

Lecture 5

Sets not countably infinite

functions mapping $\{1, 2, 3, \dots\} \rightarrow \{0, 1\}$
The class of all sets of integers

The diagonal function.

If f_1, f_2, f_3, \dots is a list of functions, then $f_D(i) = f_i(i) + 1 \pmod{2}$ is not on the list.

Can diagonalize over a list of computer programs and hence over all computable partial functions.

$$f_D(i) = \begin{cases} p_i(i) + 1 & \text{if } p_i(i) \text{ halts} \\ 0 & \text{otherwise} \end{cases}$$

is a non computable function.

Halting problem is not computable. Otherwise $f_D(i)$ would be computable.

There exists a theorem that is true but not provable. Otherwise we could solve the halting problem.