

Finite Automata

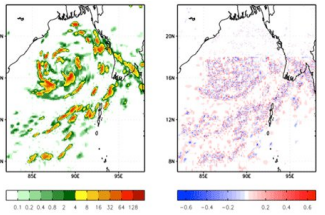
CS 2800: Discrete Structures, Spring 2015

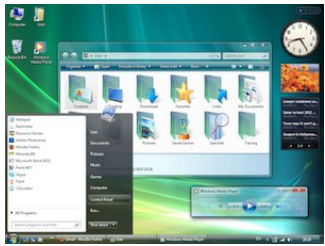
Sid Chaudhuri



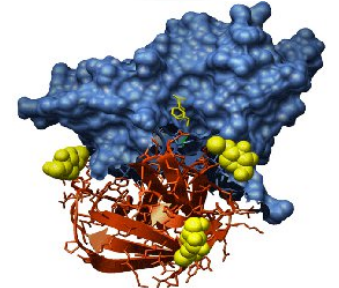
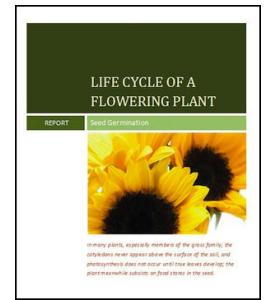
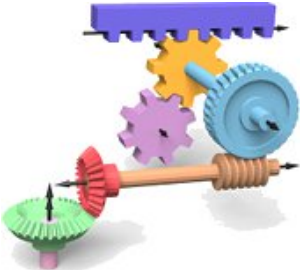
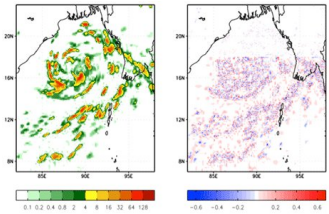


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<body>  
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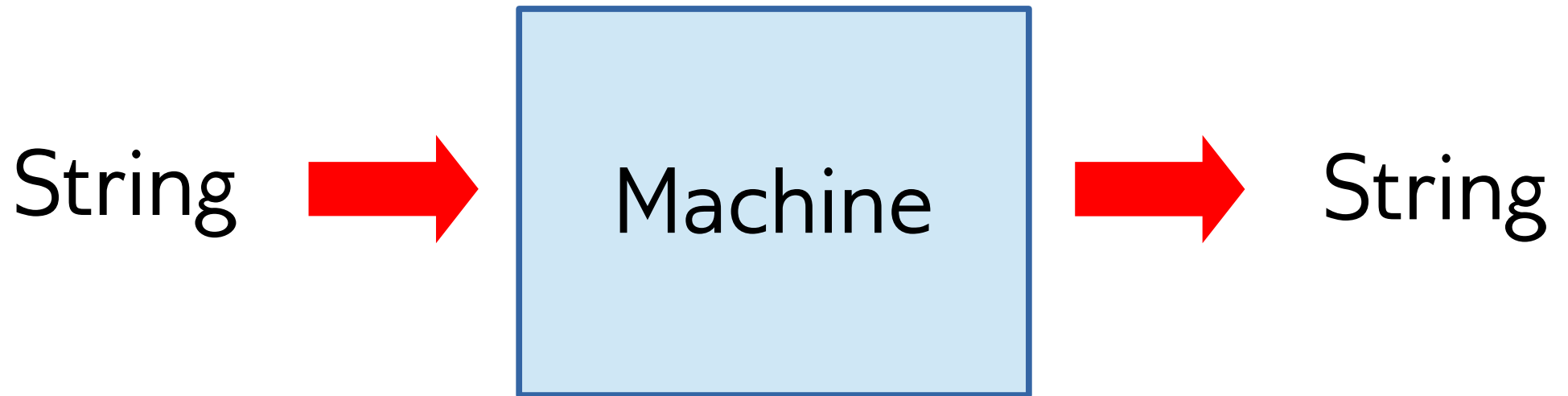




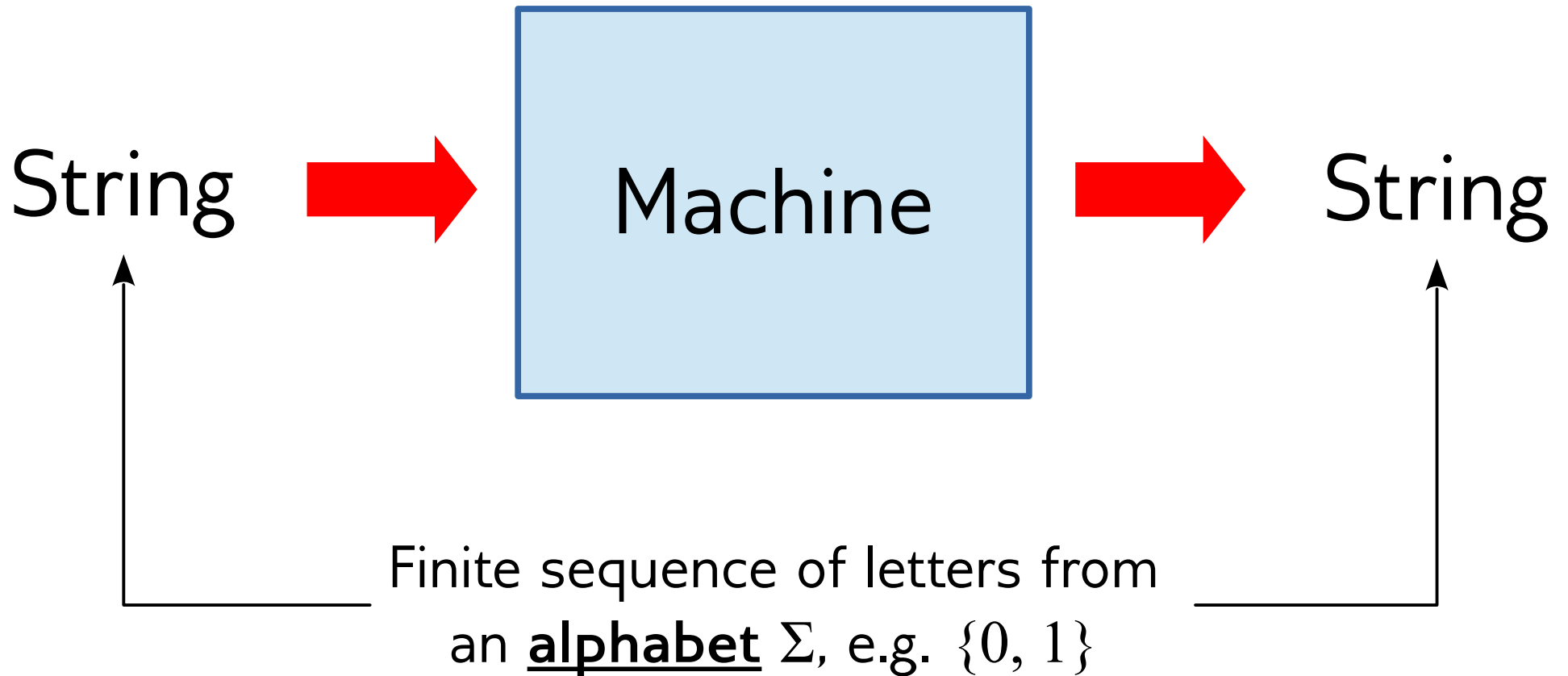
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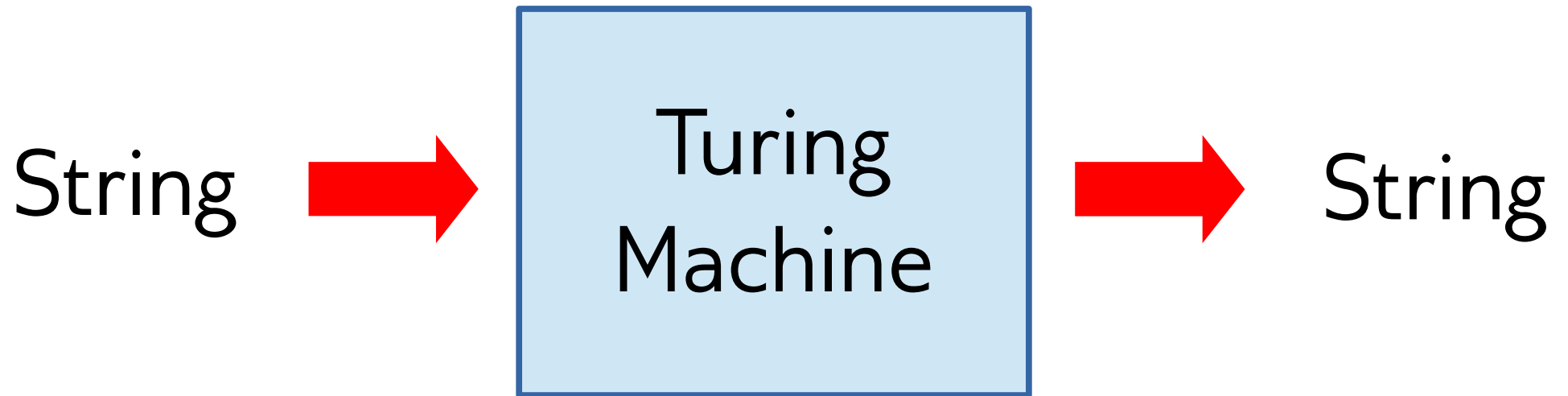
A simplified model



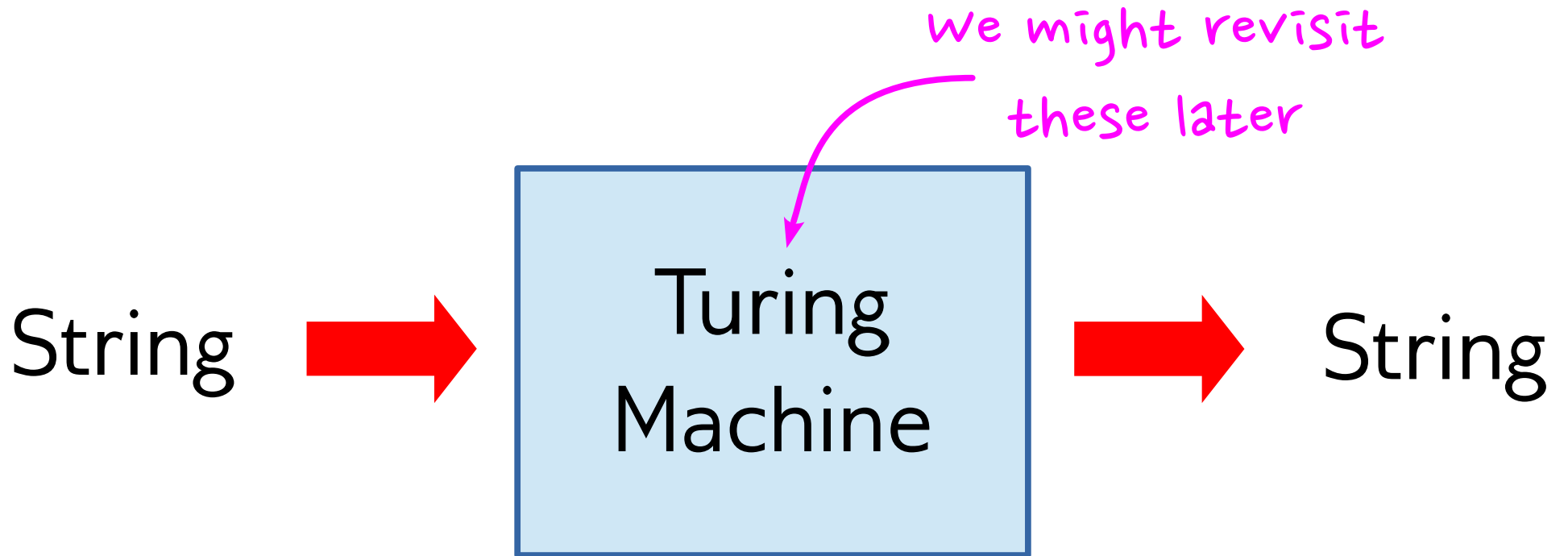
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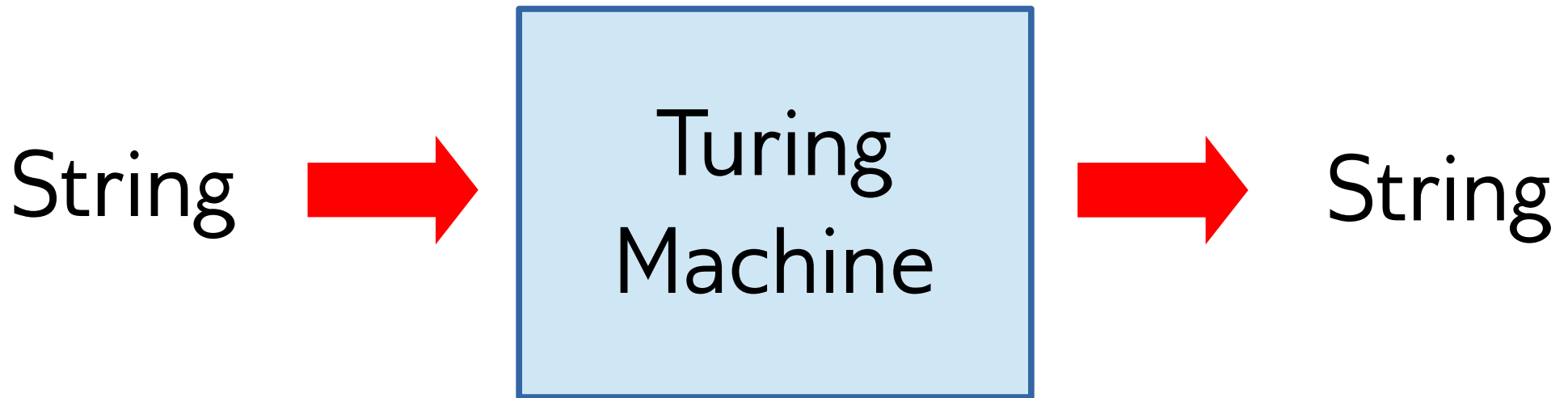
A general-purpose computer



