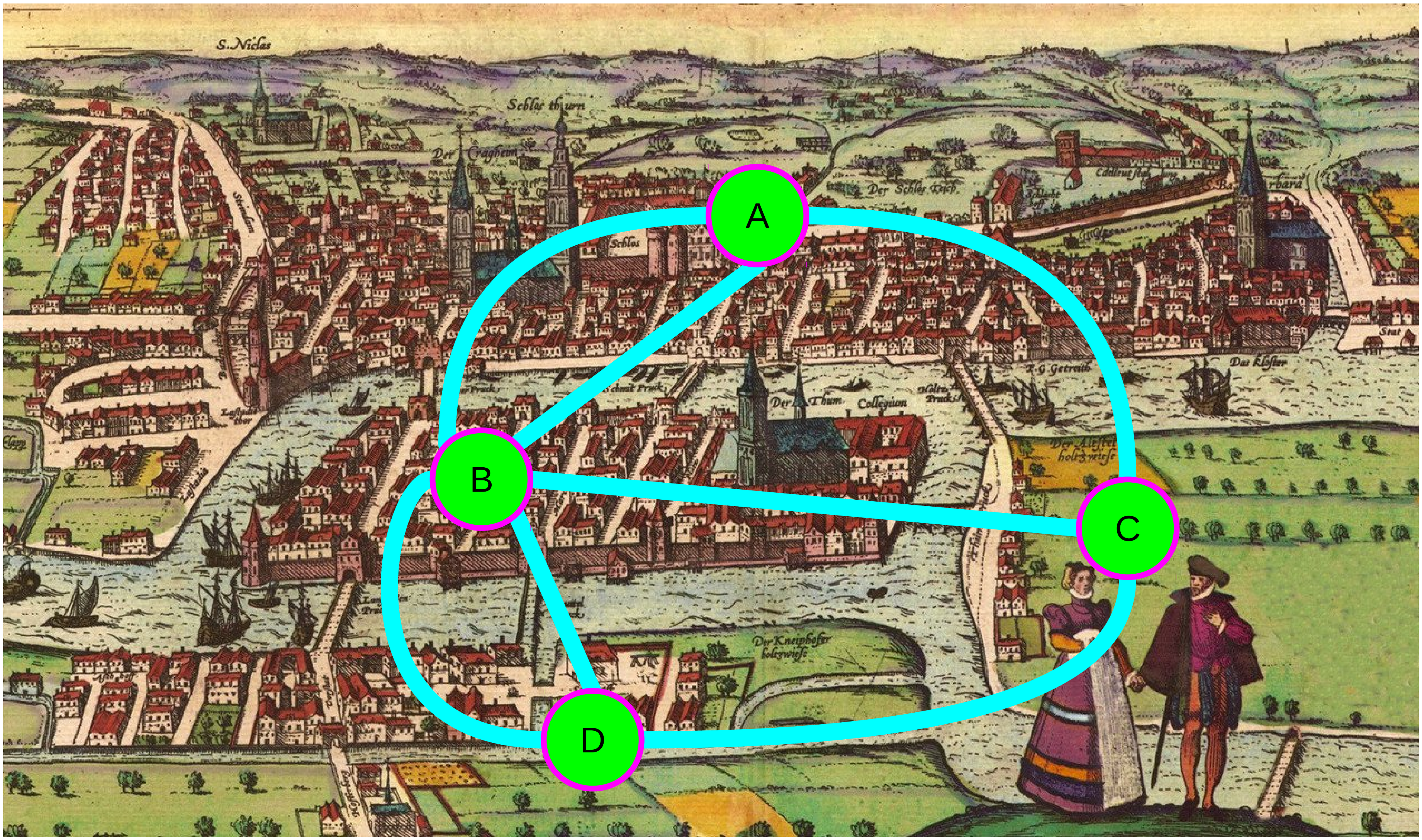
