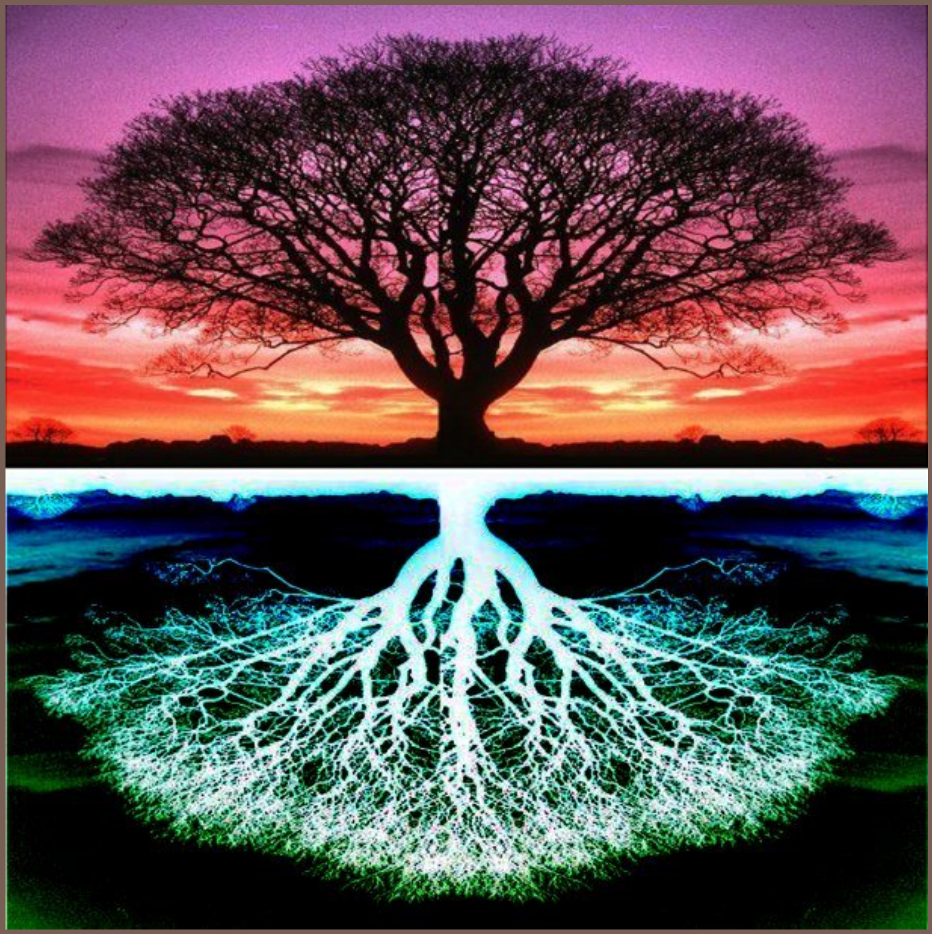


TREES

Lecture 12
CS2110 – Spring 2018

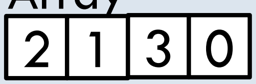
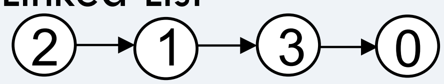
Important Announcements

- A4 is out now and due two weeks from today. Have fun, and start early!

Data Structures

- There are different ways of storing data, called data structures
- Each data structure has operations that it is good at and operations that it is bad at
- For any application, you want to choose a data structure that is good at the things you do often

Example Data Structures

Data Structure	add(val x)	lookup(int i)
Array 	$O(n)$	$O(1)$
Linked List 	$O(1)$	$O(n)$