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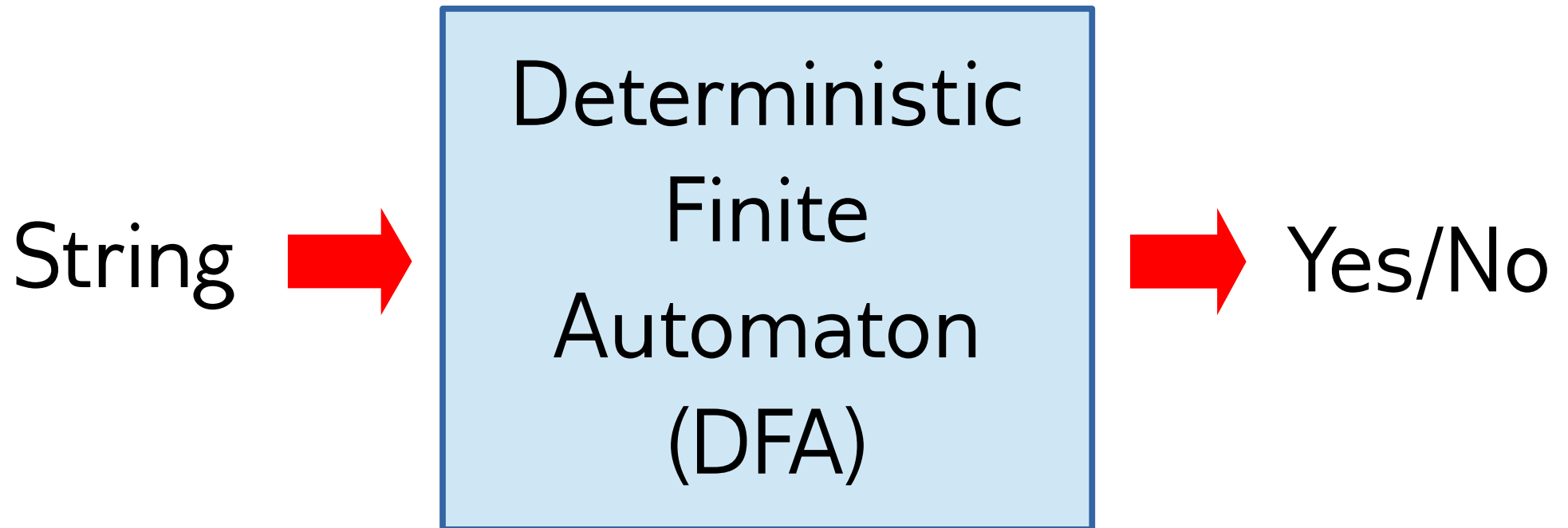


Church-Turing Thesis: Any “effective/mechanical/real-world” calculation can be carried out on a Turing machine

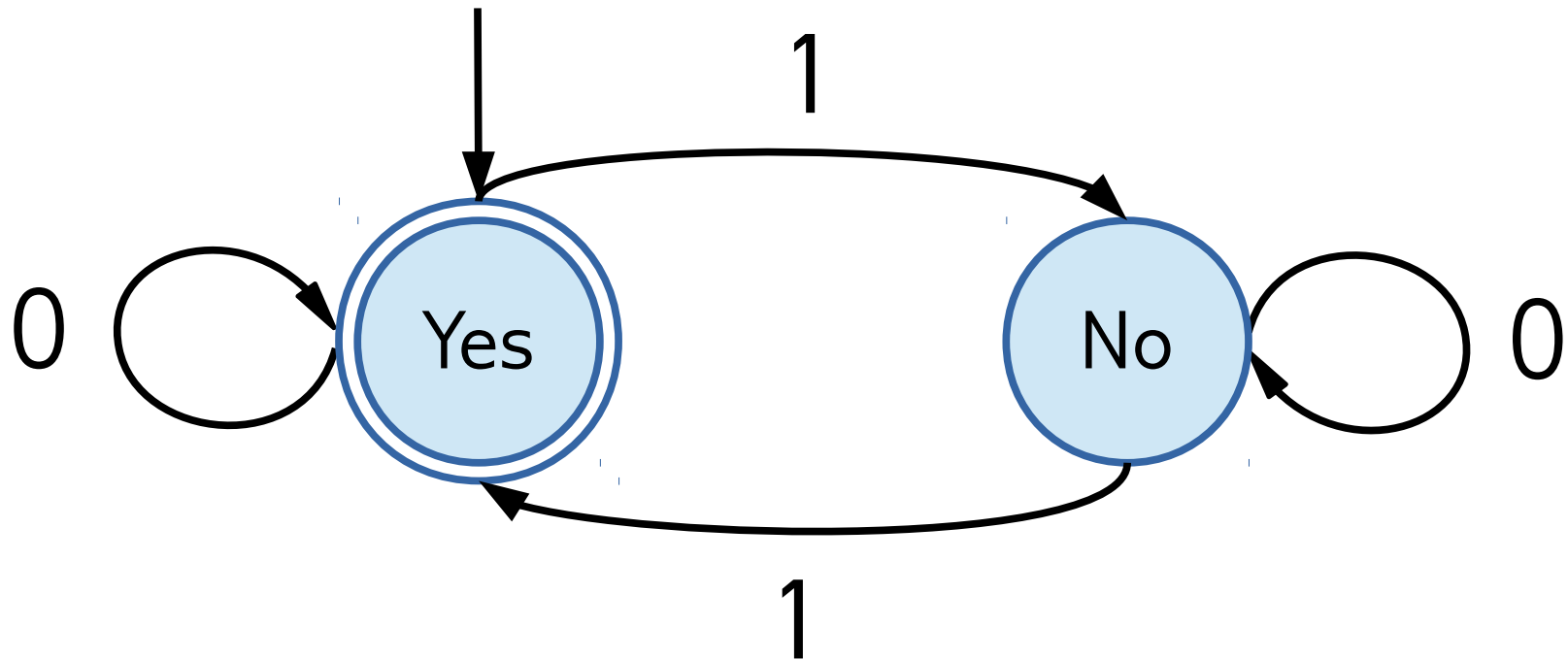


Alan Turing, 1912 - 1954

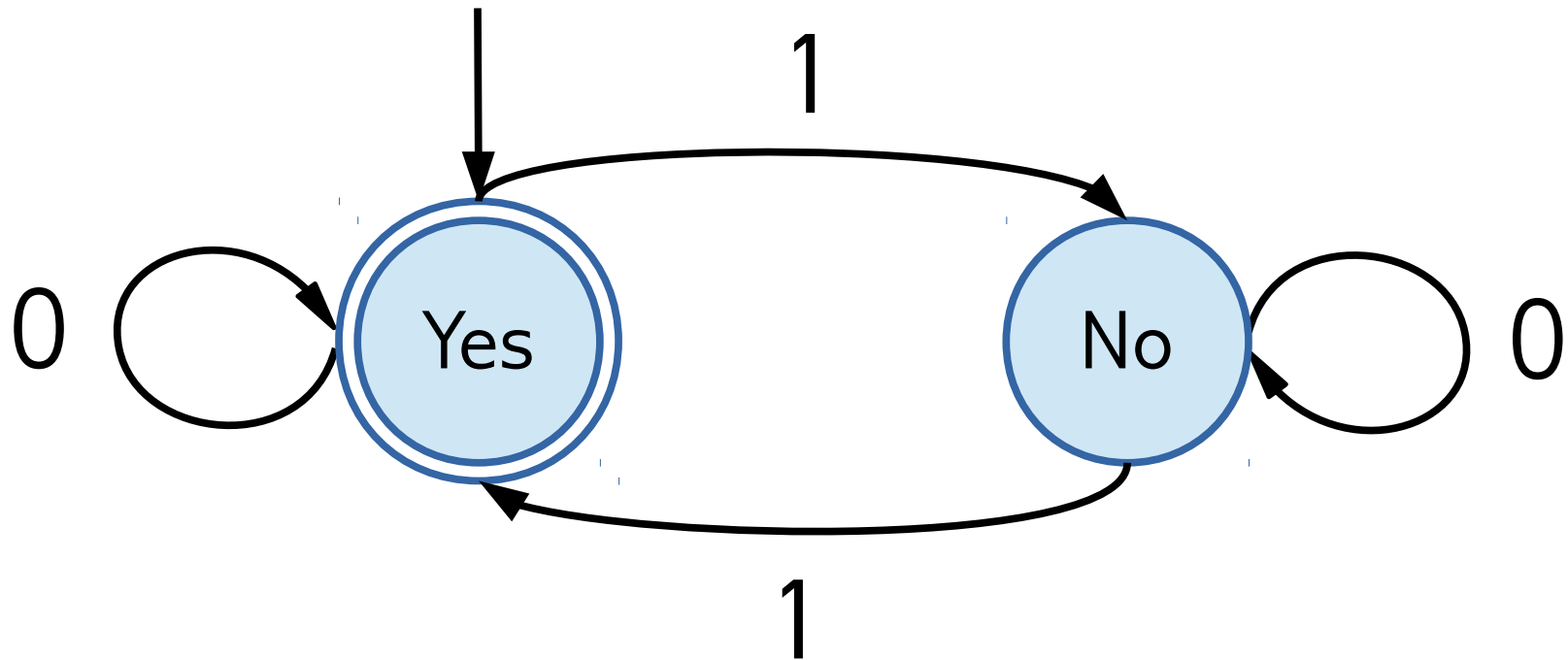
A simple “computer”



