CS 2800: **Discrete Structures**

Spring 2015

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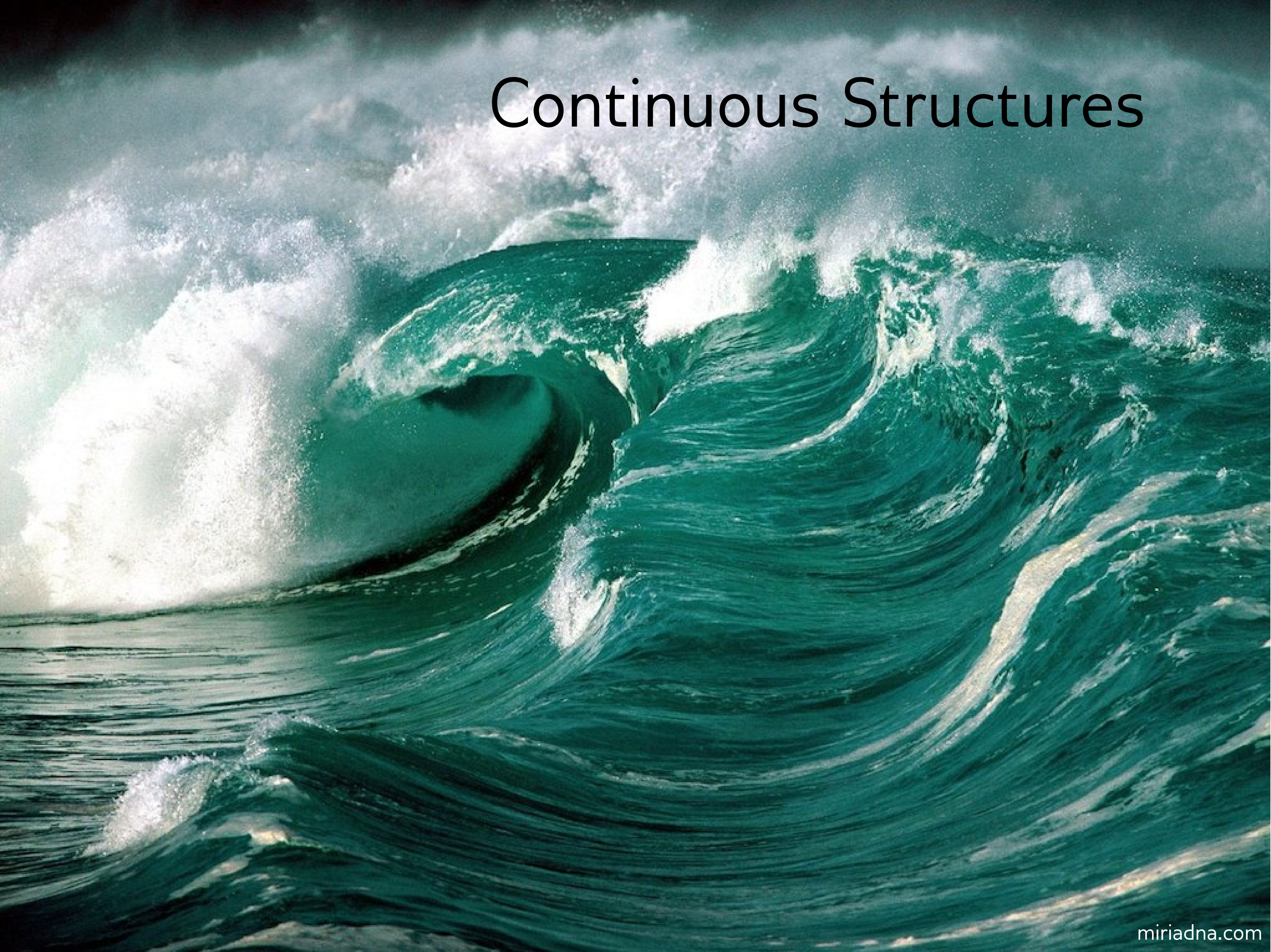
Mike George

Discrete Structures

Continuous Structures




A Discreet Structure
indieflix.com


A Discreet Structure
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Things we can count with the integers

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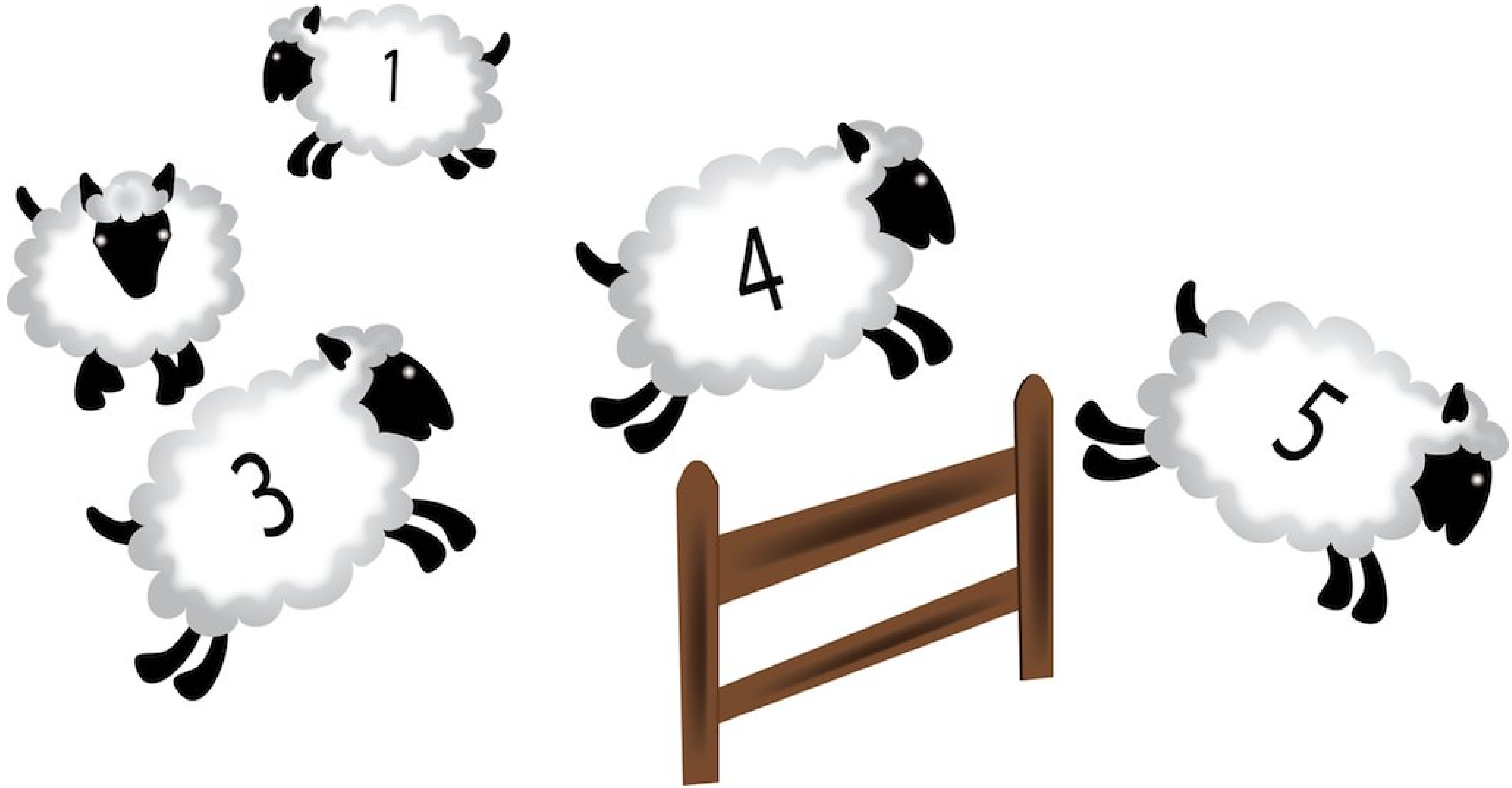
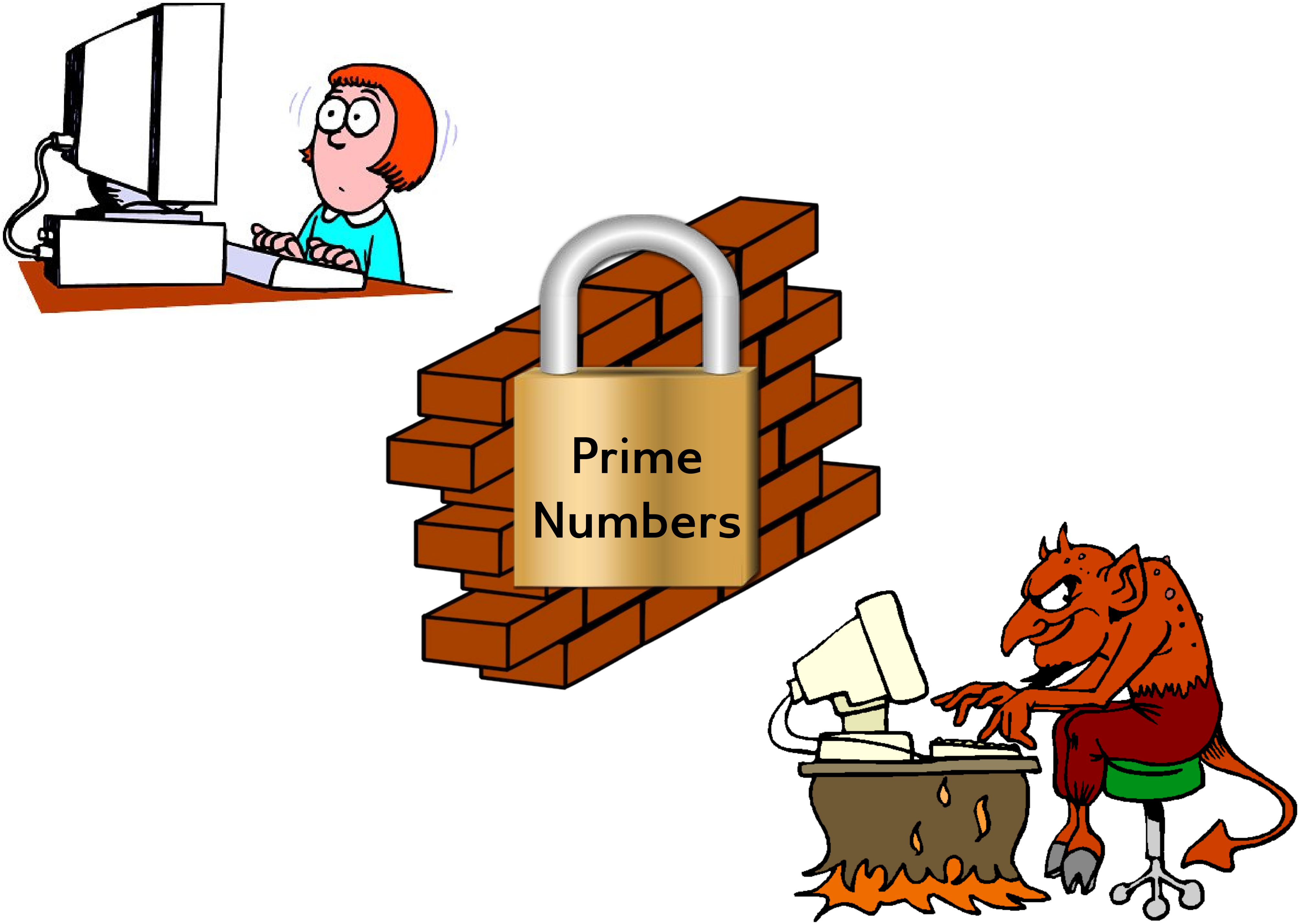


