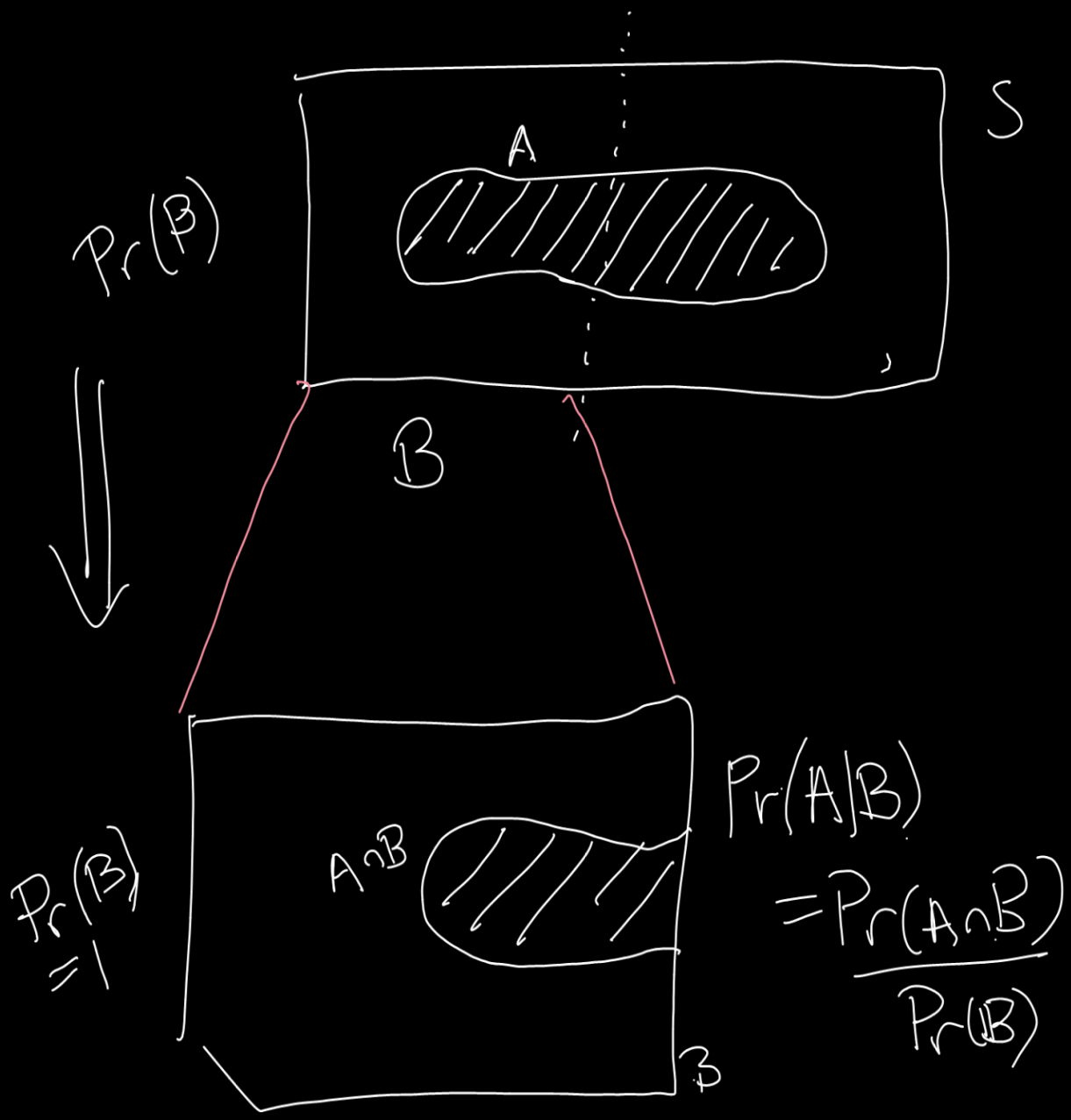