An example

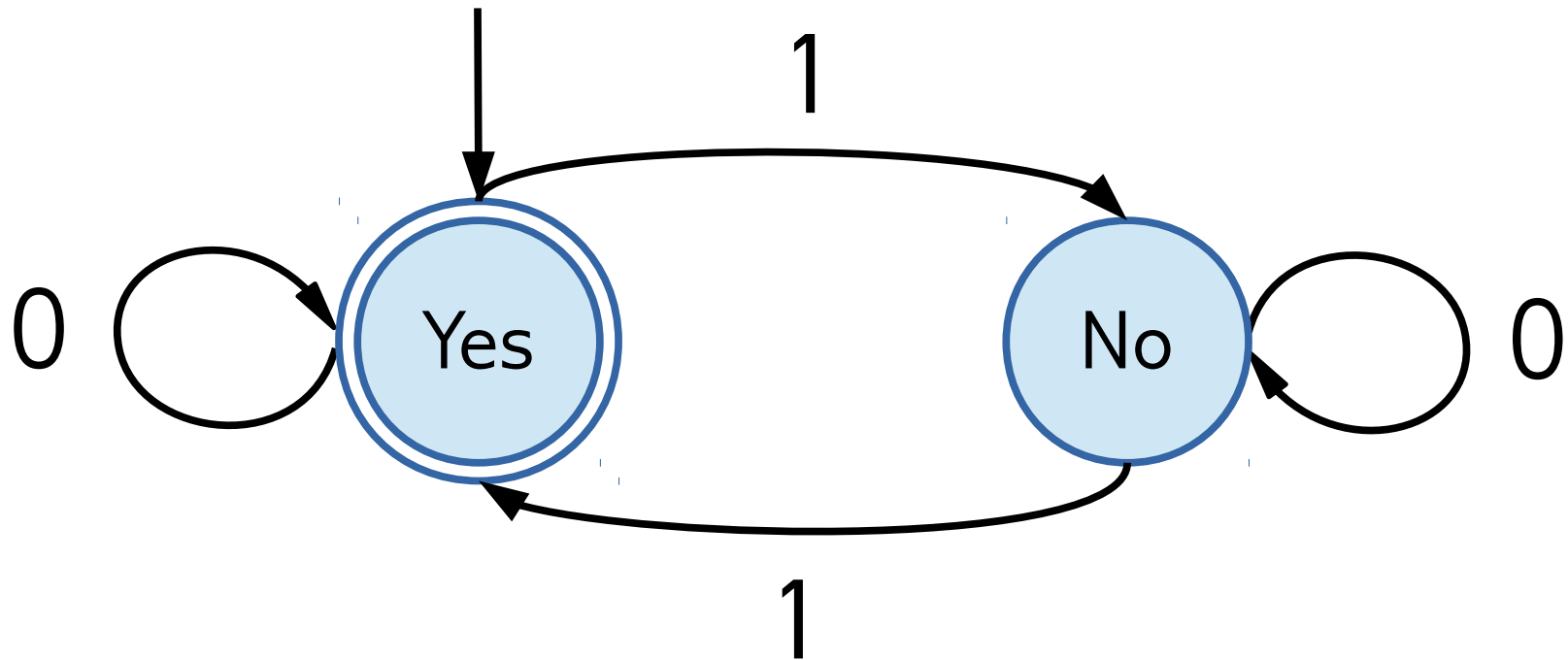


An example



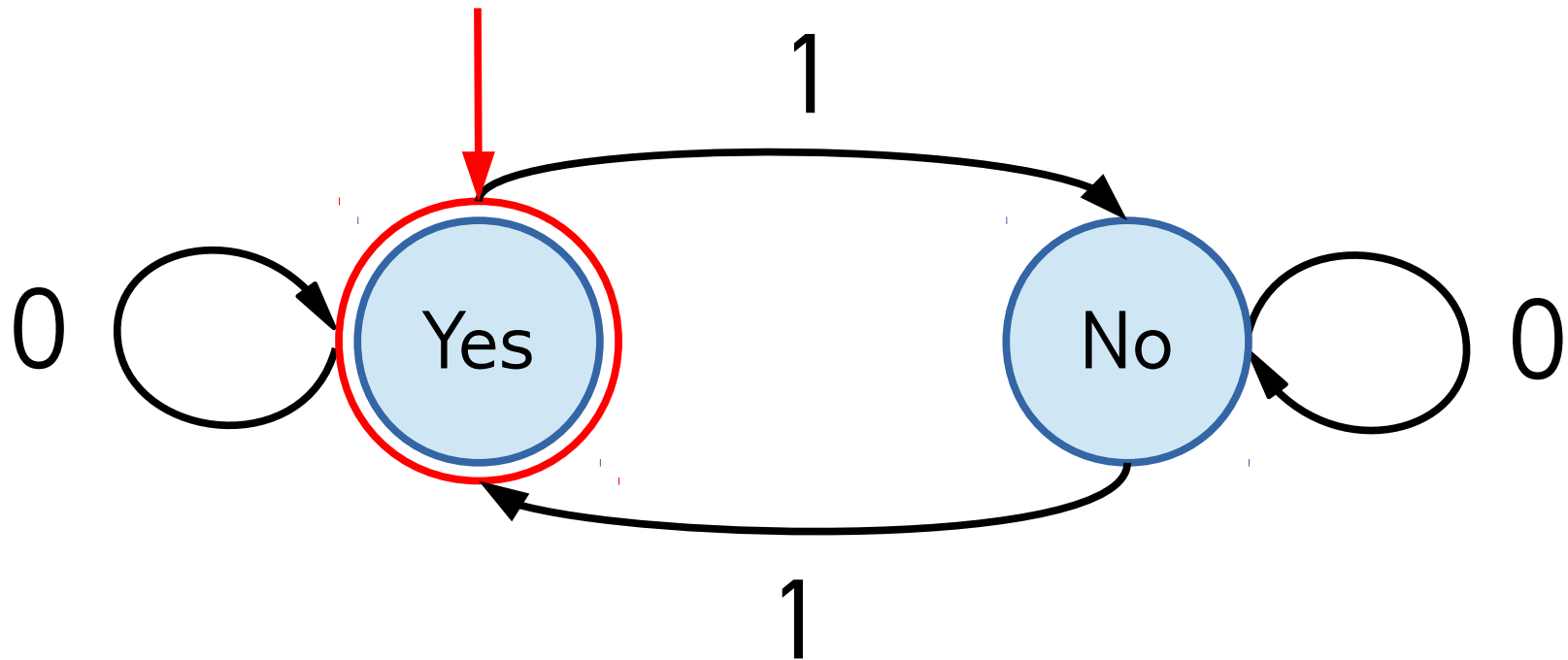
(Binary input)