Prime Numbers

A number with exactly two divisors:

1 and **itself**

2, 3, 5, 7, 11, 13, 17...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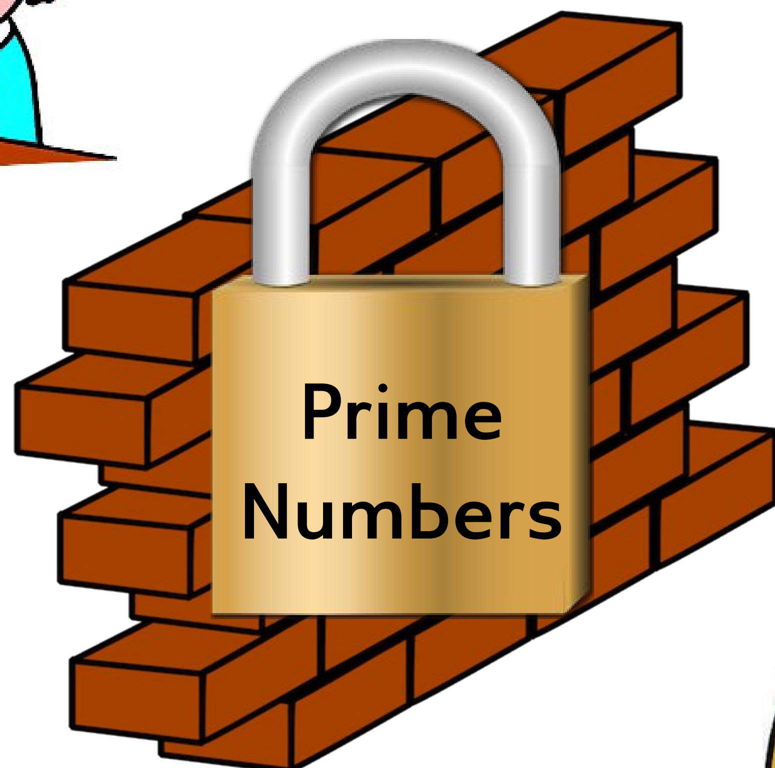
Numbers


Prime
Numbers

How many prime numbers exist?

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1,000?

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1,000,000?

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1,000?

1,000,000?

An infinite number?

How many prime numbers exist?

1,000?

1,000,000?

An infinite number

Euclid's Proof of Infinitude of Primes

(~300BC)

Euclid's Proof of Infinitude of Primes

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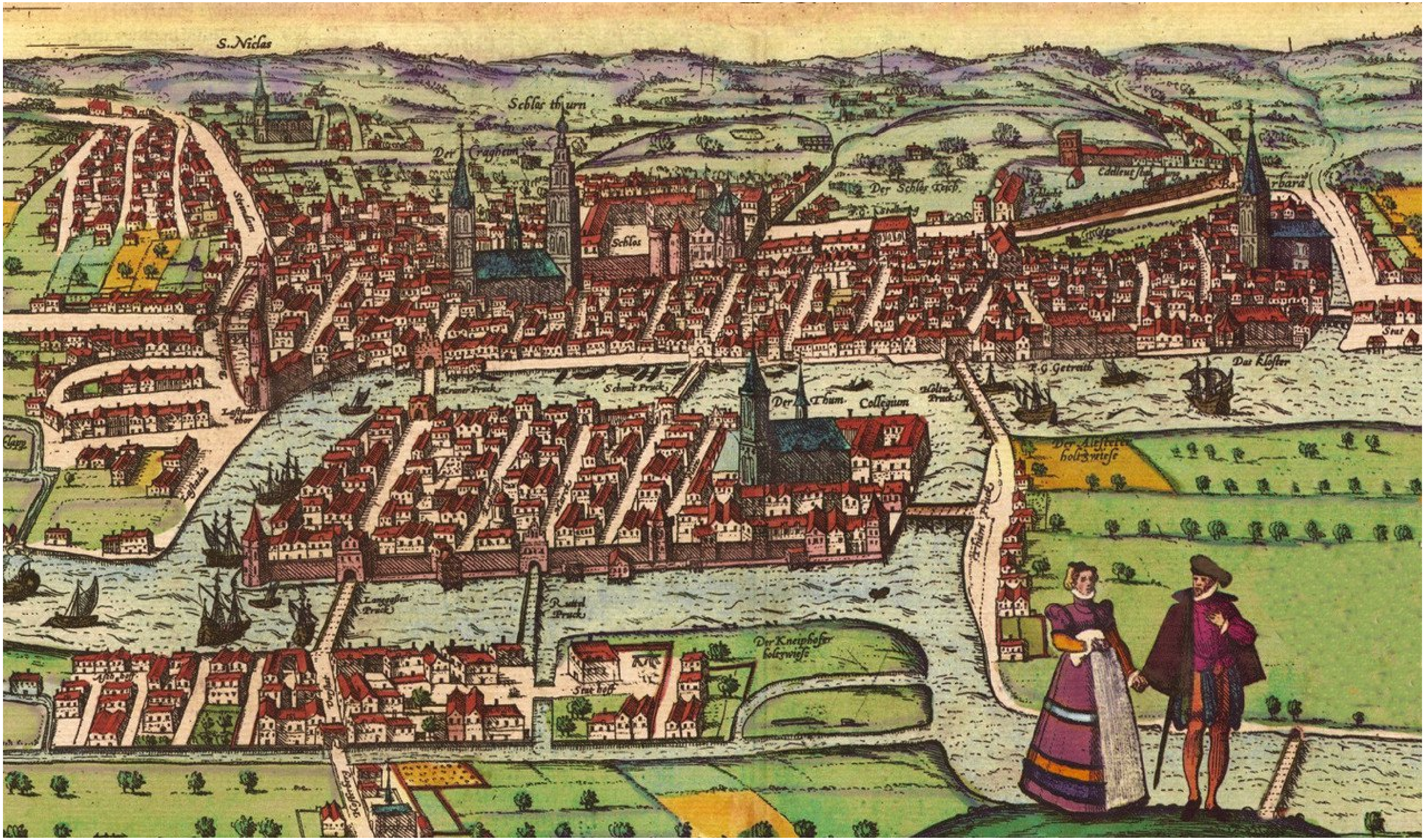
- So n has a **prime divisor greater than p**

Contradiction!!!