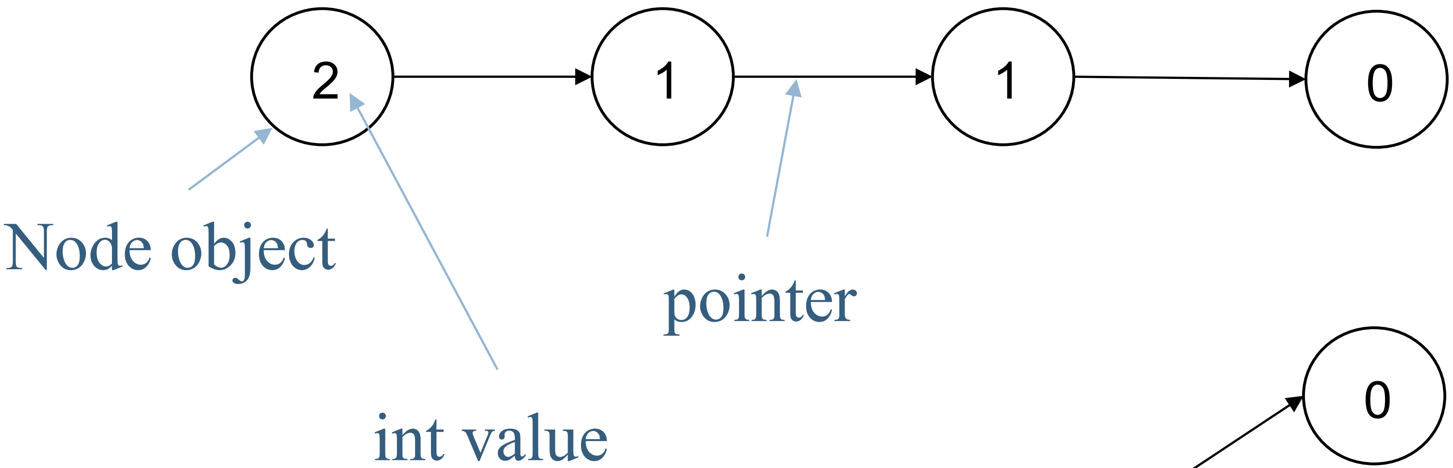
The Problem of Search

Example Data Structures

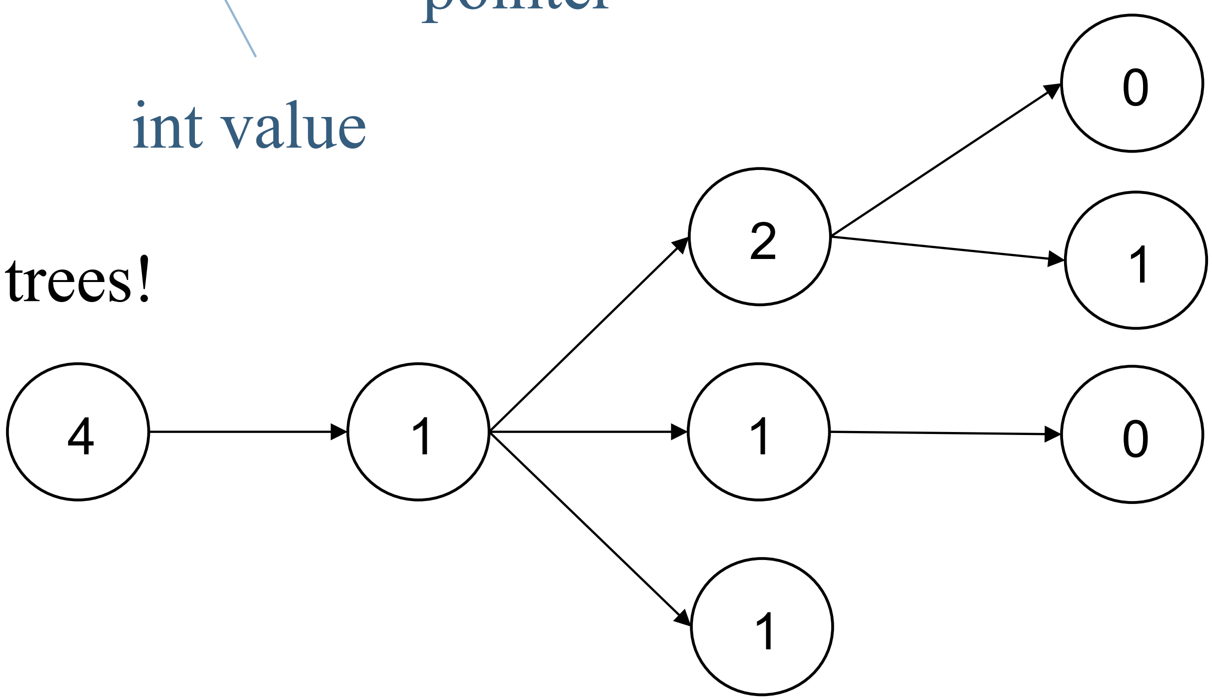
Data Structure	add(val x)	lookup(int i)	search(val x)
Array 2 1 3 0	$O(n)$	$O(1)$	$O(n)$
Linked List $2 \rightarrow 1 \rightarrow 3 \rightarrow 0$	$O(1)$	$O(n)$	$O(n)$

Tree

Singly linked list:



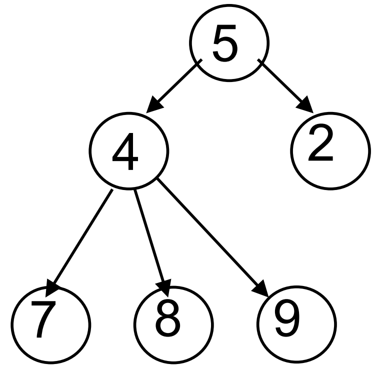
Today: trees!