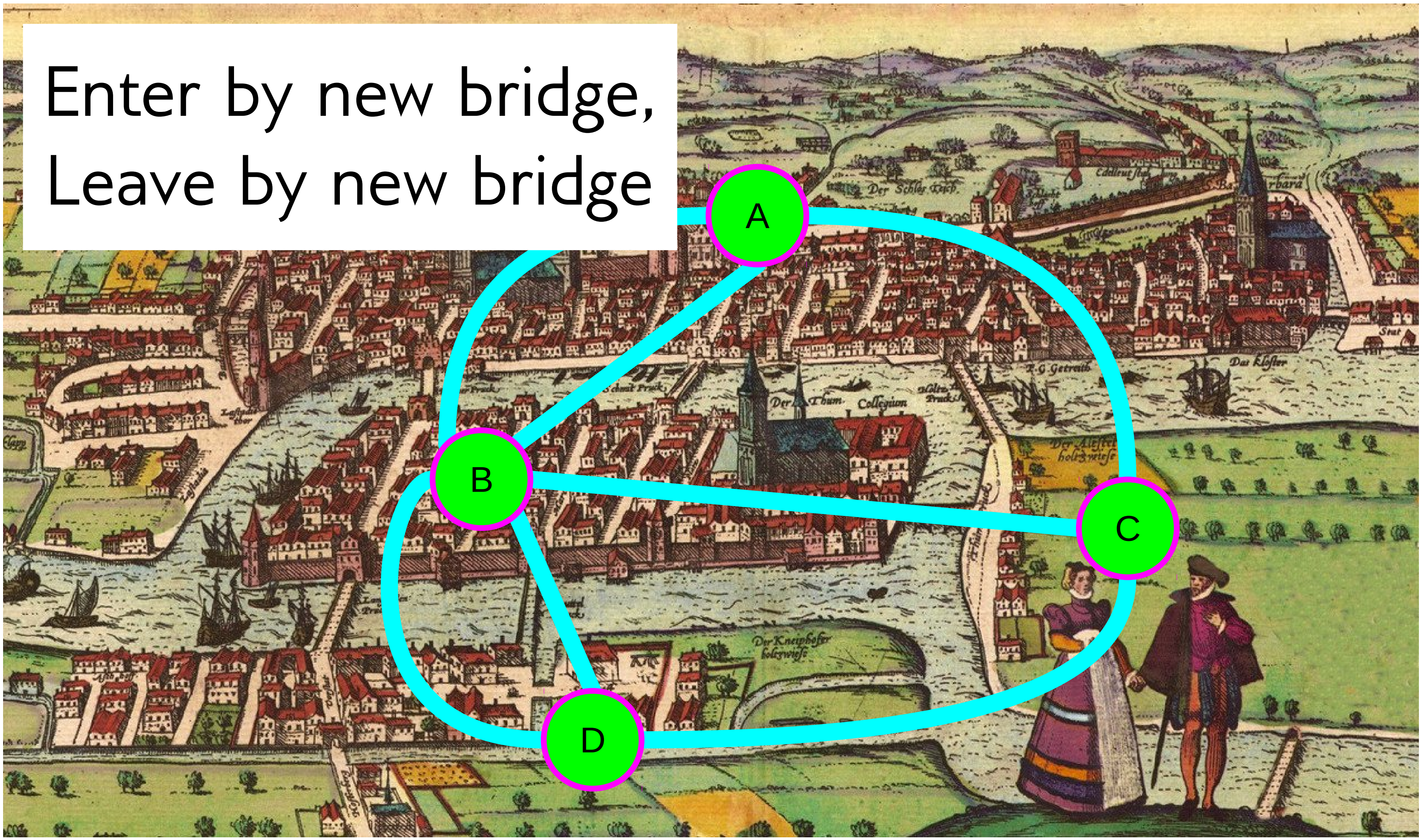
Bridges of Königsberg