Discrete Structures

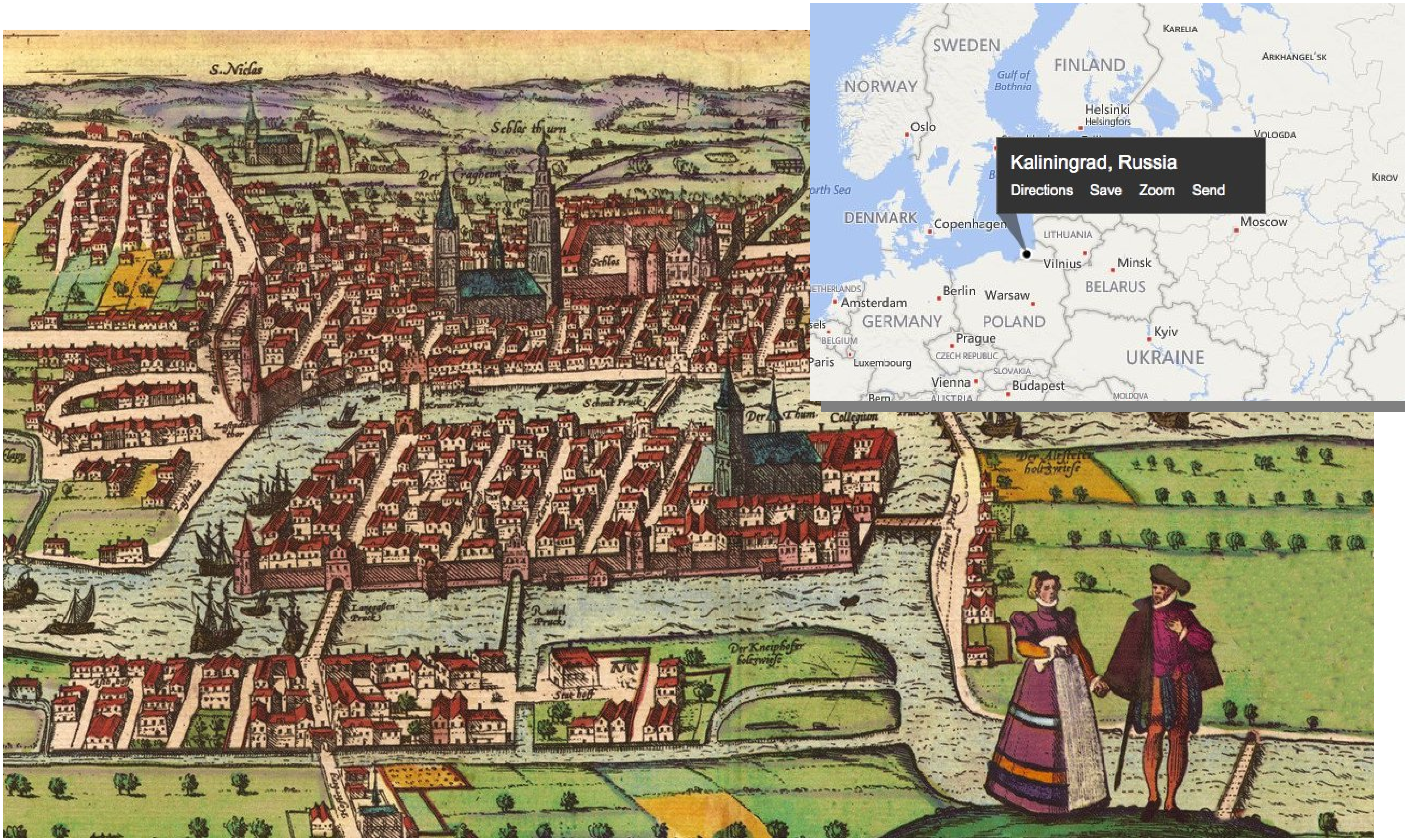
- Number theory
- Proof systems
- Sets, functions, relations
- Counting and probability

Bridges of Königsberg



Braun & Hogenberg, “Civitates Orbis Terrarum”, Cologne 1585. Photoshopped to clean up right side and add 7th bridge.

Bridges of Königsberg



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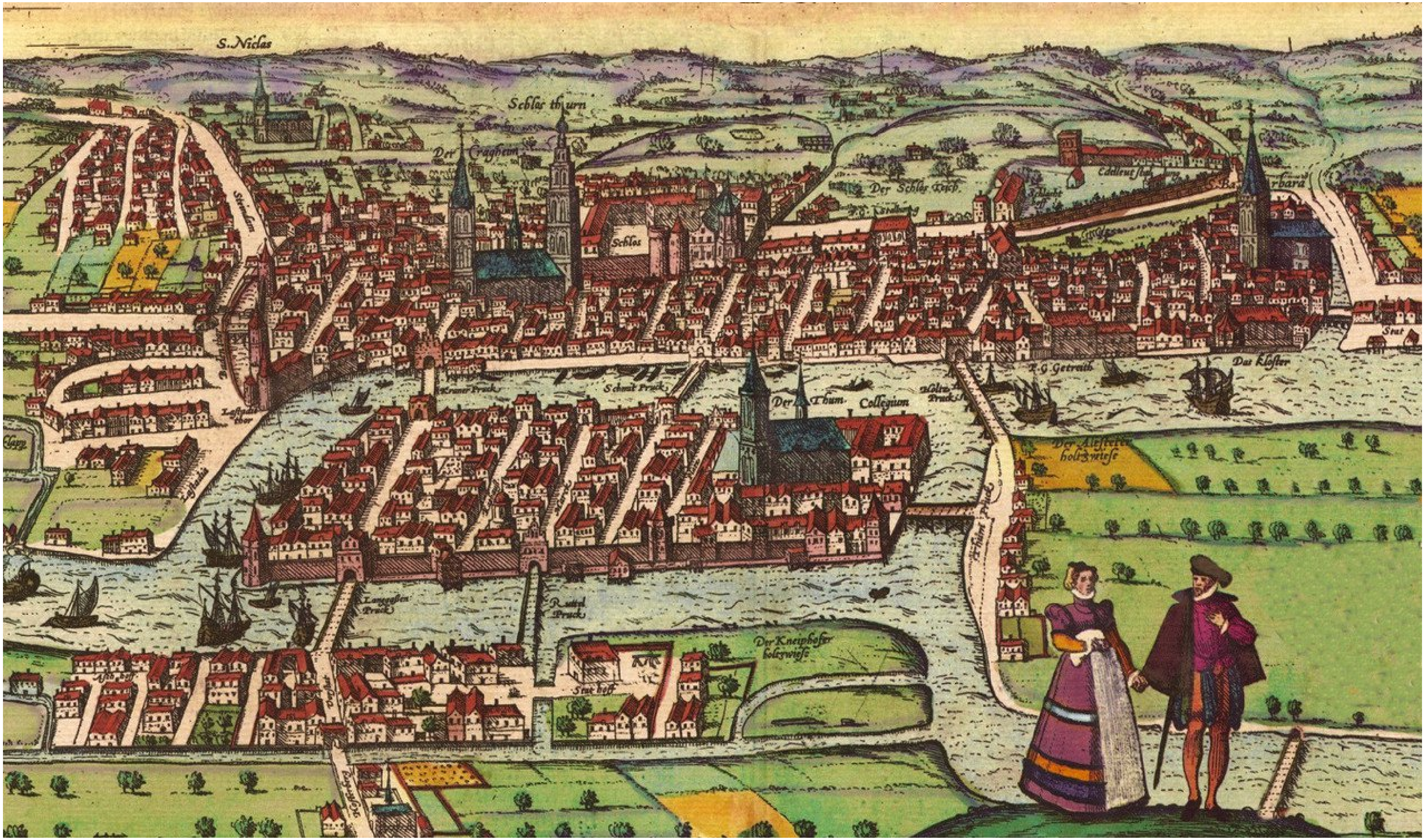