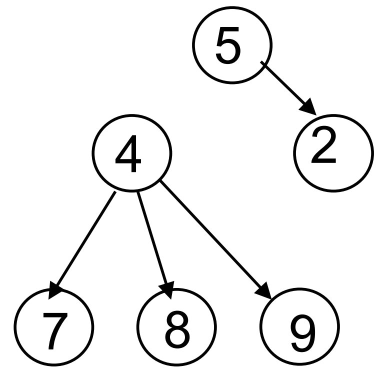
Tree Overview

Tree: data structure with nodes, similar to linked list

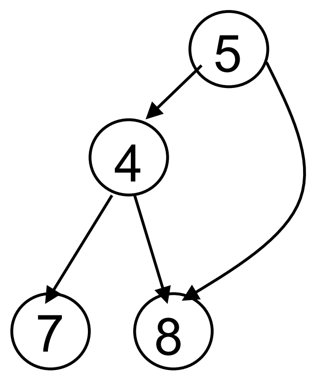
- Each node may have zero or more *successors* (children)
- Each node has exactly one *predecessor* (parent) except the *root*, which has none
- All nodes are reachable from *root*



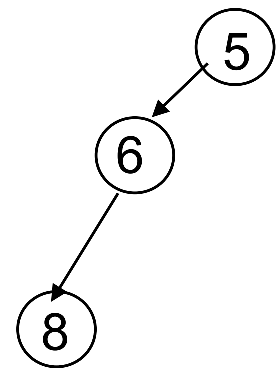
A tree



Not a tree

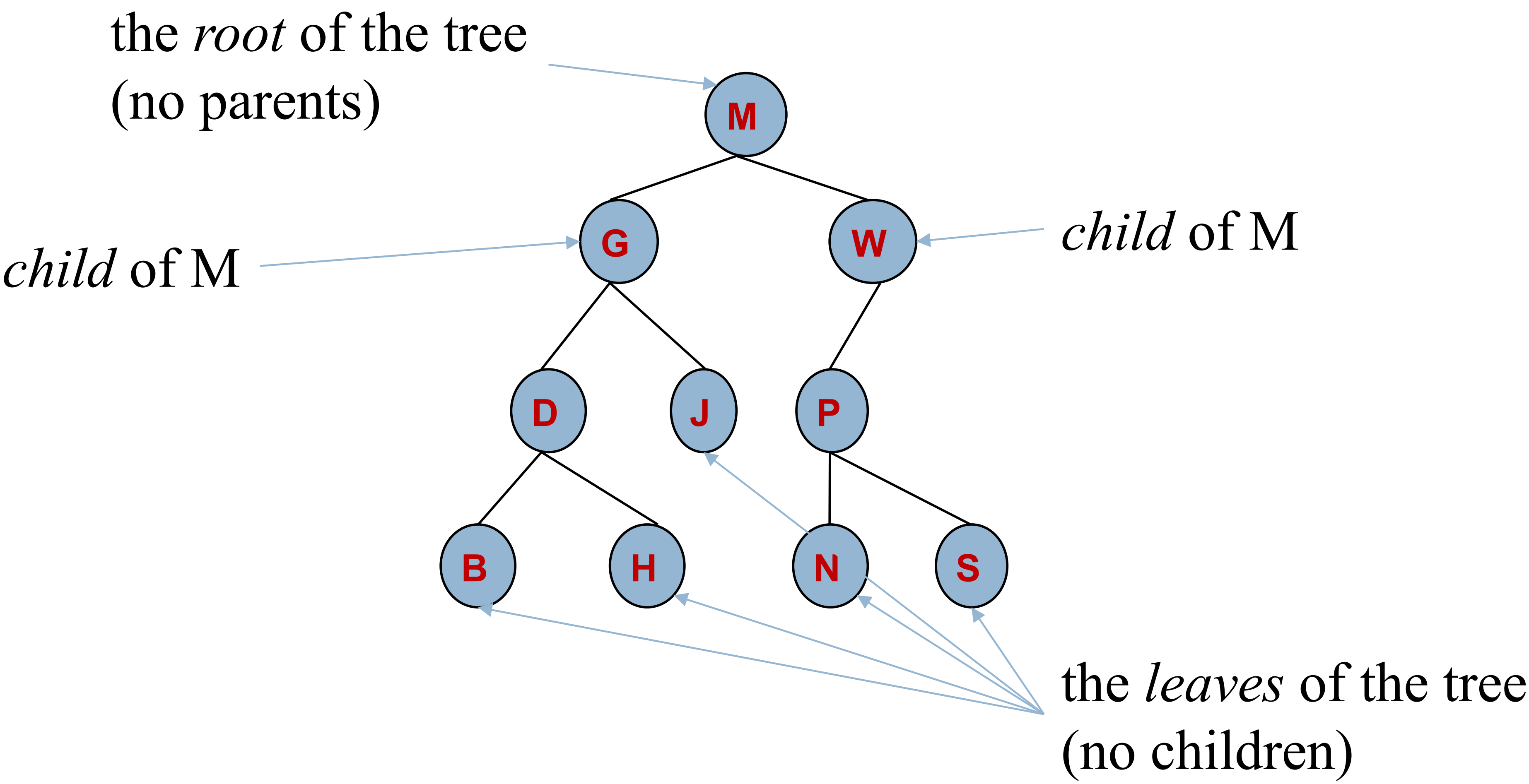


Not a tree

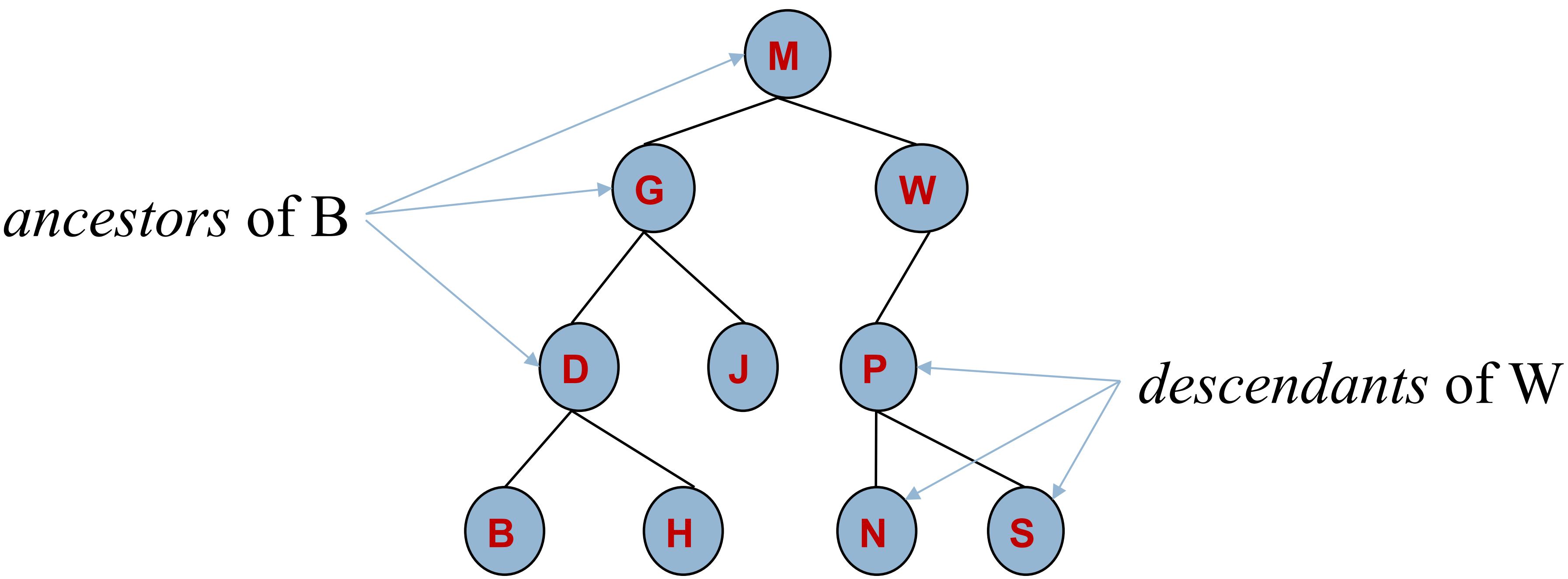


A tree

Tree Terminology

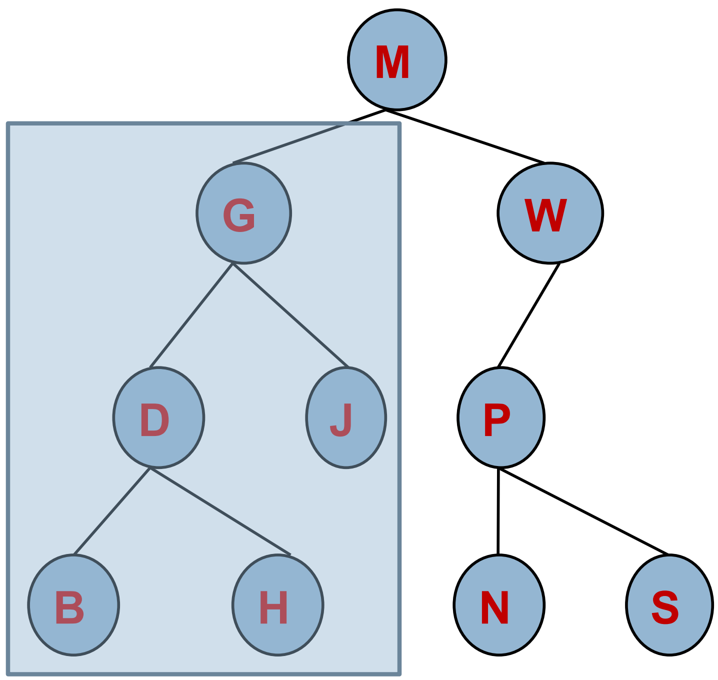


Tree Terminology



Tree Terminology