An example



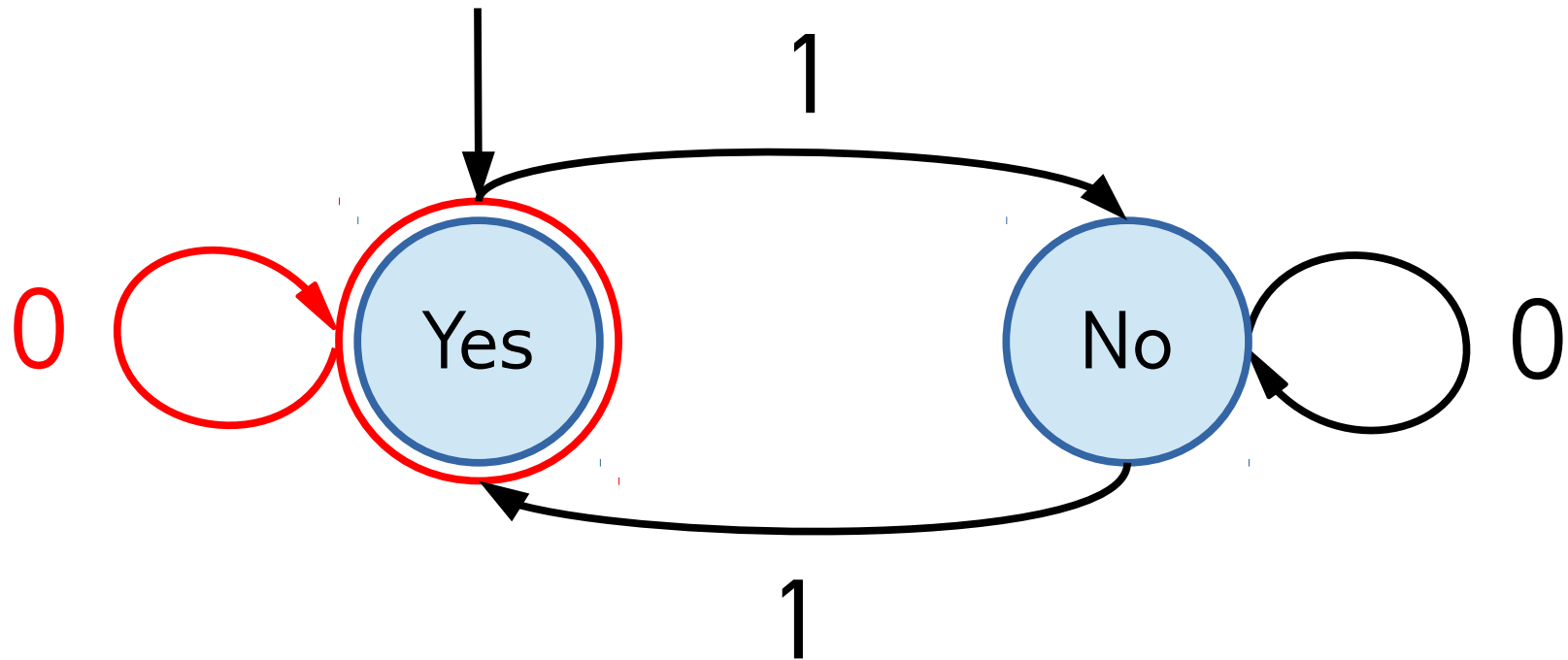
Input: 01001

An example



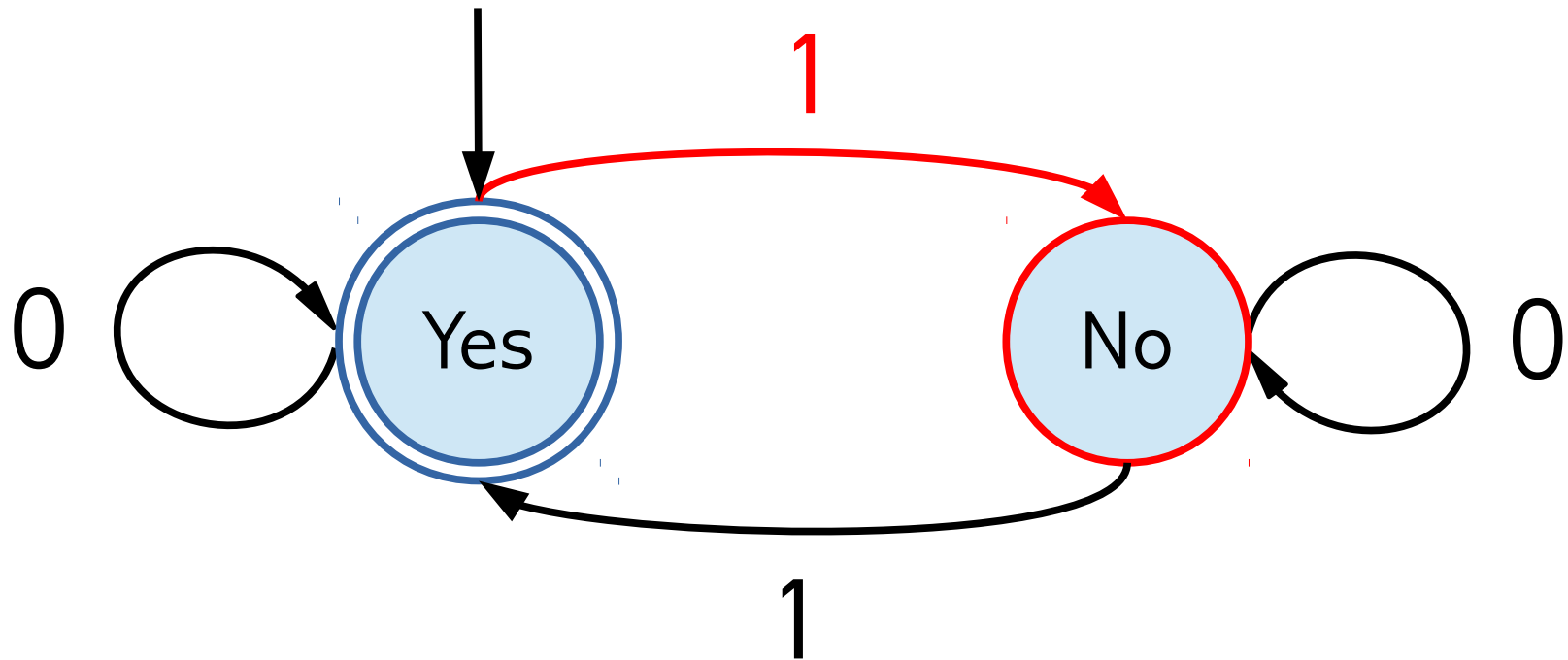
Input: 01001

An example



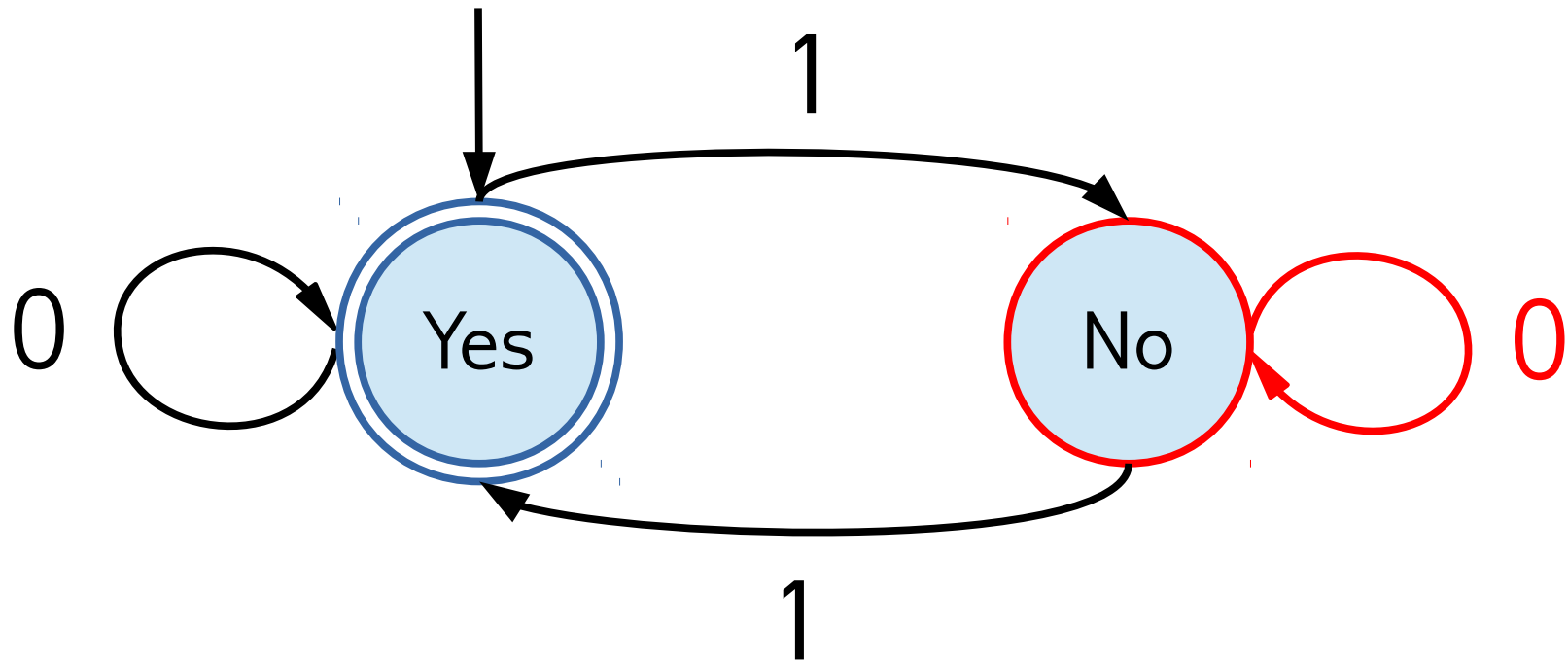
Input: 01001

An example



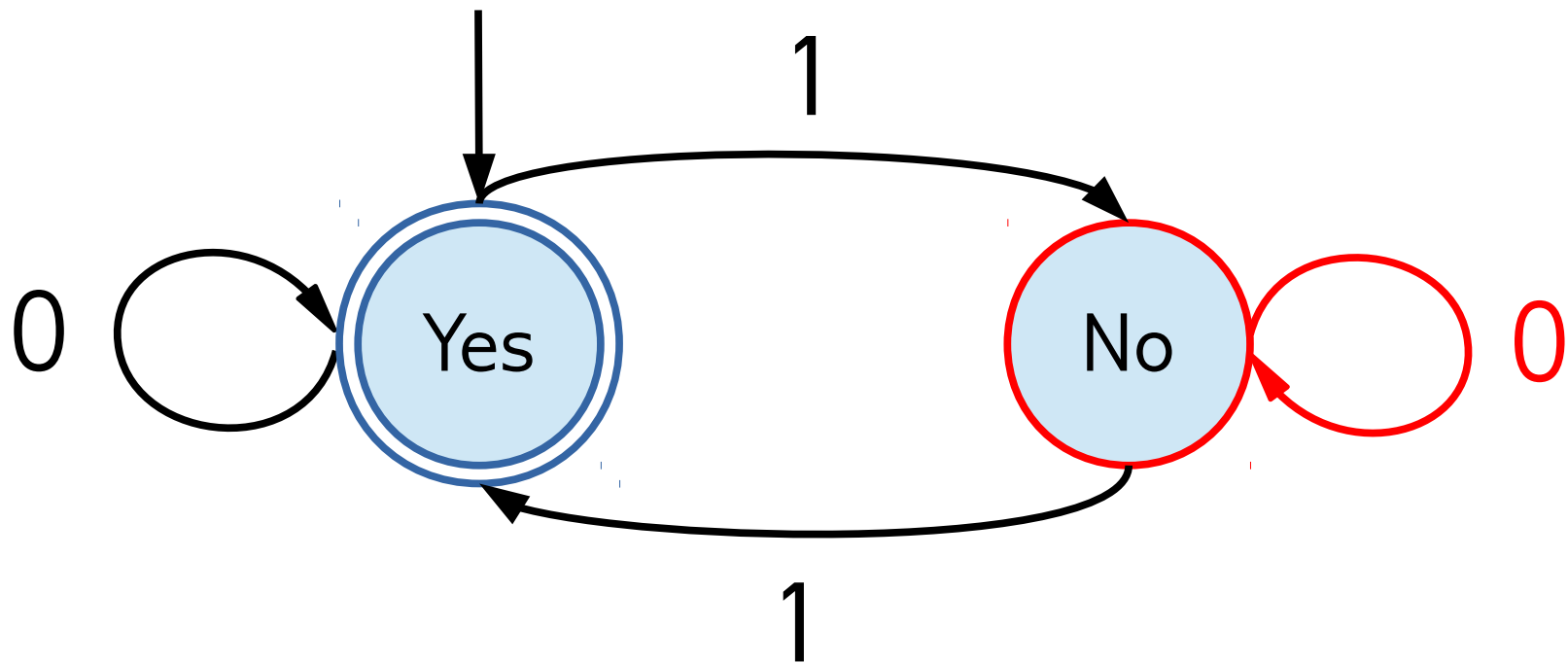
Input: 01001

An example



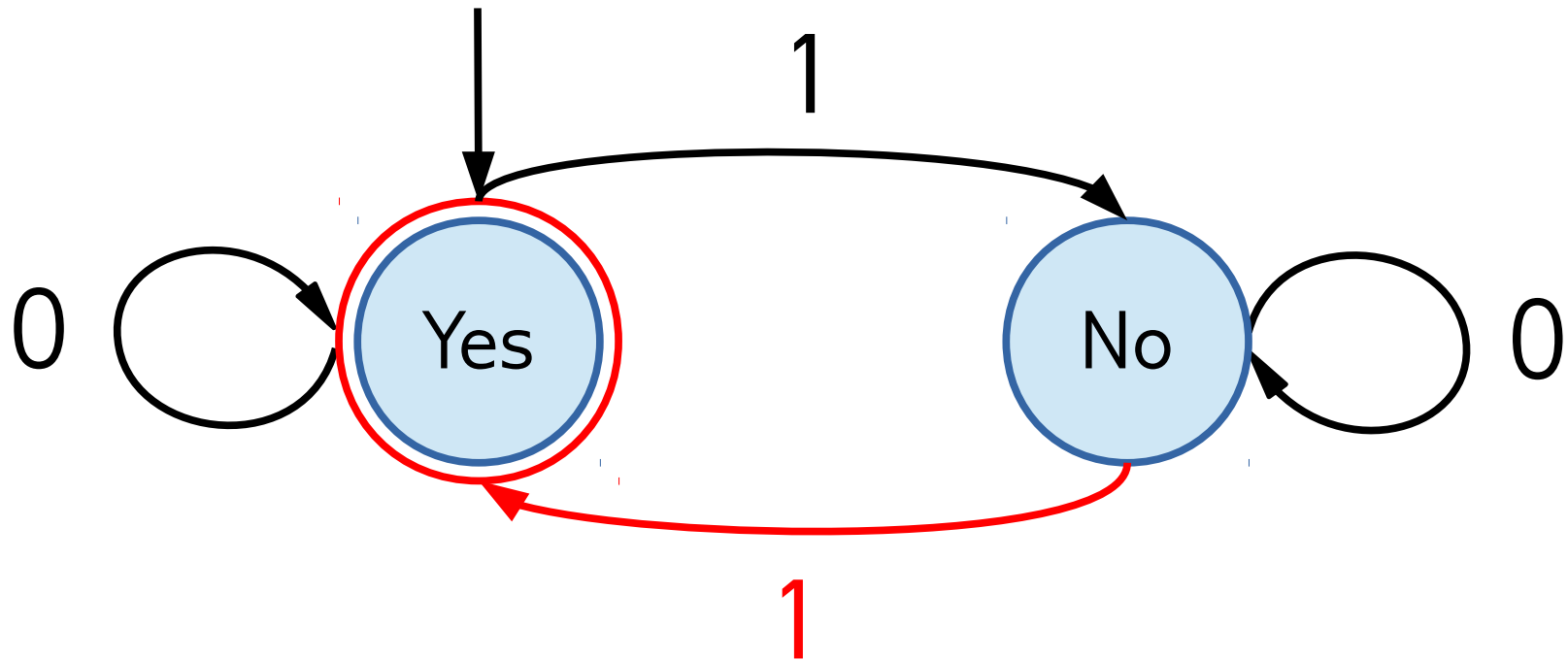
Input: 01001

An example



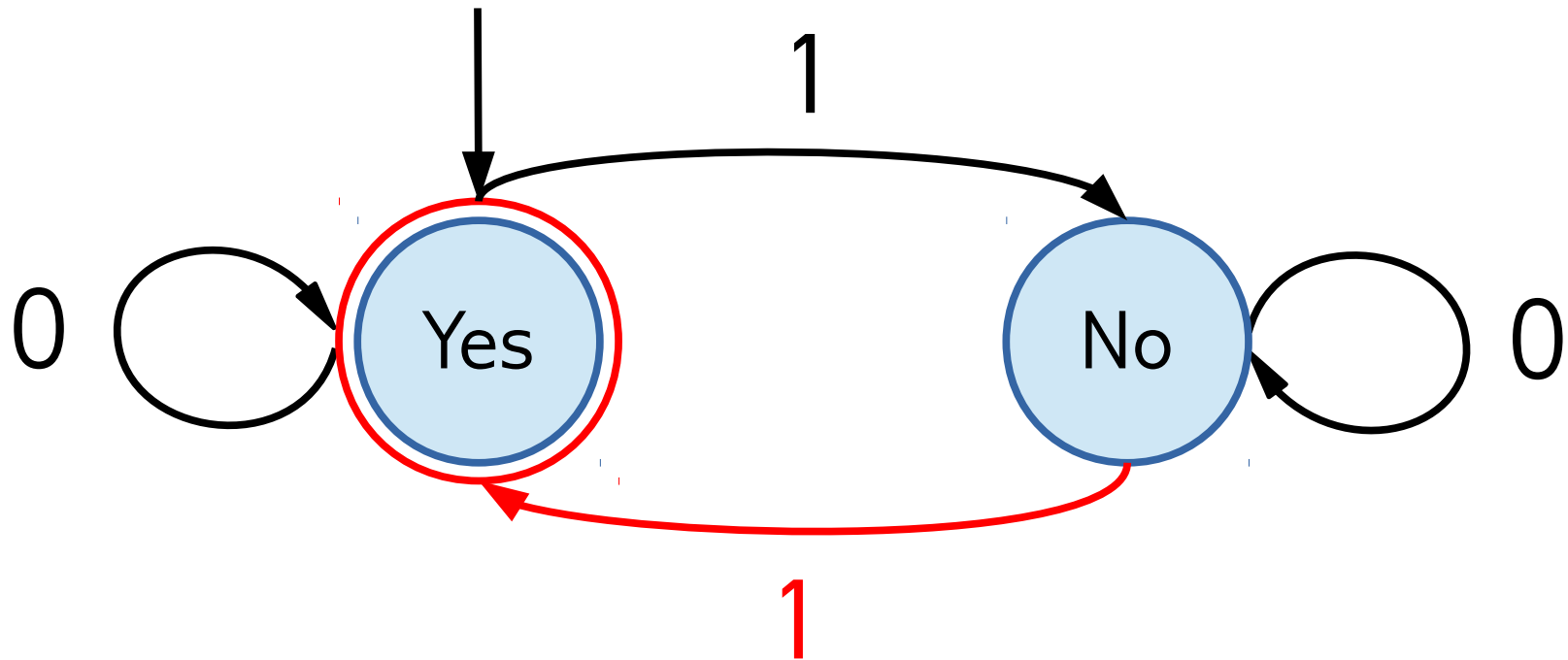
Input: 01001

An example



Input: 0100**1**

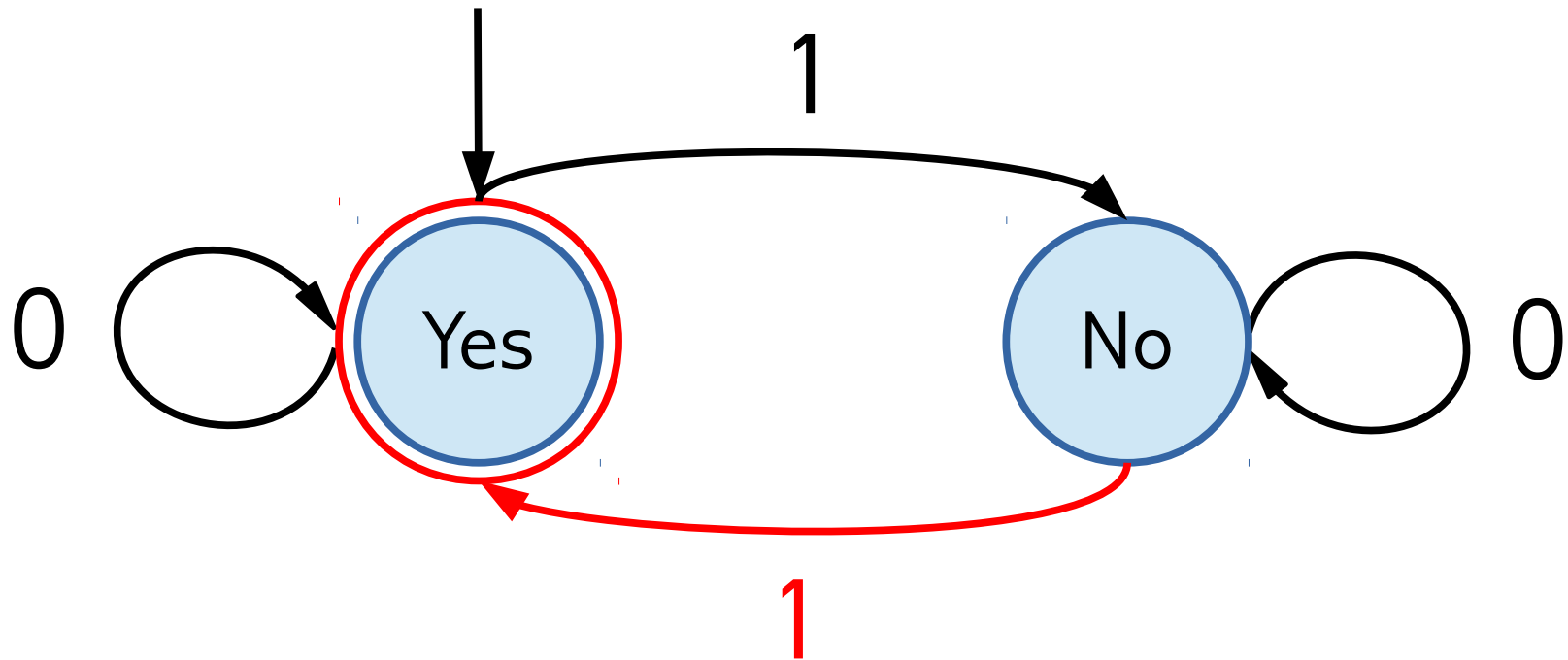
An example



Input: 01001

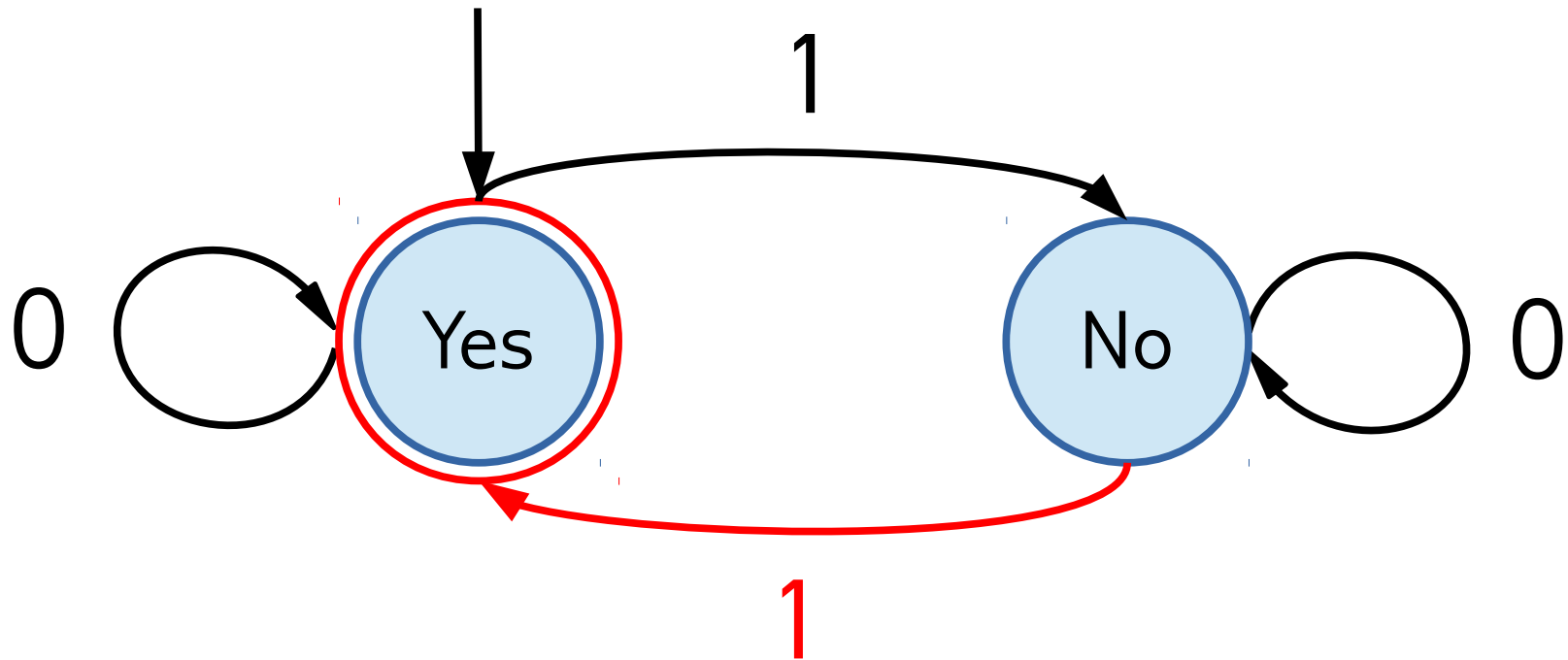
Output: **Yes!**

An example



In general, on what binary strings does this DFA return Yes?

An example