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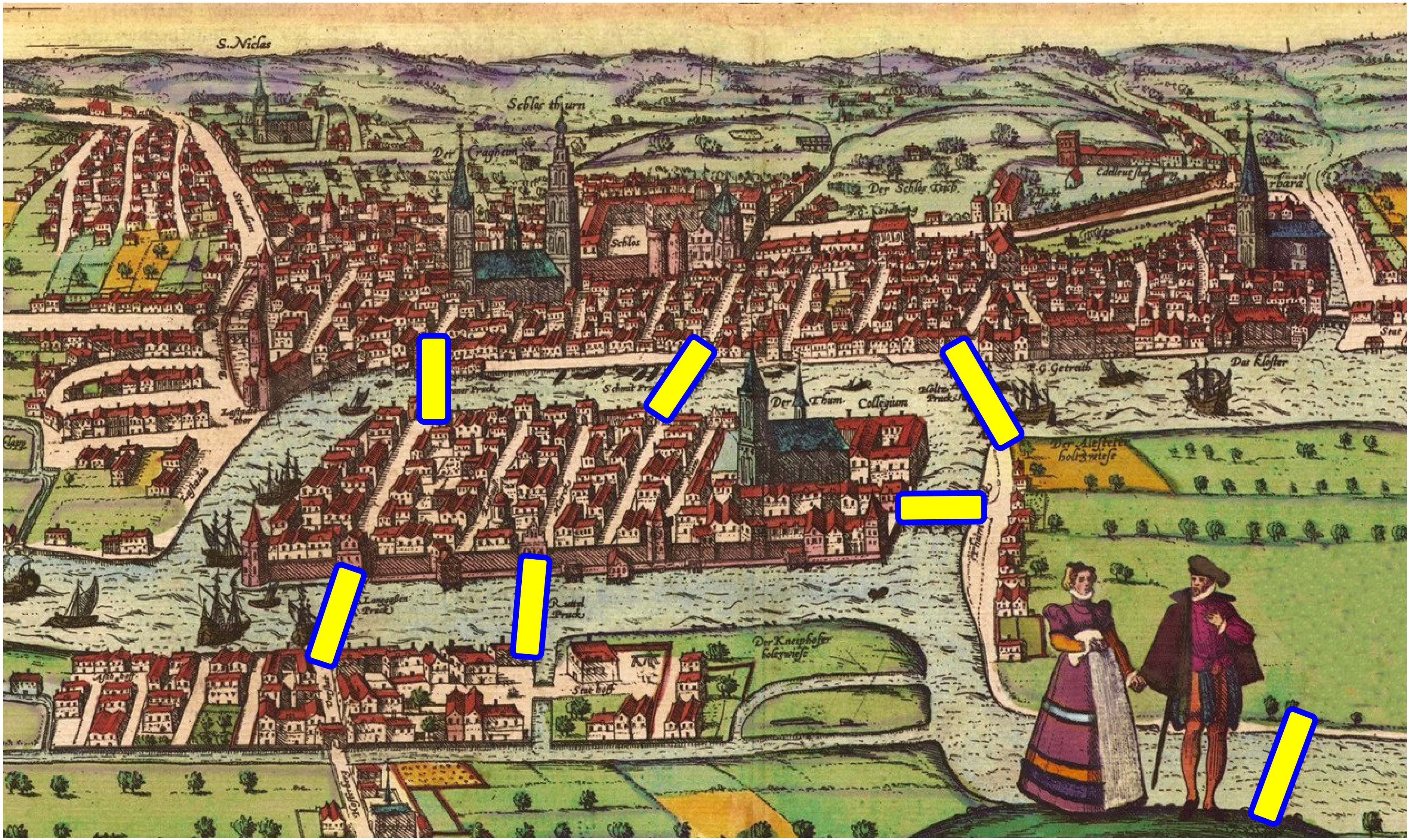

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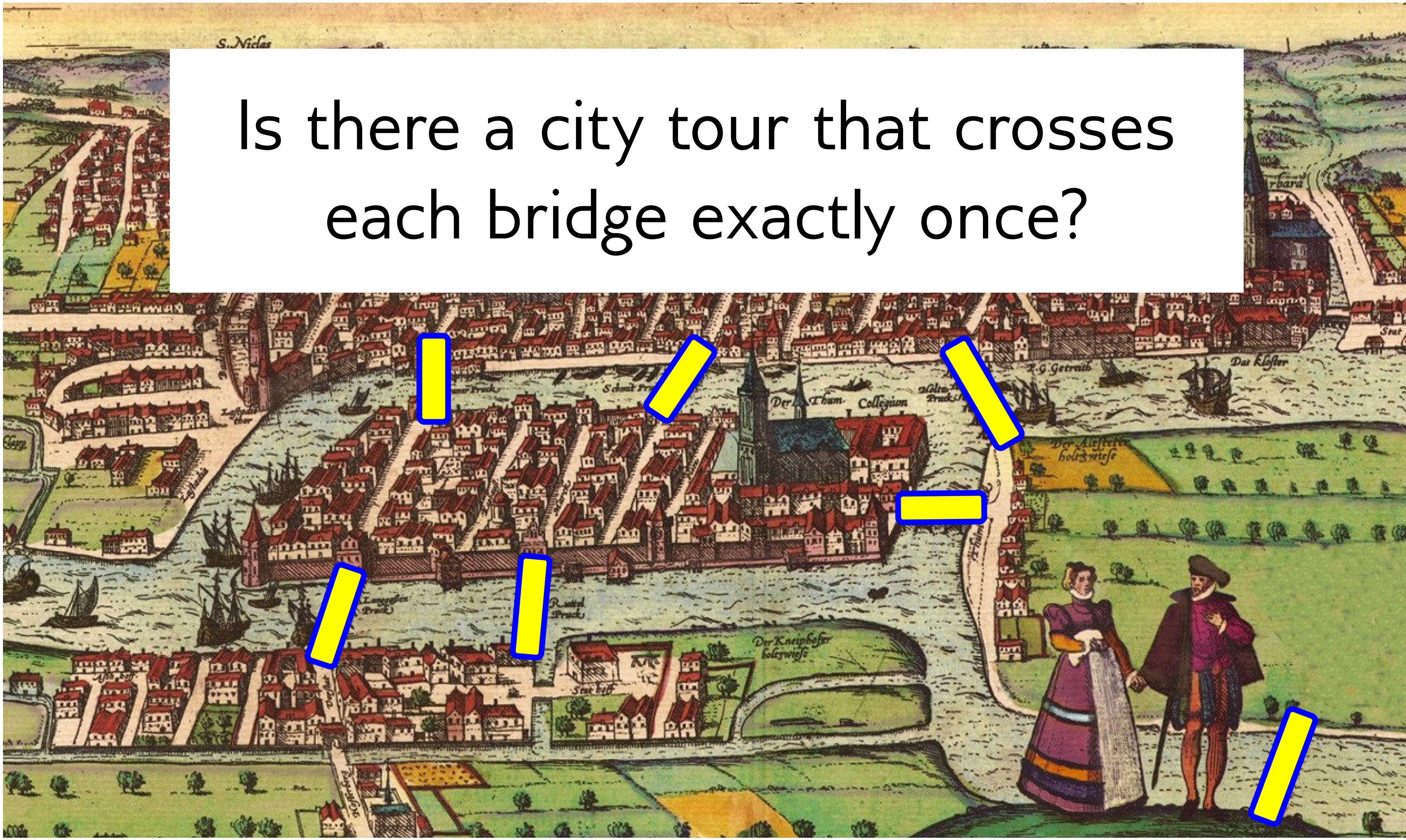
Bridges of Königsberg

Is there a city tour that crosses each bridge exactly once?



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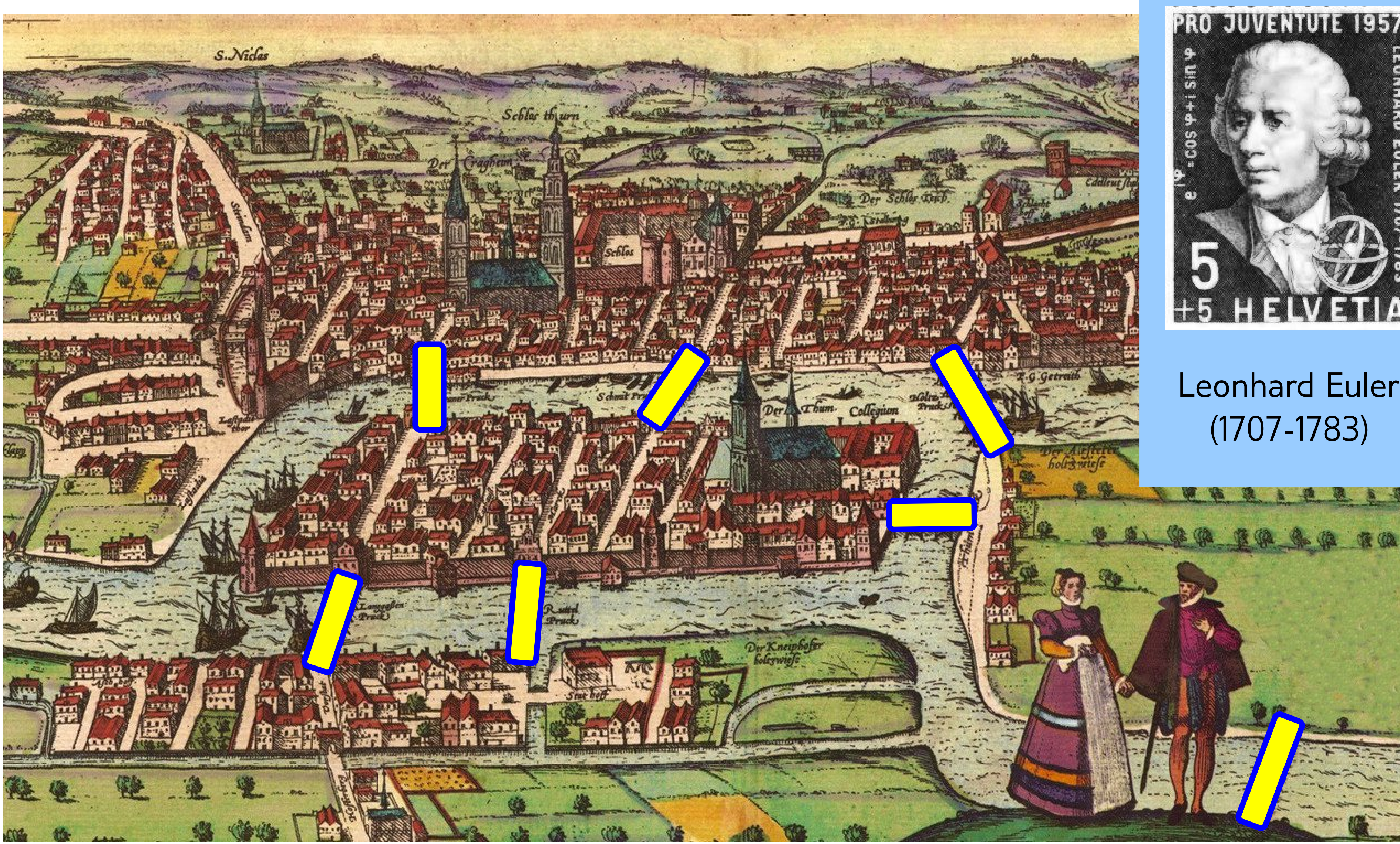
Bridges of Königsberg



