

1. Consider the following experiment. To decide where to travel to, a traveler throws a dart at a map of a region. The region contains four states, named  $a$ ,  $b$ ,  $c$ , and  $d$ . Assume that the probability of hitting a given state is proportional to the area of the state, and that the probability of hitting one of the four states is one (that is, the traveler never misses the map).

Let  $X$  be the variable giving the cost of a ticket to the selected state, and let  $Y$  be the variable giving the population of the selected state. Facts about the states are contained in the following table:

State	Area	Cost	Population
$a$	1000 km <sup>2</sup>	\$350	900,000
$b$	1000 km <sup>2</sup>	\$1000	10,000,000
$c$	3500 km <sup>2</sup>	\$575	500,000
$d$	2000 km <sup>2</sup>	\$350	500,000

- (a) What is the expected value of  $X$ ?
- (b) What is the variance of  $Y$ ?
- (c) Are  $X$  and  $Y$  independent? Prove your answer.
2. Let  $X$  and  $Y$  be random variables on a probability space  $(S, P)$ . Show that if  $X$  and  $Y$  are independent, then  $Var(X + Y) = Var(X) + Var(Y)$ . *Note:* this is an easy proof to find in lots of places. You may consult them if you wish, but make sure that your submitted work is your own.