1. Draw a DFA that recognizes strings in $\{a, b\}^*$ having an even number of as and an odd number of bs.

2. The set S of strings of balanced parentheses is defined inductively by the following three rules:

    i. $\epsilon \in S$

    ii. If $x \in S$ then the string "(x)" is in S

    iii. If x and y are in S, then $xy \in S$

    For example, the string $(()())() \in S$, but $))(()$ is not.

    (a) Justify the statement $(()())() \in S$ using the rules above.

    (b) Use structural induction to prove that for all $x \in S$, x has the same number of left parentheses as right parentheses.

    (c) Use the pumping lemma to show that S is not DFA-recognizable.

3. (Optional, not graded) get ready to enjoy a relaxing break!