Leonhard Euler
(1707-1783)

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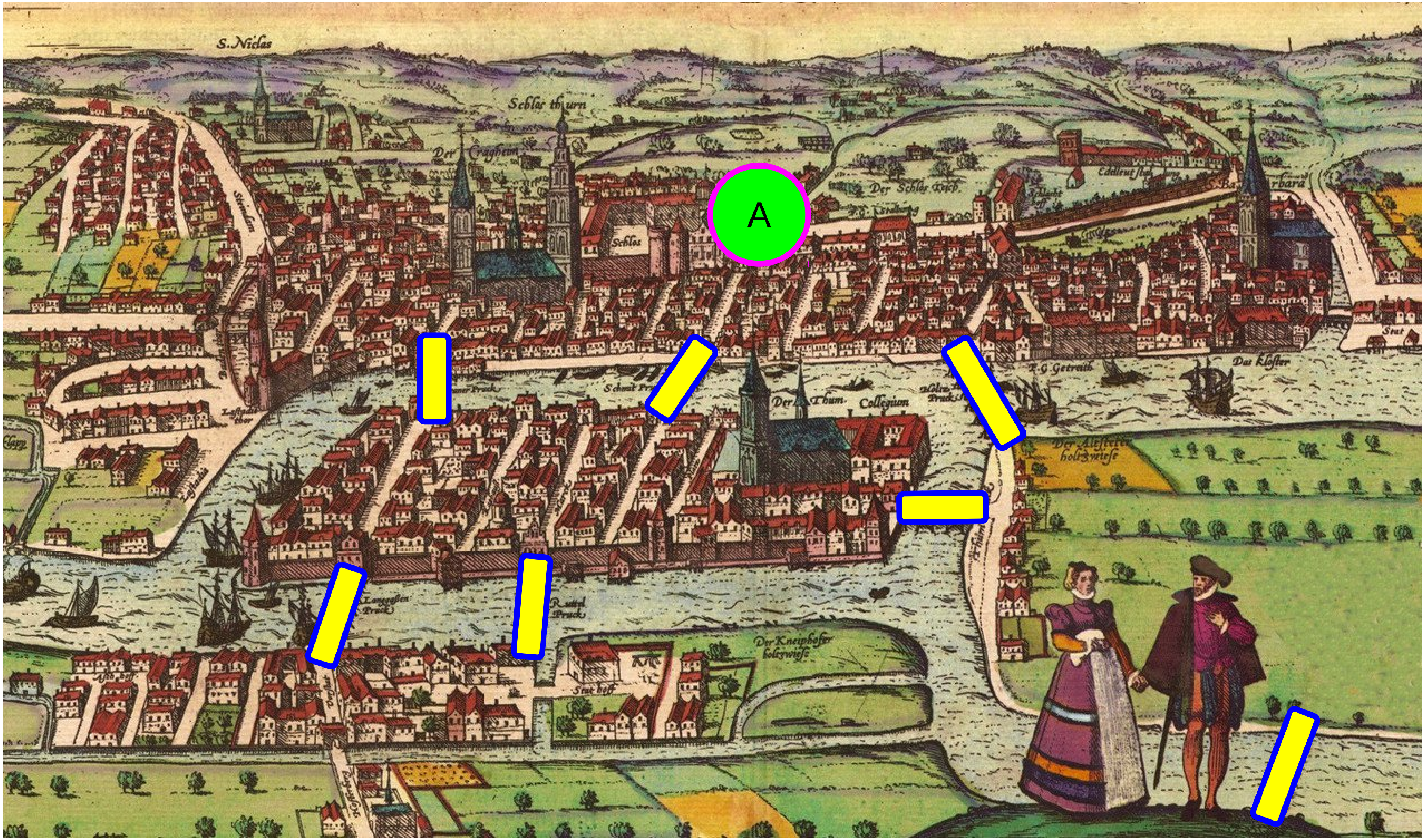
Bridges of Königsberg



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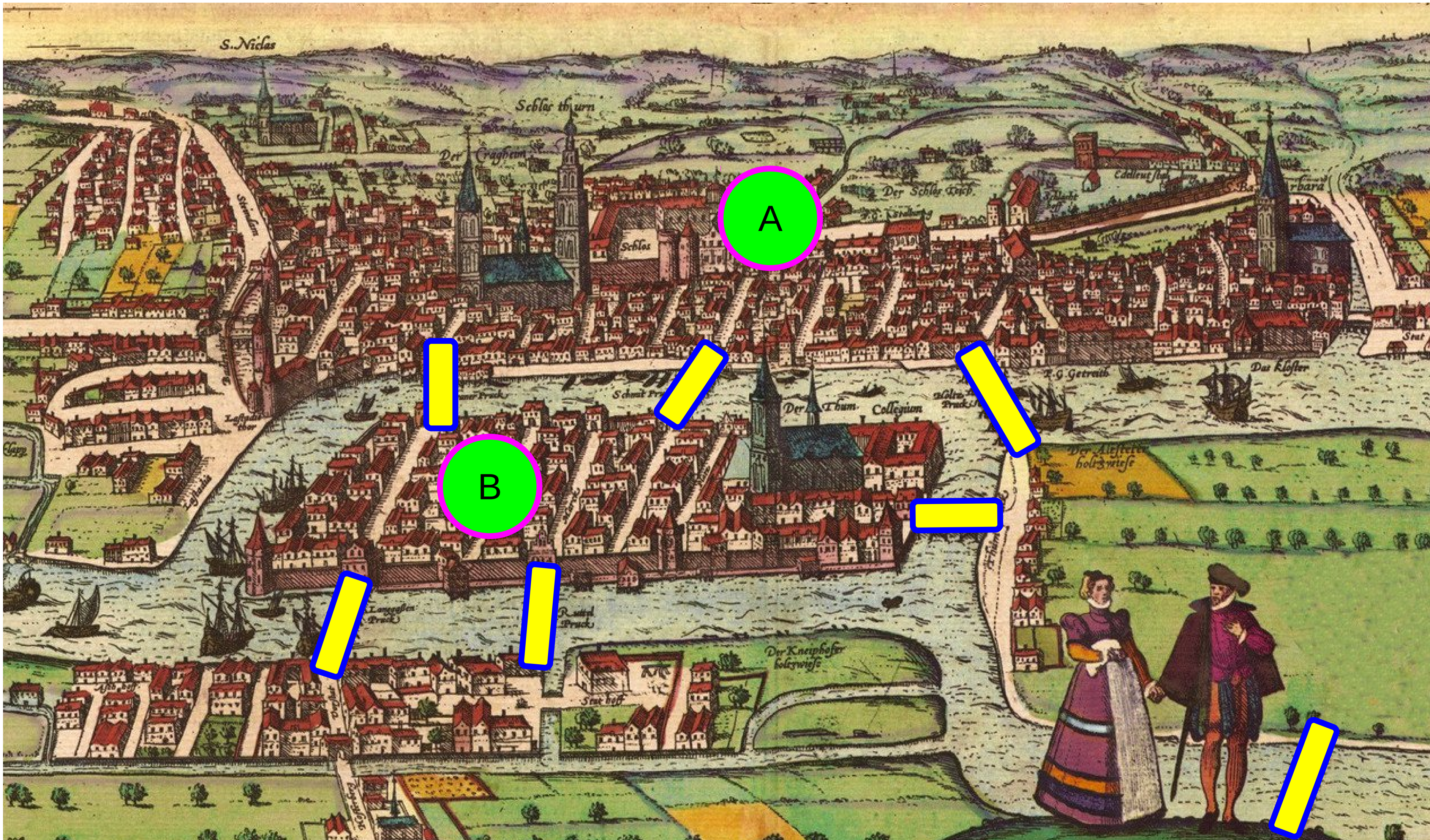