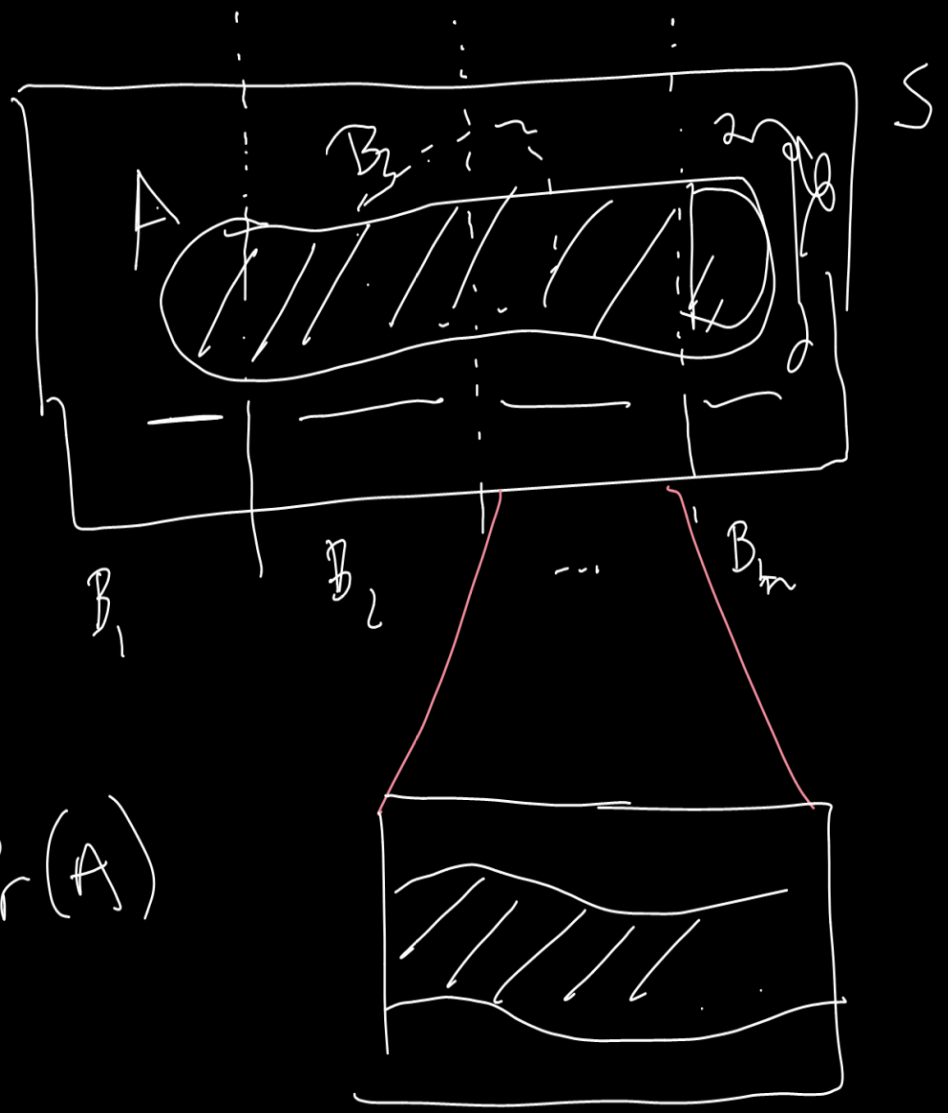
CS 2800 today