Braun & Hogenberg, “Civitates Orbis Terrarum”, Cologne 1585. Photoshopped to clean up right side and add 7th bridge.

Bridges of Königsberg

Enter by new bridge,
Leave by new bridge

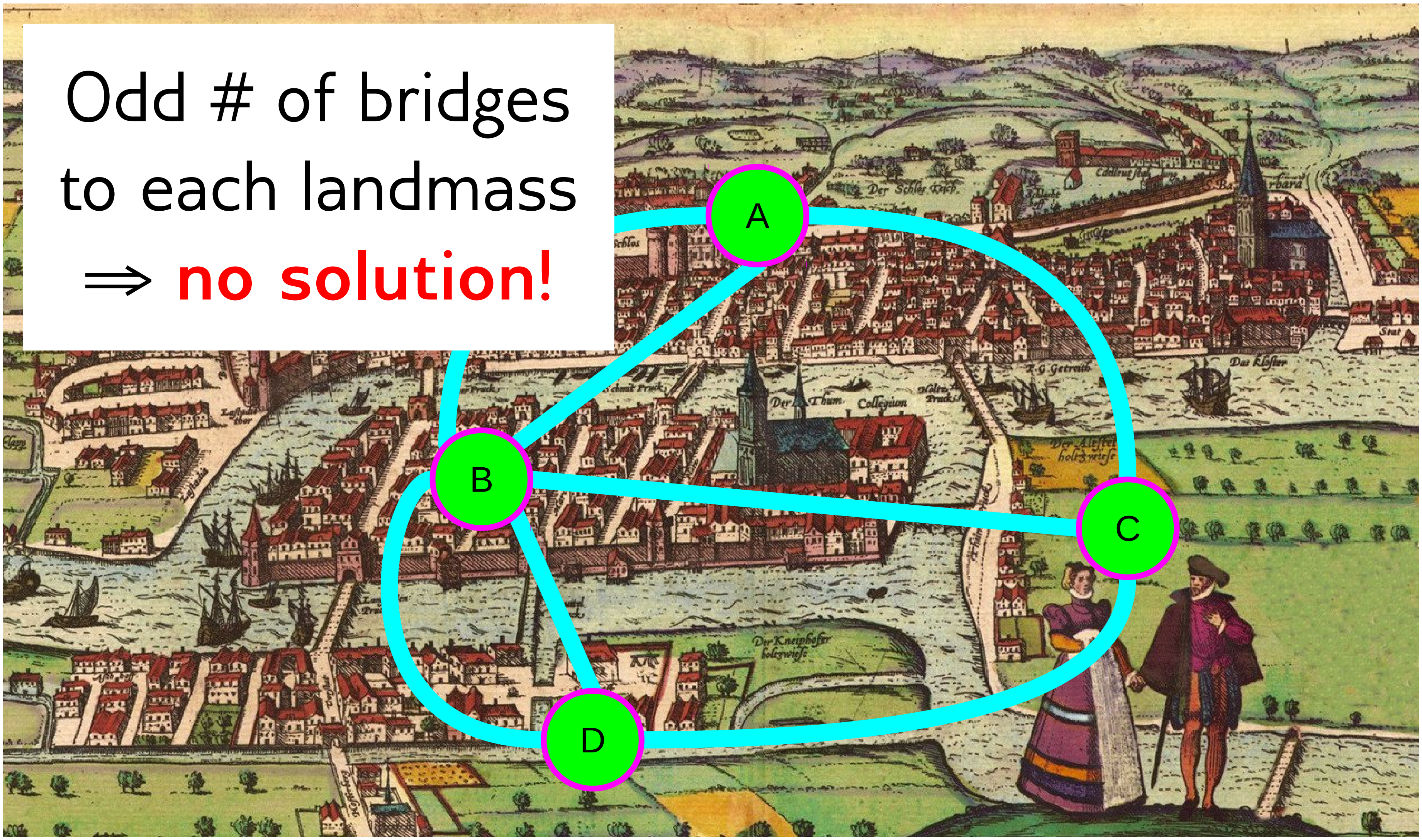


Braun & Hogenberg, “Civitates Orbis Terrarum”, Cologne 1585. Photoshopped to clean up right side and add 7th bridge.