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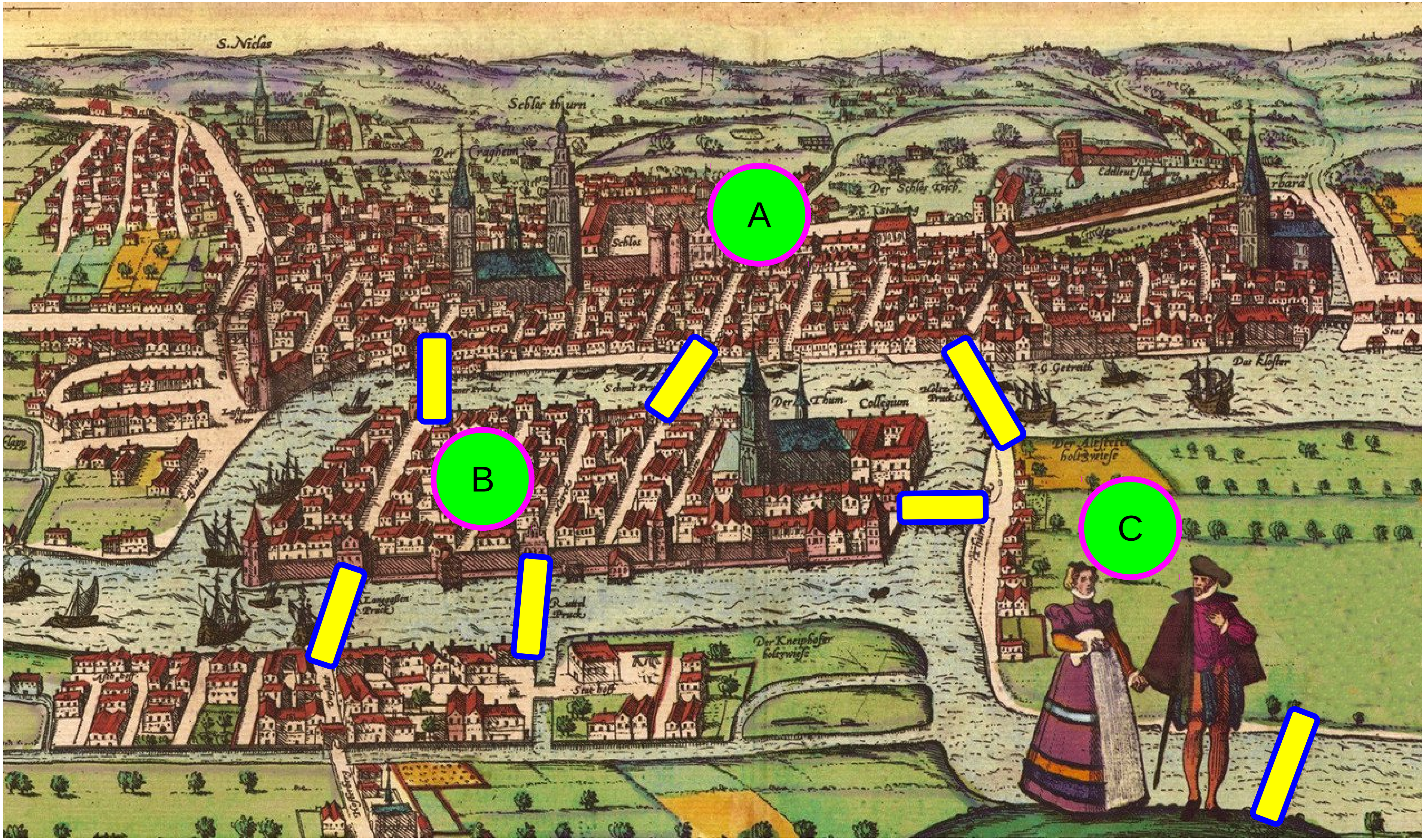

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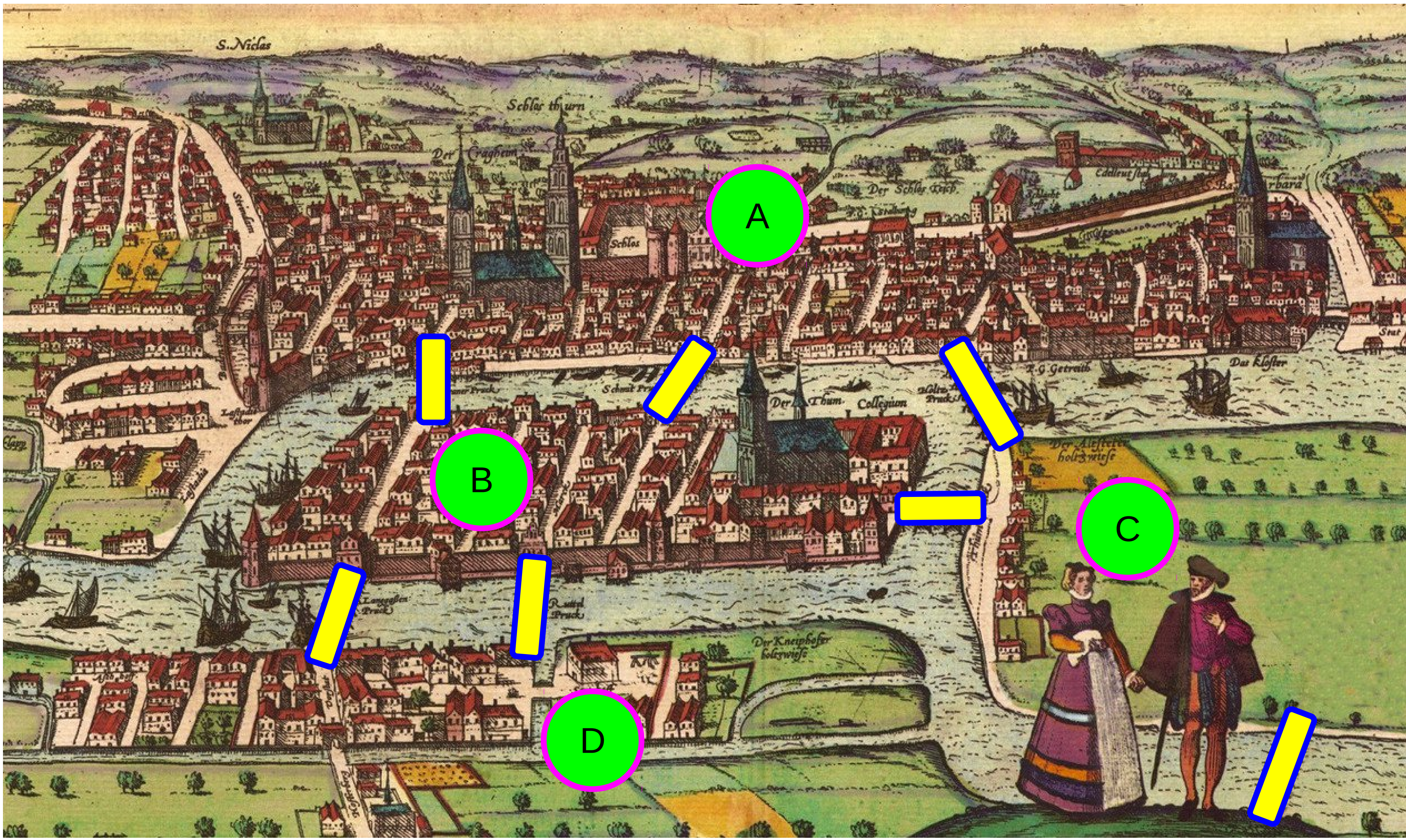