- Bayes Rule
- Random Variables


S
A
Pr(B)
B
A∩B
Pr(B)
= 1
B
Pr(A|B)
= Pr(A∩B) / Pr(B)

Law of total prob.



$$Pr(A) = Pr(A|B_1)Pr(B_1) + Pr(A|B_2)Pr(B_2) + \dots + Pr(A|B_n)Pr(B_n)$$

If B_i disjoint and $\bigcup B_i = S$.

Bayes Rule

know $Pr(B|A)$

want $Pr(A|B)$

$$Pr(A|B) = \frac{Pr(B|A)Pr(A)}{Pr(B)}$$

$$Pr(A|B)Pr(B) = Pr(A \cap B)$$

$$= Pr(B|A)\,Pr(A)$$

$$= \frac{Pr(B|A)Pr(A)}{\sum Pr(B|A_i)Pr(A_i)}$$

A = have disease

B = positive test

$A_1 = A, A_2 = \overline{A}$

know $Pr(A)$

$= Pr(\overline{B}|A)$ false neg

want $Pr(A|B)$ $= Pr(B|\overline{A})$ positive

Random Variables

Definition: A (real valued) random variable X on sample sp. S is a function $X: S \to \mathbb{R}$

Ex: S: set of outcomes of M. Hall games

$X: S \to \mathbb{R}$

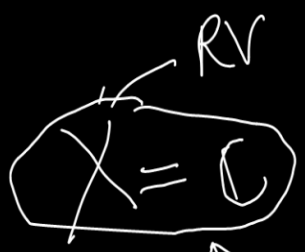
$X(\text{goat}) = 0 \quad X(\text{car}) = 1$

$X(\text{goat}) = \text{cost of goat} \quad X(\text{car}) = \text{price of car.}$

X: Net value/cost of an outcome.

Ex: if S = outcomes from 10 flips, #H (# of heads) is a RV.

RV ⟺ Events.



$X = c$

(a) outcome?

(b) event

(c) RV

(d) none

(e) ?

Set of all outcomes s on which $X(s) = c$.

$$\text{“}X = c\text{”} = \{s \in S \mid X(s) = c\}$$

$$\Pr(X = c)$$

$X \leq c$?

$X : S \to \mathbb{R}$ ↖ RVs

$Y : S \to \mathbb{R}$ ↙

$X + Y$

event ?

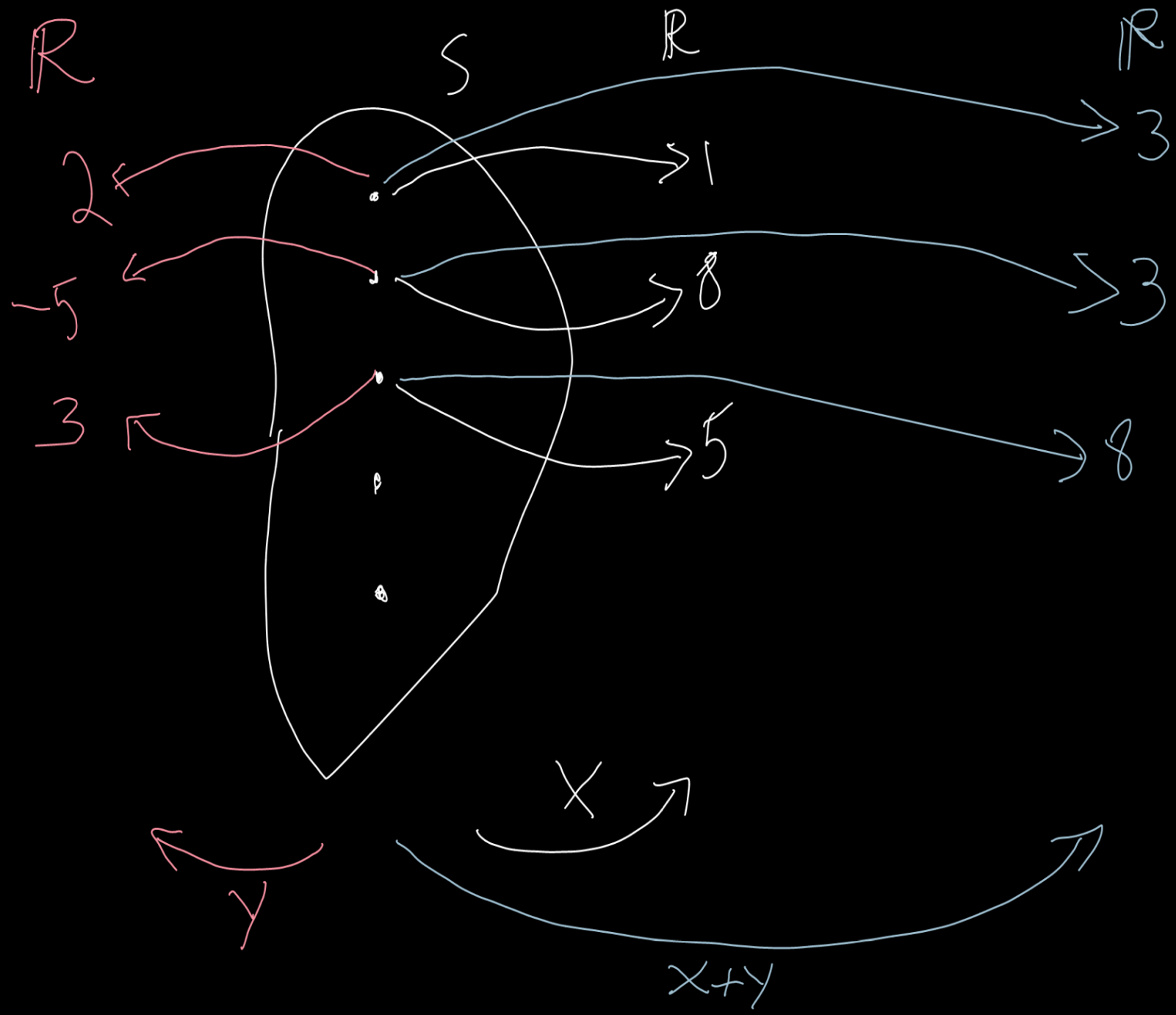
random var. ?