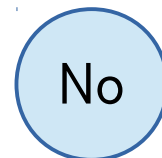
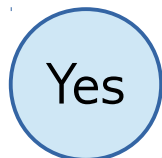
Ans: All strings with an even number of 1's

Deterministic Finite Automaton

- A DFA is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$

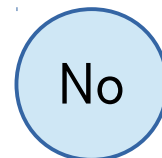
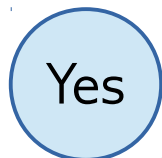
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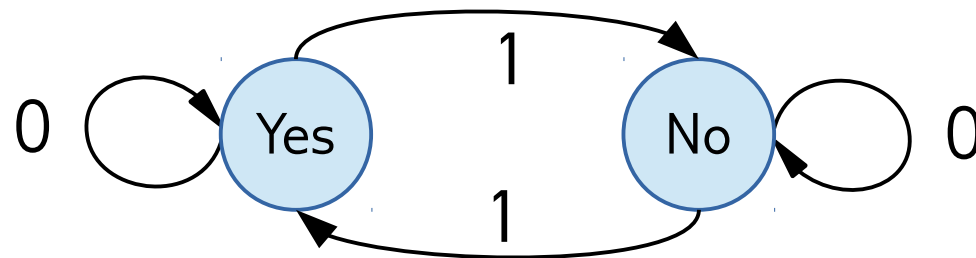
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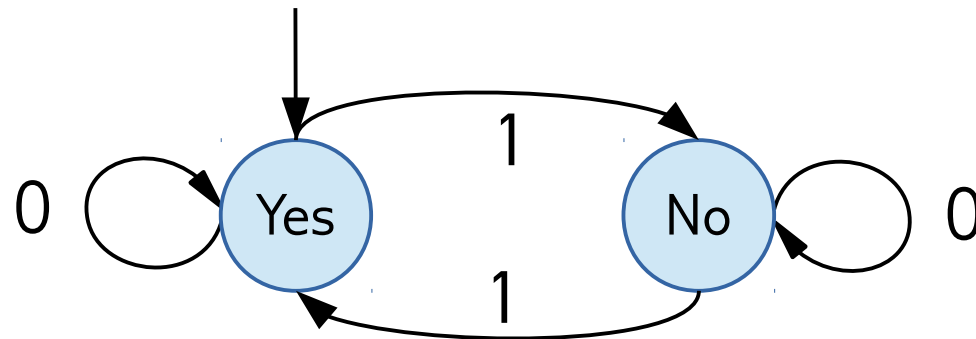
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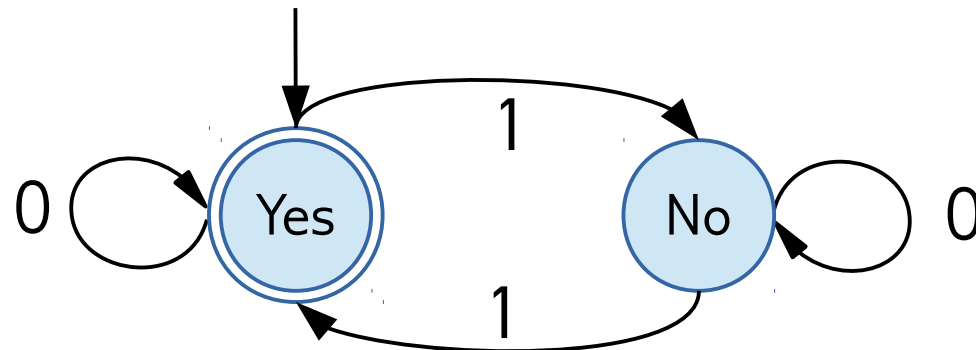
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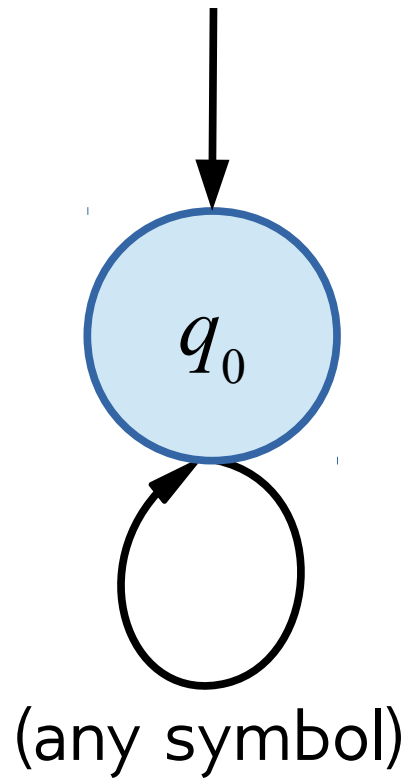