Bridges of Königsberg

Odd # of bridges to each landmass

$\Rightarrow$ **no solution!**

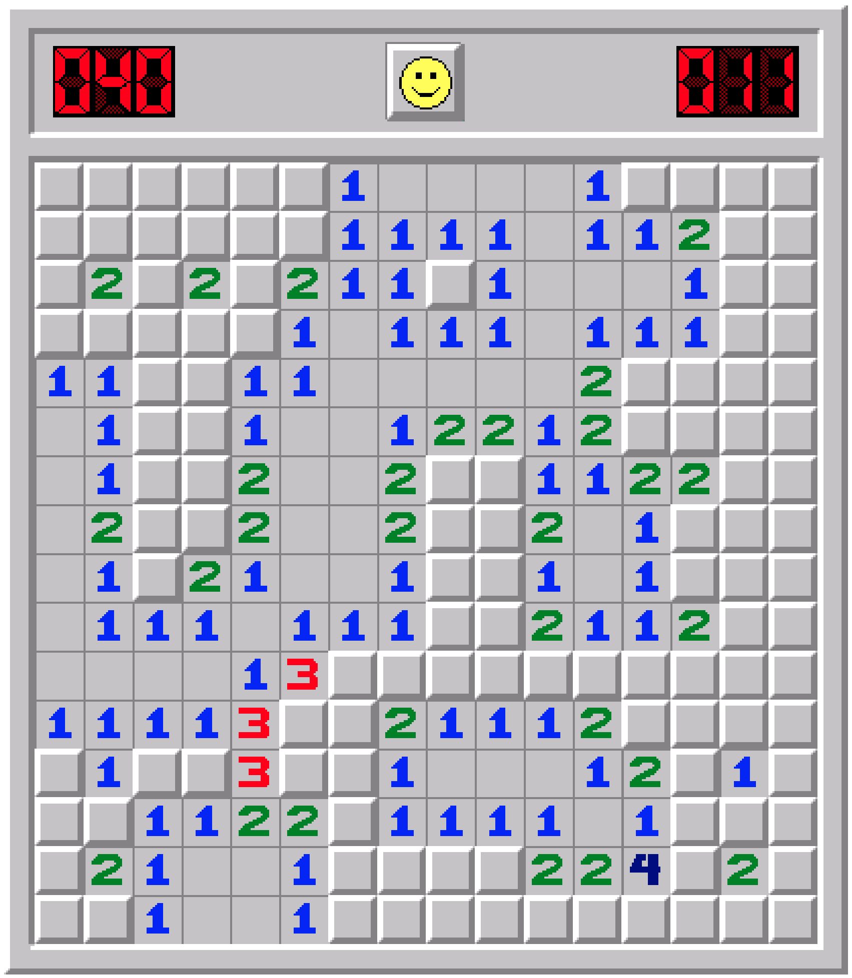


Braun & Hogenberg, "Civitates Orbis Terrarum", Cologne 1585. Photoshopped to clean up right side and add 7th bridge.

Bridges of Königsberg

- Cross each bridge once: Euler Path
  - Easy for a computer to calculate
- Visit each landmass once: Hamiltonian Path
  - Probably very hard for a computer to calculate
  - If you can find an efficient solution, you will get $1M and undying fame (answers "P = NP?")
  - (Will also break modern crypto, collapse the banking system, revolutionize automated mathematics and science, bring about world peace…)

You'll also be terrific at Minesweeper



Discrete Structures

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- Proof systems
- Sets, functions, relations
- Counting and probability
- Graph theory
- Models of computation, automata, complexity

This sentence is false.

This sentence is false.

If true, it is false

If false, it is true

This sentence is false.

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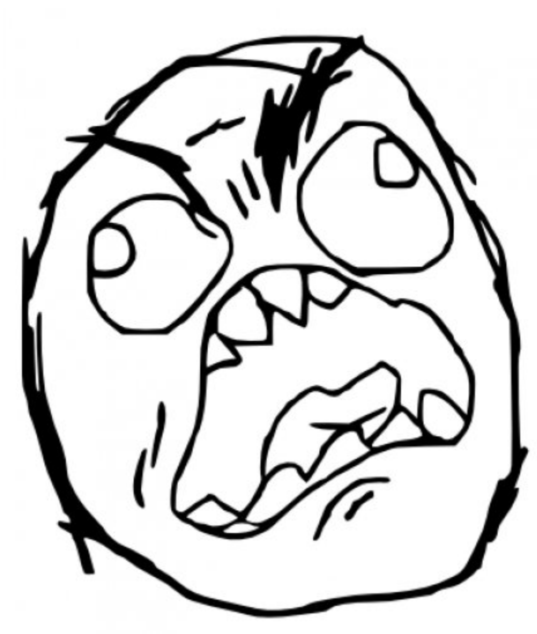
If false, it is true



Discrete Structures

- Number theory
- Proof systems
- Sets, functions, relations
- Counting and probability
- Graph theory
- Models of computation, automata, complexity
- Logic
- Decidability, computability

warrenphotographic.co.uk, auntiedogmasgardenspot.wordpress.com

One running theme of the course:

- How to prove things
- How to write good proofs

That's what we'll be staring with.