other ?

$\sin(X)$

$X \pm Y : S \to \mathbb{R}$

$(X+Y)(s) := X(s) + Y(s)$

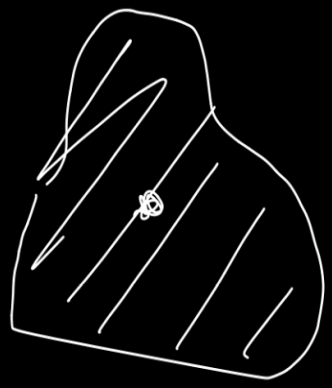

R
S
R
R
2
-5
3
1
8
5
3
3
8
X
Y
X+Y

Expected Value

idea: weighted average

Def^n The Expected Value of X $\left(E(X)\right)$

is $\sum_{s \in S} X(s) \cdot Pr(\{s\})$



Ex: S students — Exam score — $1/\#$students if equiprob.

$=$ Average.

Alt. Exp. Value

Claim

$$E'(X) := \sum_{x \in \mathbb{R}} x \cdot \Pr(X = x) = E(X) = \sum_{s \in S} X(s) \Pr(\{s\})$$

$$\{ s \in S \mid X(s) = x \} = \bigcup_{s \mid X(s) = x} \{s\}$$

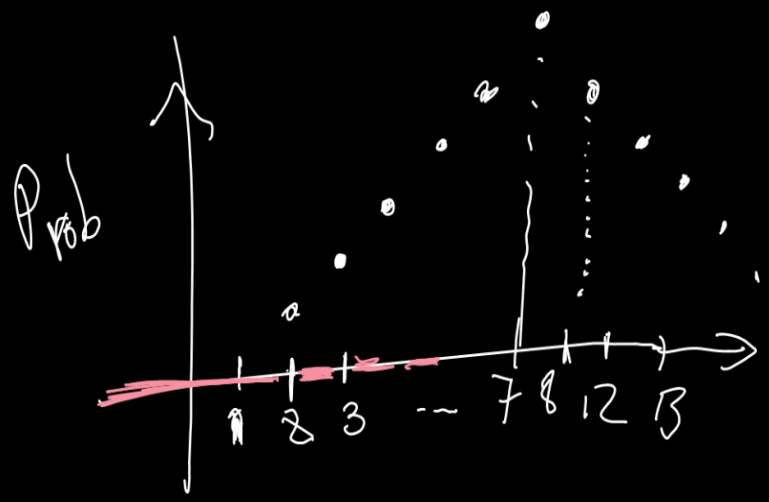
PDF

Prob. density fun.
distr. fun.

$X =$
Ex: sum of two dice

Defn $PDF_X : \mathbb{R} \to \mathbb{R}$

$$PDF_X(x) = Pr(X = x)$$



Def'n: $CDF : \mathbb{R} \to \mathbb{R}$

$$CDF_X(x) = Pr(X \leq x)$$