Deterministic Finite Automaton

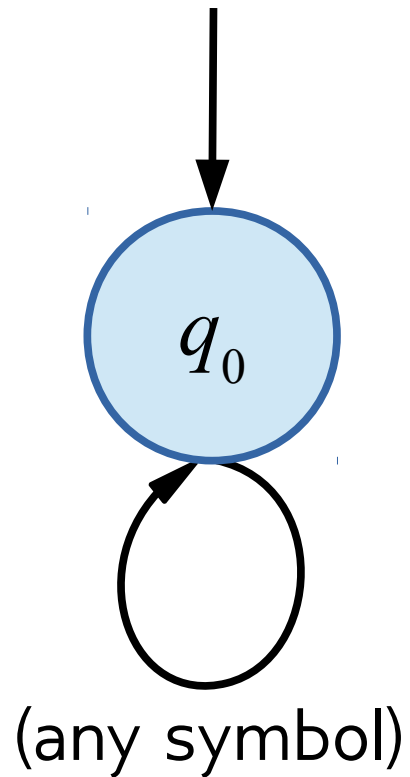
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 - $F \subseteq Q$ is the set of **final/accepting states**



What does this DFA accept?

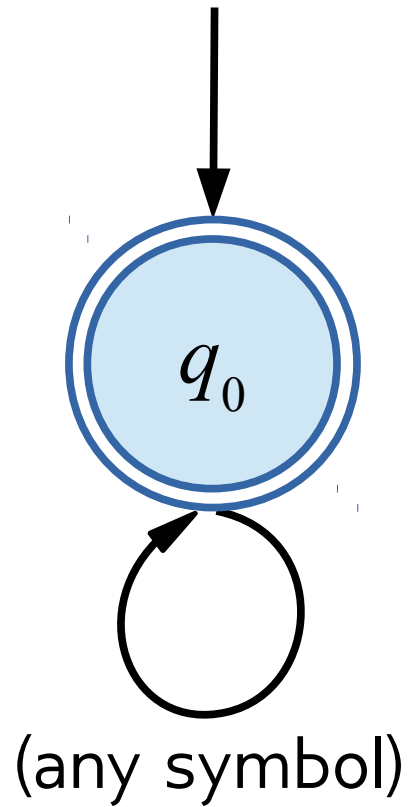


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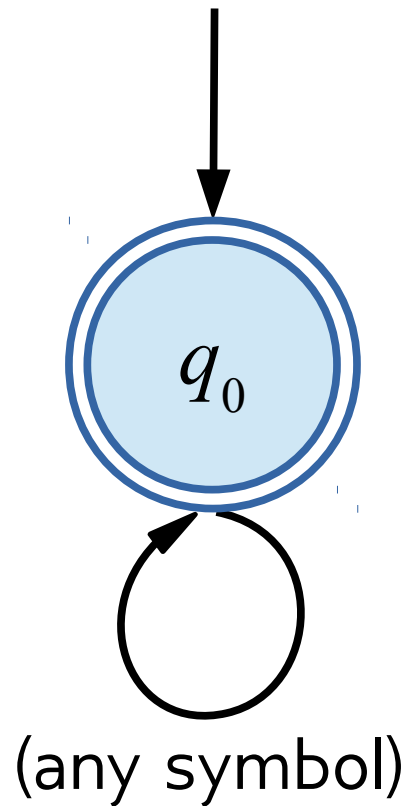


Answer: No strings

What does this DFA accept?

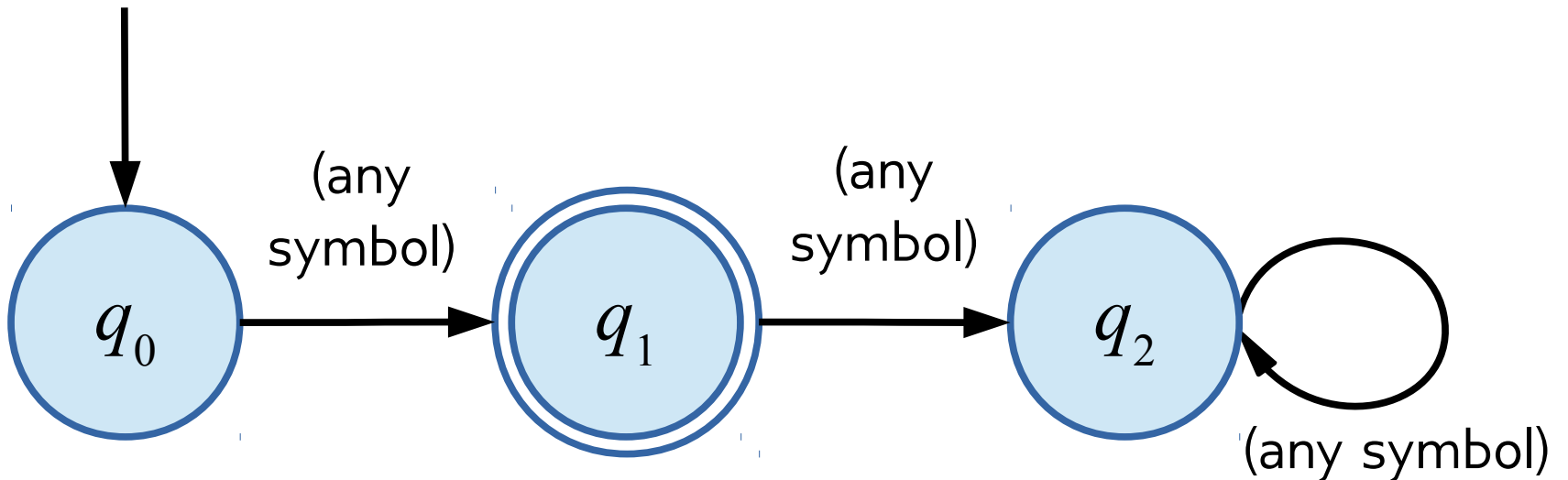


What does this DFA accept?

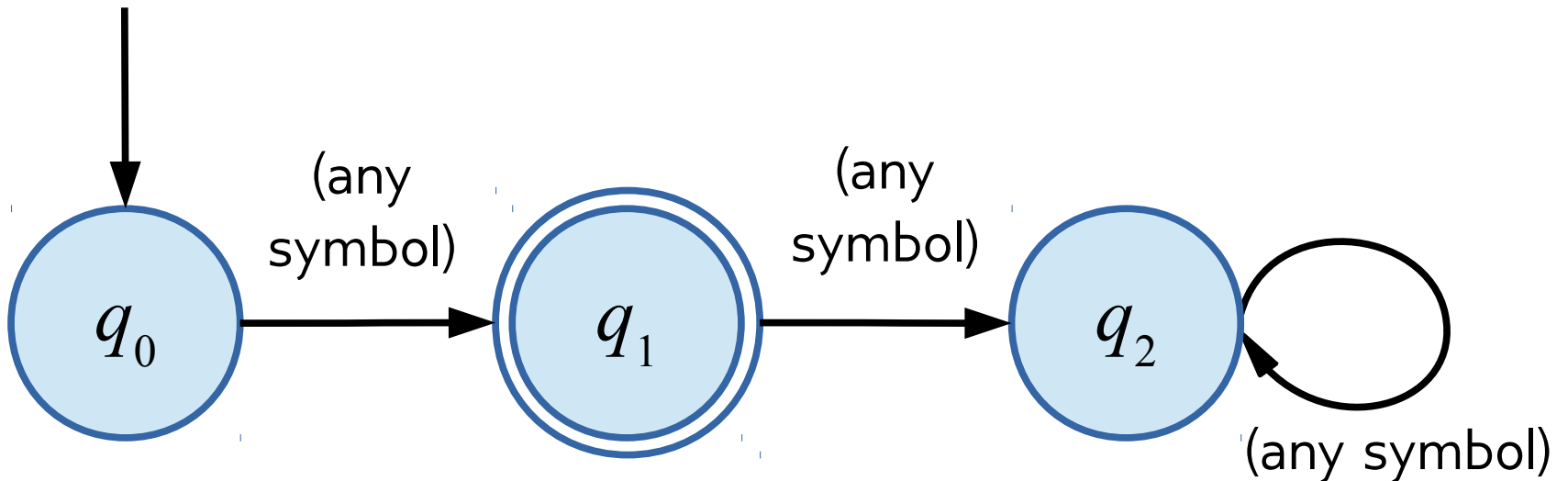


Answer: All strings

What does this DFA accept?

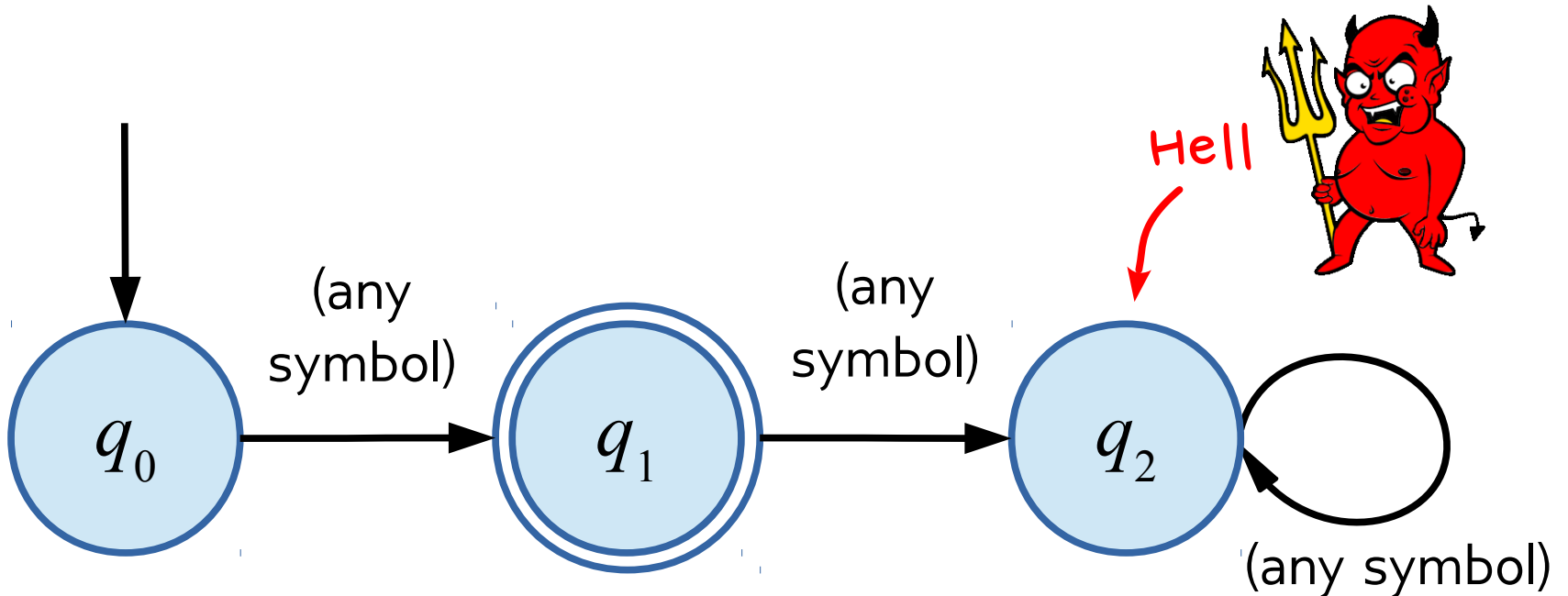


What does this DFA accept?