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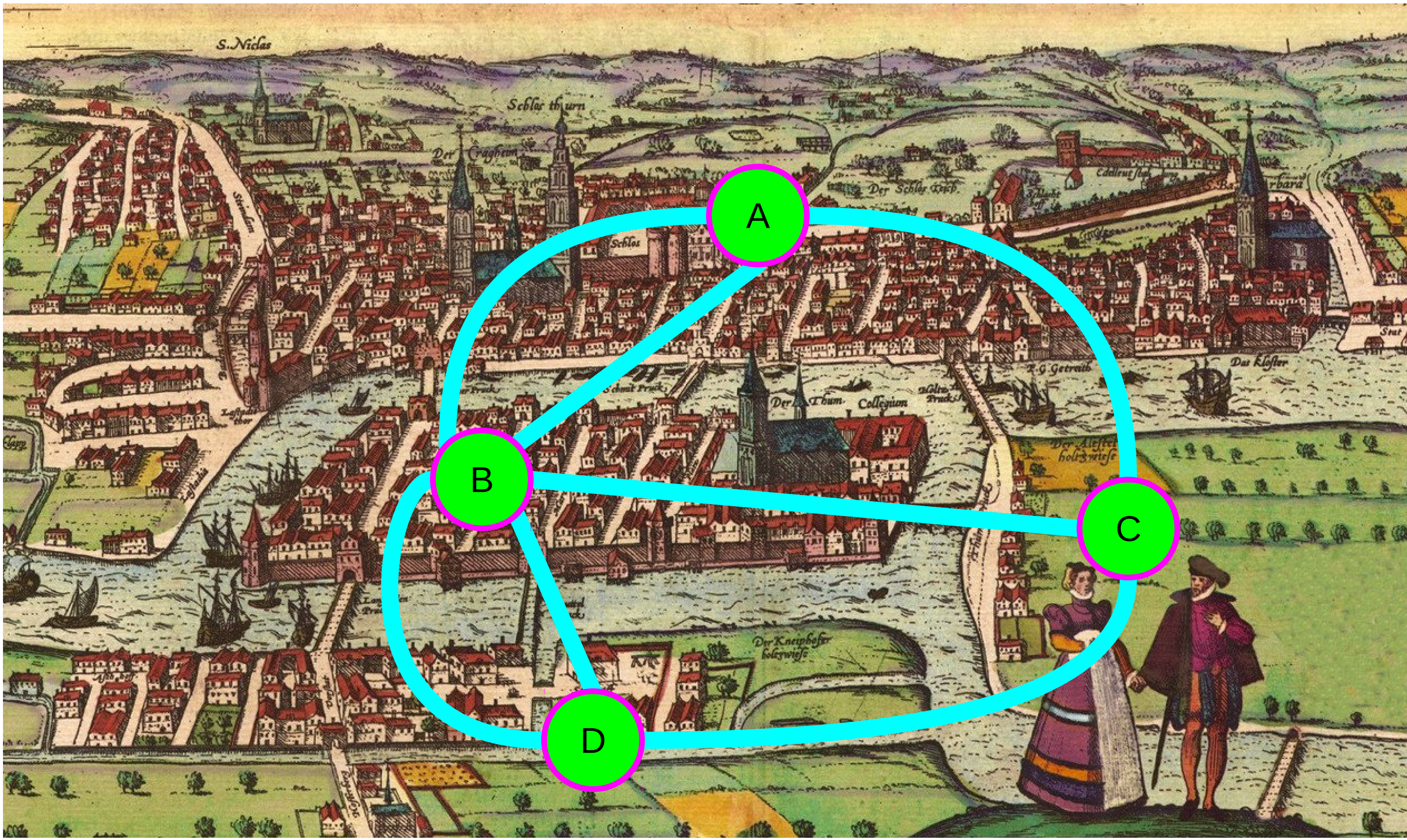

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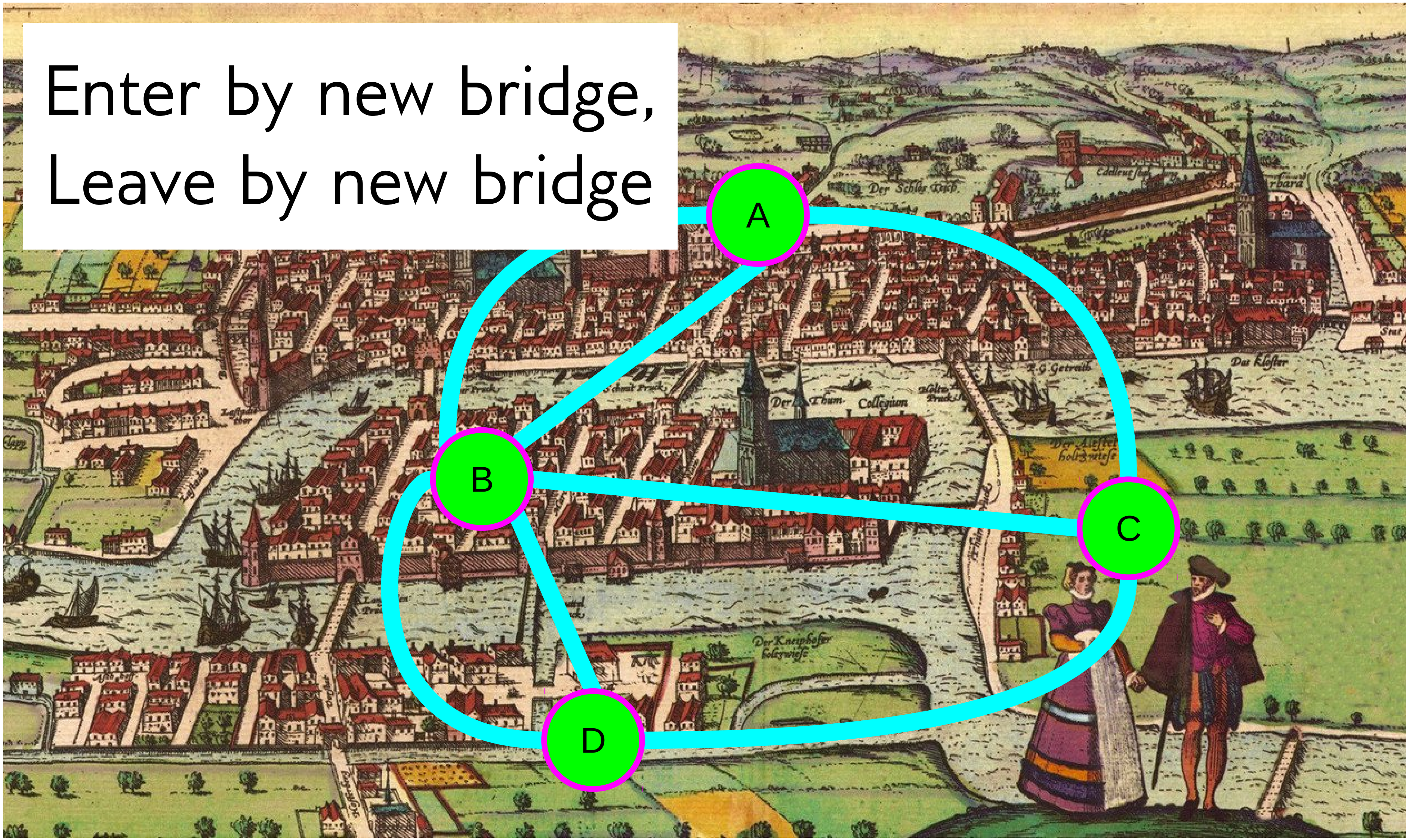
Bridges of Königsberg

Enter by new bridge,
Leave by new bridge



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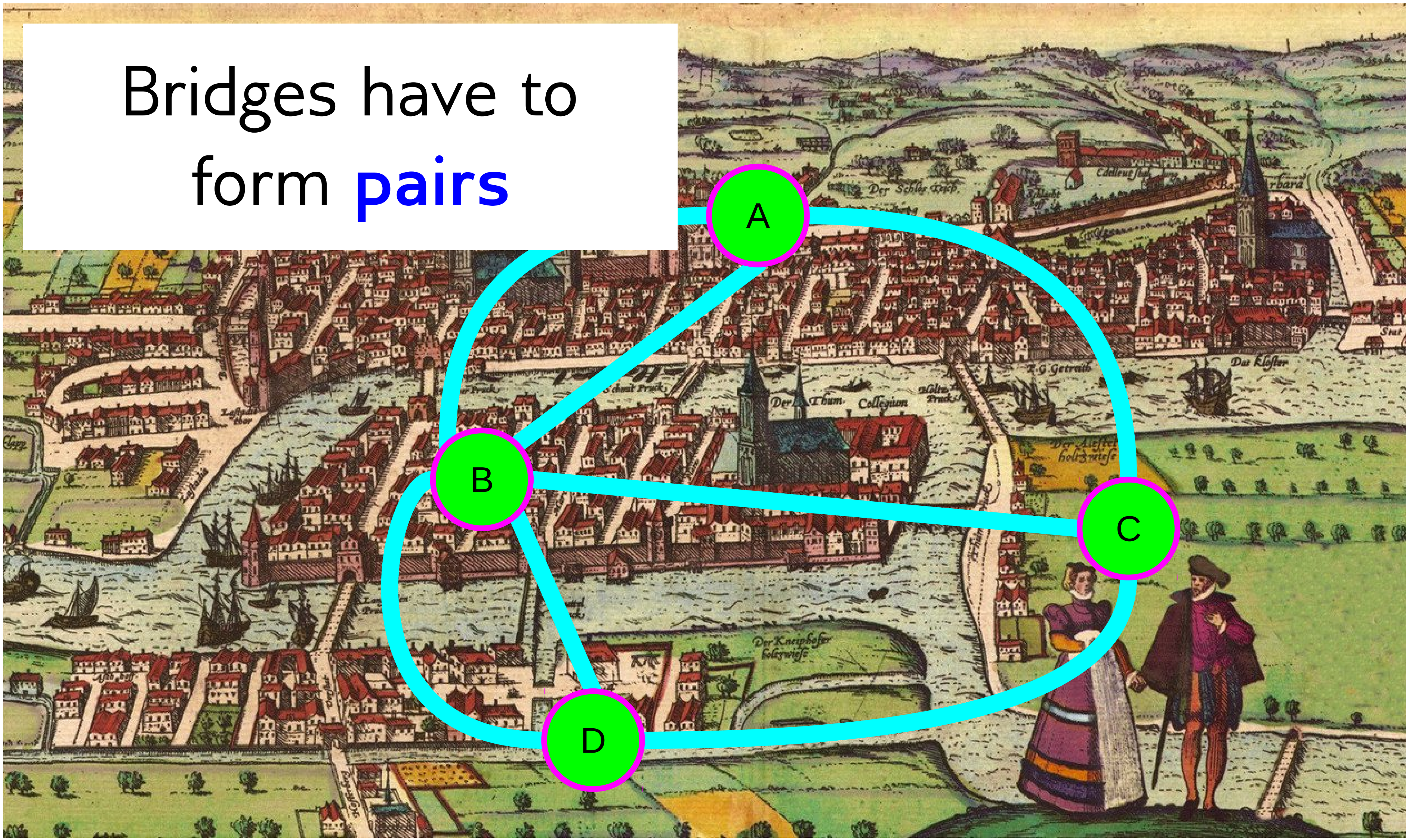
Bridges of Königsberg

