

## Lectures 9 and 10

Random variables such as role of a dice.

Probability distributions

Problem if uniform distribution for a countably infinite set

$$a + a + a^2 + \dots + a^n = \frac{1-a^{n+1}}{1-a}$$

Joint probability  $\text{Prob}(A, B)$

Conditional probability  $\text{Prob}(A|B)$ .

$$\text{Prob}(A|B) = \frac{\text{Prob}(A \cap B)}{\text{Prob}(B)}$$

Independence

If events  $A$  and  $B$  are independent

$$\text{Prob}(A \cap B) = \text{Prob}(A)\text{Prob}(B)$$

Bayes rule

$$\text{Prob}(B|A) = \frac{\text{Prob}(A|B)\text{Prob}(B)}{\text{Prob}(A)}$$

Expectation of a random variable  $\sum_x xp(x)$

Linearity of expectation  $E(x + y) = E(x) + E(y)$

Variance  $\sigma(x) = E(x - E(x))^2$

$$\sigma(x) = E(x^2) - E(x)^2$$

$\text{Var}(x + y) = \text{Var}(x) + \text{Var}(y) + 2E(xy)$

If  $x$  and  $y$  are independent  $E(xy) = E(x)E(y)$  and  $\sigma^2(x + y) = \sigma^2(x) + \sigma^2(y)$

Standard deviation is square root of variance.