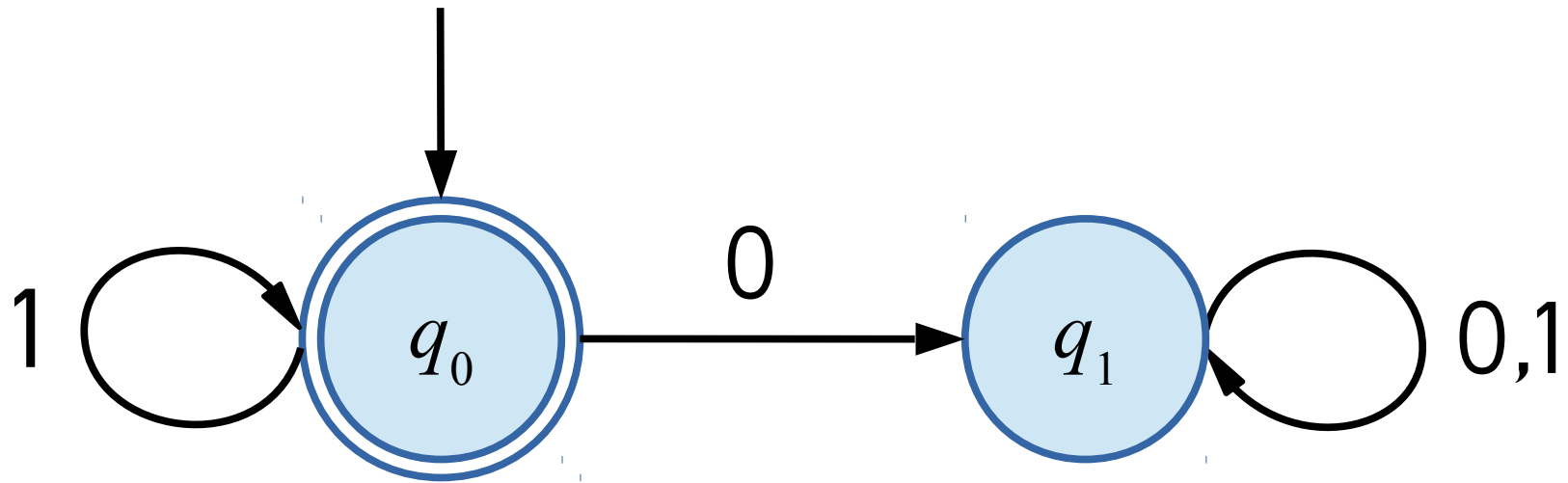
Answer: Strings of length 1

What does this DFA accept?

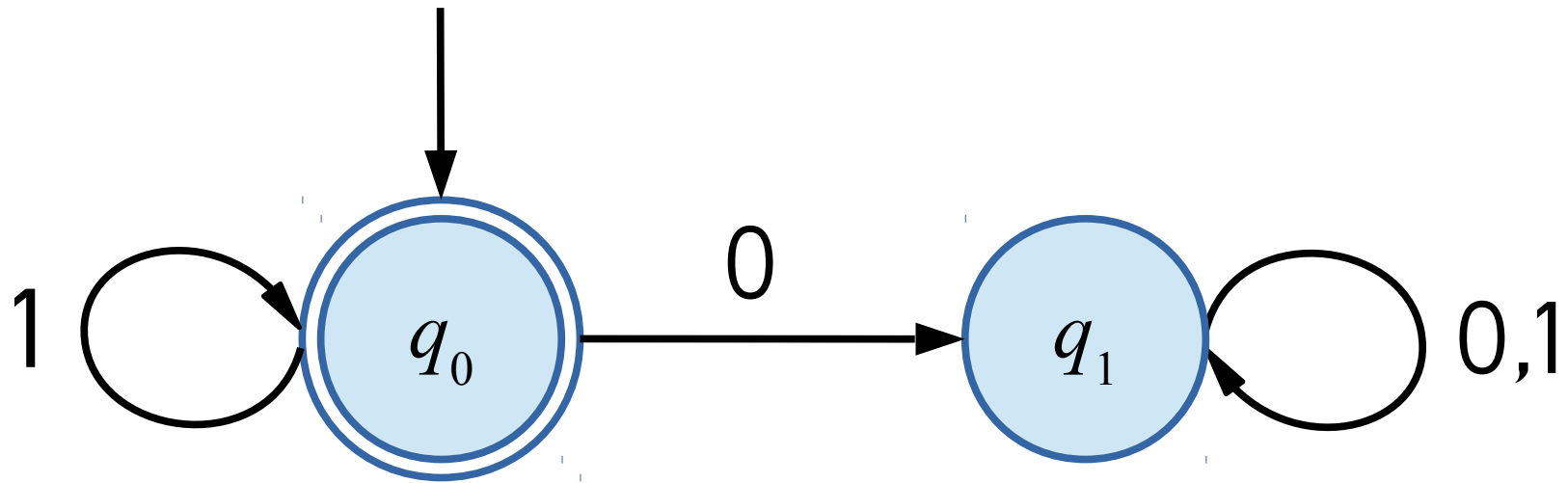


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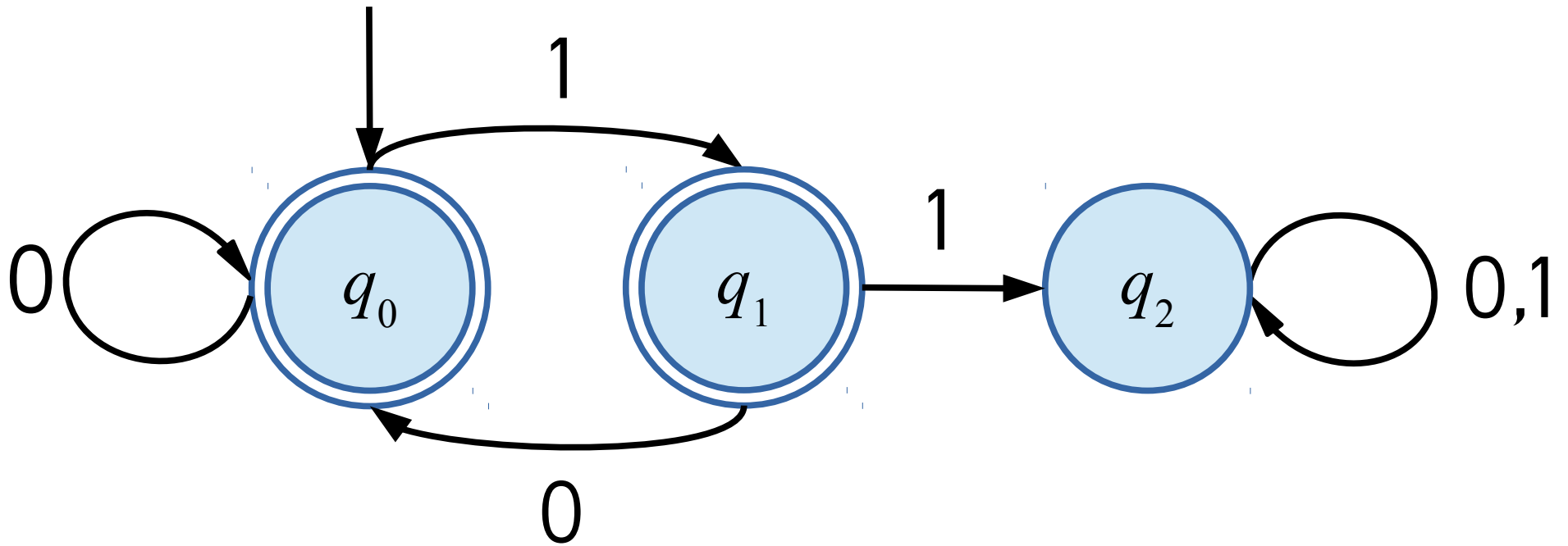


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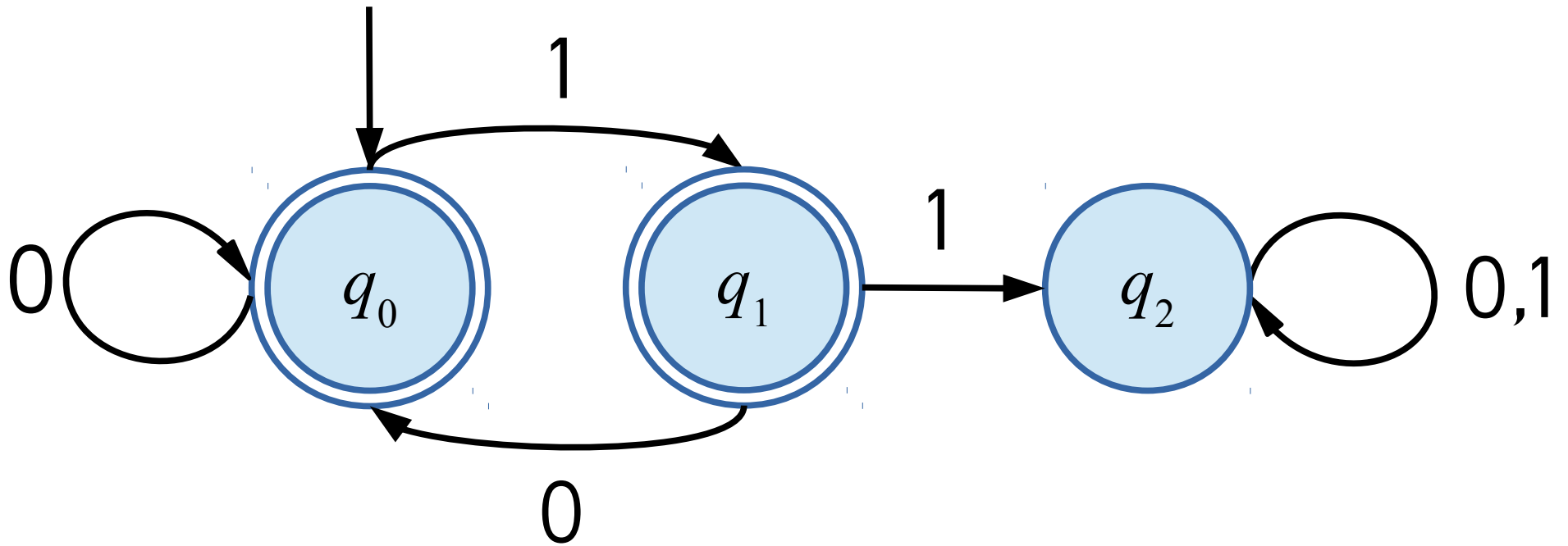


Answer: Strings containing only 1's

What does this DFA accept?



What does this DFA accept?