Bridges have to form **pairs**



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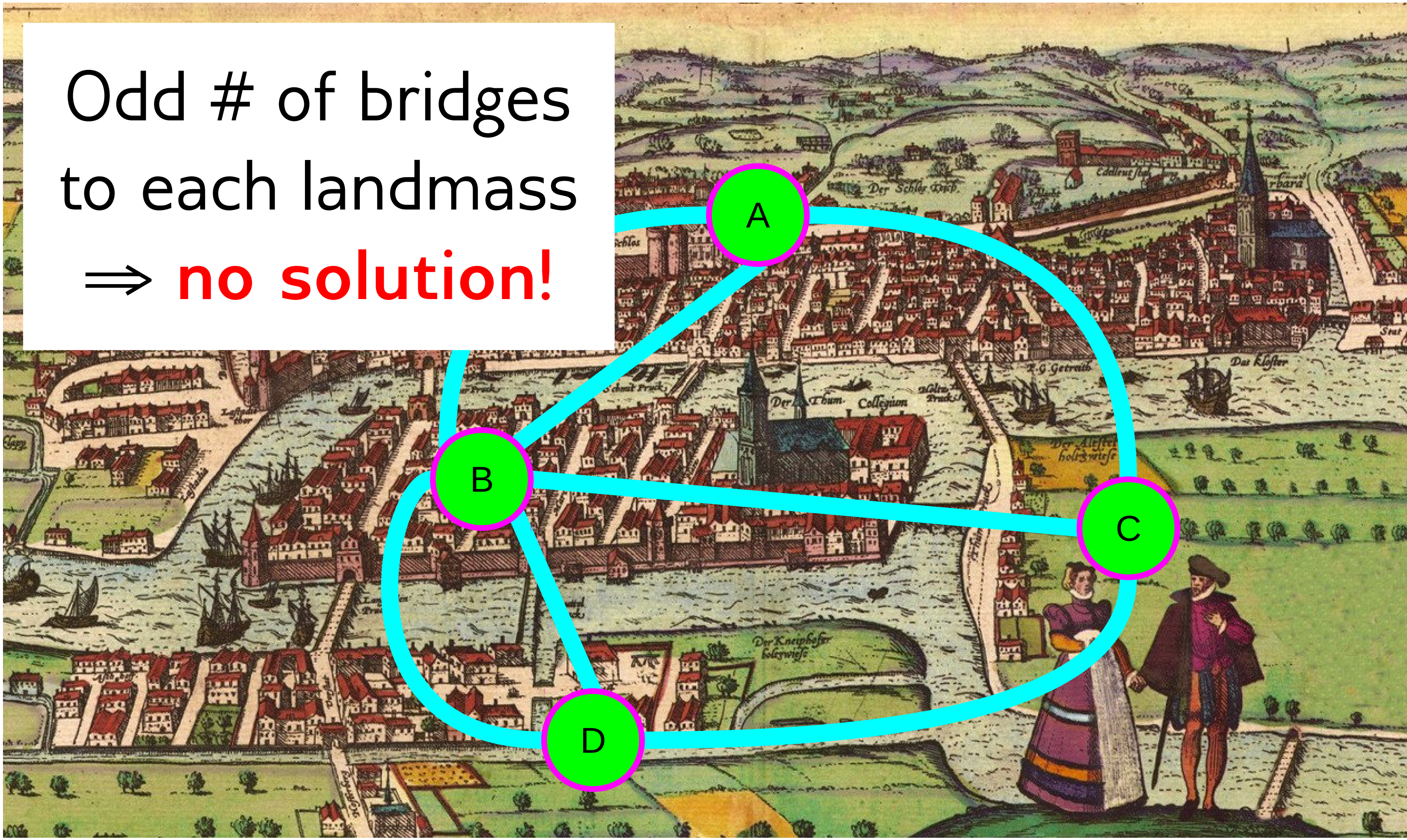
Bridges of Königsberg

Odd # of bridges
to each landmass
⇒ **no solution!**



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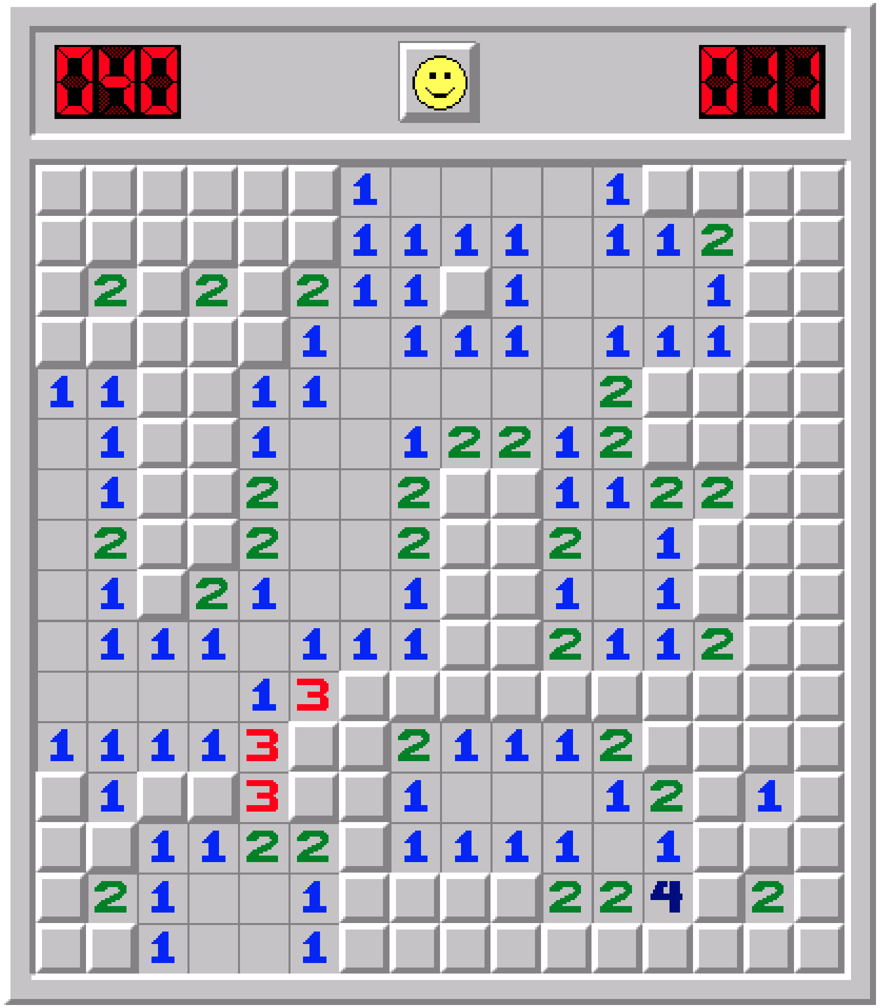
Bridges of Königsberg

- Cross each bridge once: Euler Path
  - Easy for a computer to calculate
- Visit each landmass once: Hamiltonian Path
  - Probably very hard for a computer to calculate
  - If you can find an efficient solution, you will get $1M and undying fame (answers "P = NP?")
  - (Will also break modern crypto, collapse the banking system, revolutionize automated mathematics and science, bring about world peace...)

You'll also be terrific at Minesweeper



Discrete Structures

- Number theory
- Proof systems
- Sets, functions, relations
- Counting and probability
- Graph theory
- Models of computation, automata, complexity

This sentence is false.

This sentence is false.

If true, it is false

If false, it is true

This sentence is false.

If true, it is false

If false, it is true

Discrete Structures

- Number theory
- Proof systems
- Sets, functions, relations
- Counting and probability
- Graph theory
- Models of computation, automata, complexity
- Logic
- Decidability, computability

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