Answer: Strings containing
no two consecutive 1's

Language

- Given alphabet Σ , a **language** L is a set of strings over the alphabet, i.e. $L \subseteq \Sigma^*$

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Set of all possible strings over Σ
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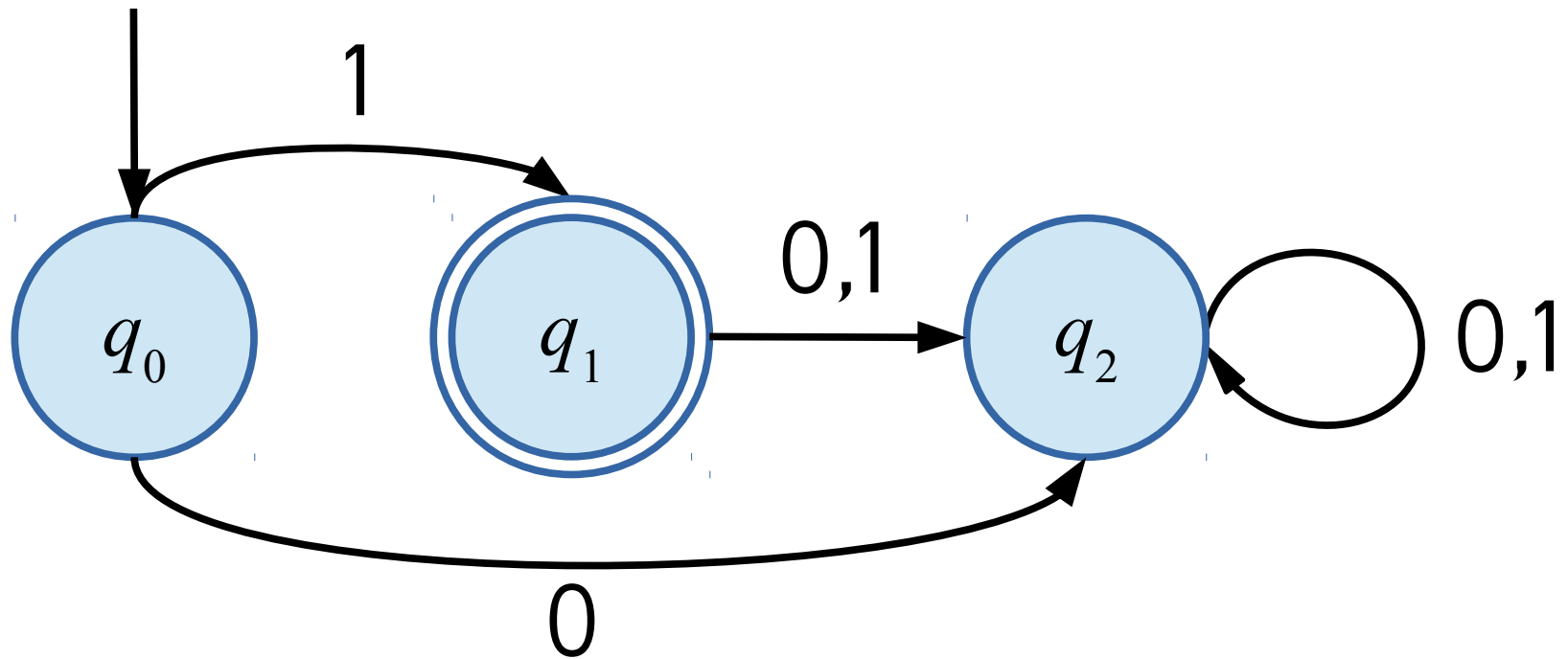


Language

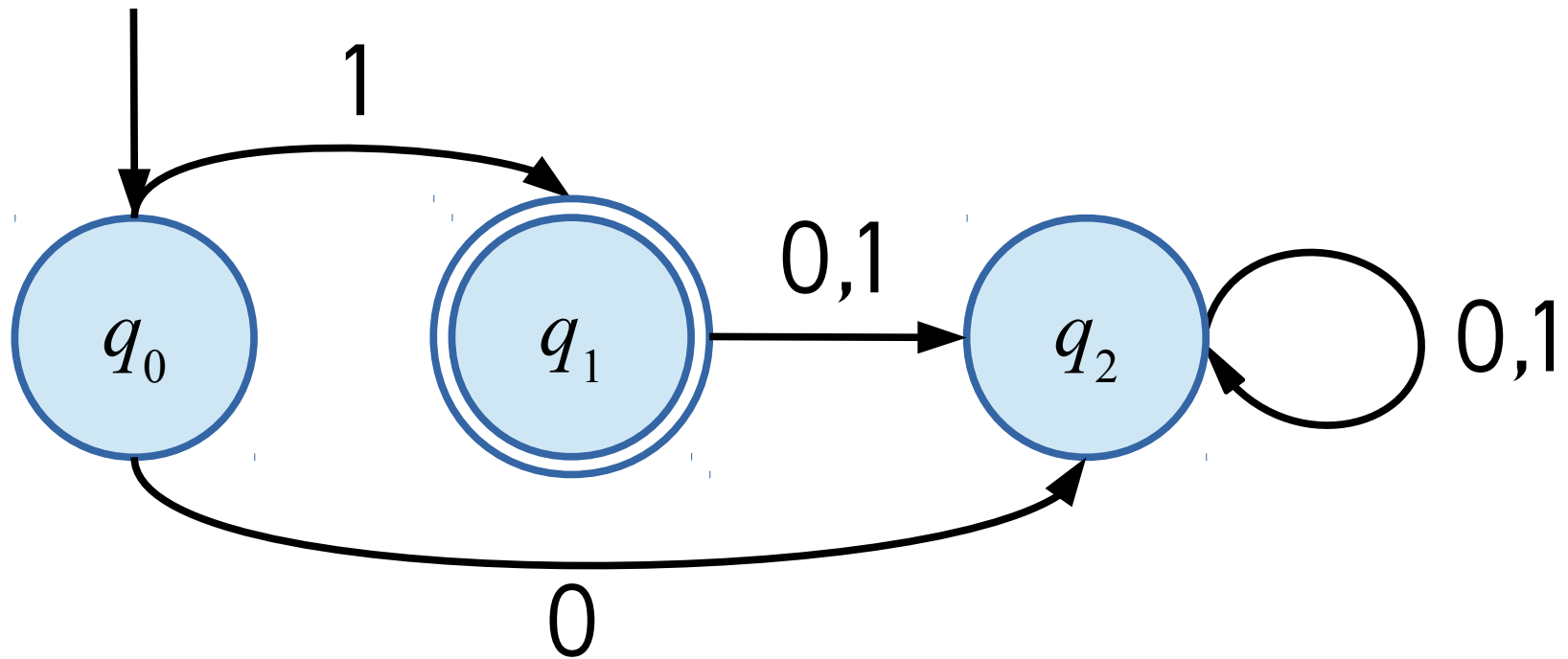
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What language does this DFA accept?

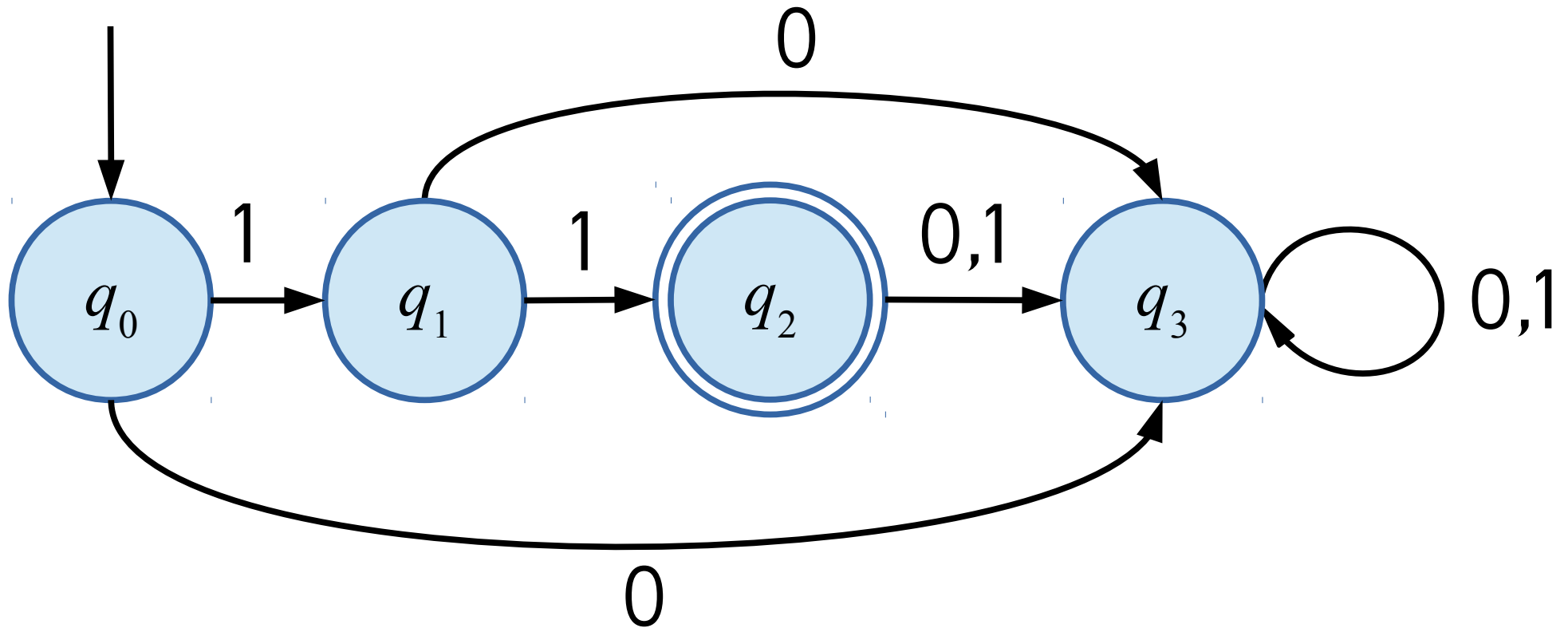


What language does this DFA accept?

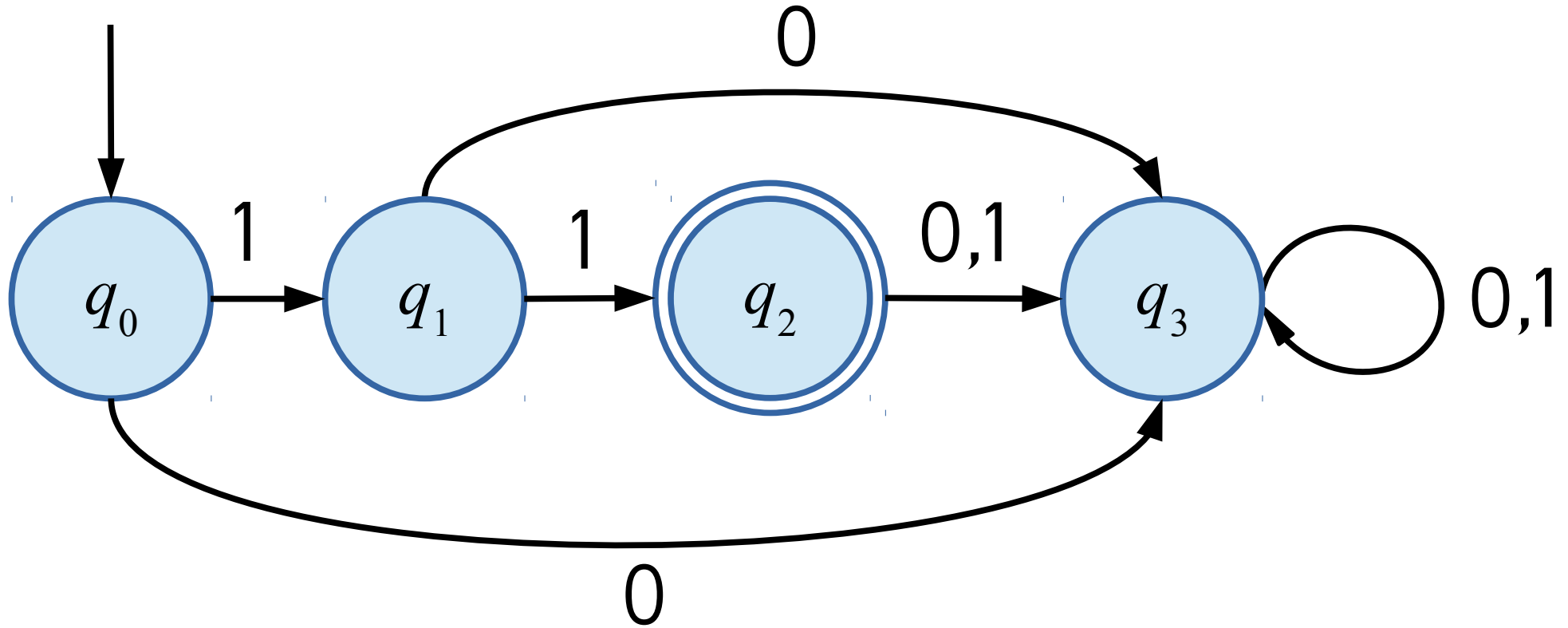


Answer: Only the string 1

What language does this DFA accept?



What language does this DFA accept?



Answer: Only the string 11

DFA's find it difficult to count

- A DFA that recognizes the language $\{1^c\}$ (the single string of c 1's) must have at least c states

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- A DFA that recognizes the language $\{1^c\}$ (the single string of c 1's) must have at least c states
 - The parent alphabet is irrelevant (but must of course contain 1)

(Proof discussion to be completed next class)