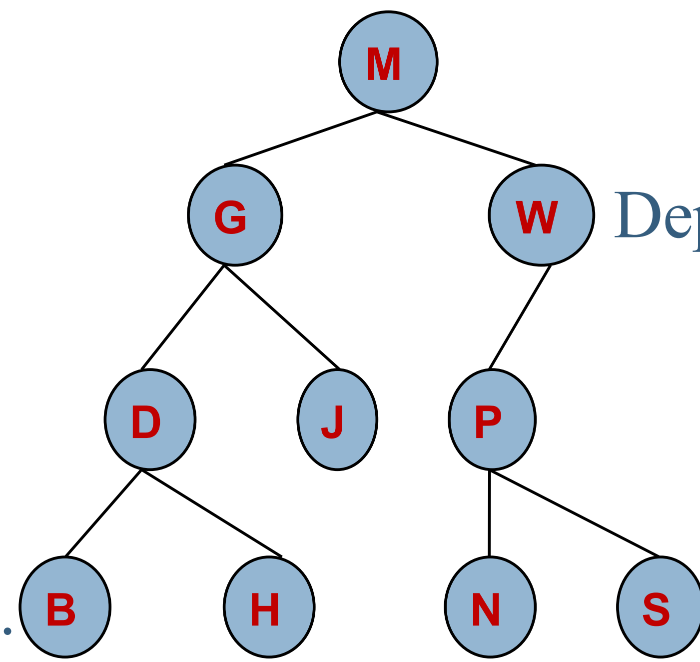
subtree of M



Tree Terminology

A node's *depth* is the length of the path to the root.

A tree's (or subtree's) *height* is he length of the longest path from the root to a leaf.

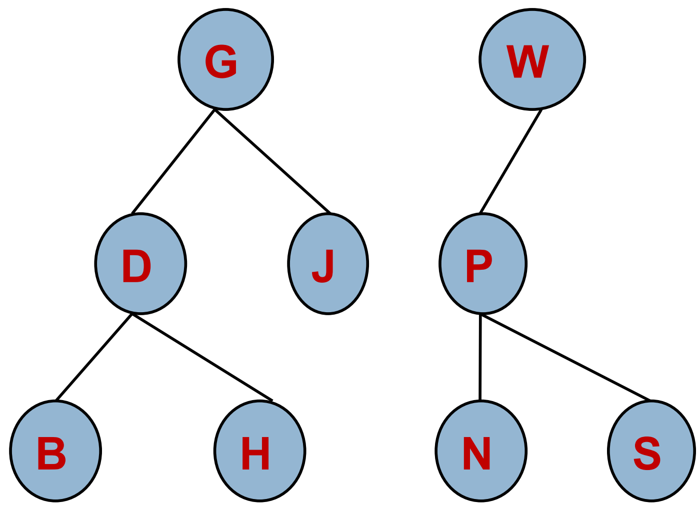


Depth 1, height 2.

Depth 3, height 0.

Tree Terminology

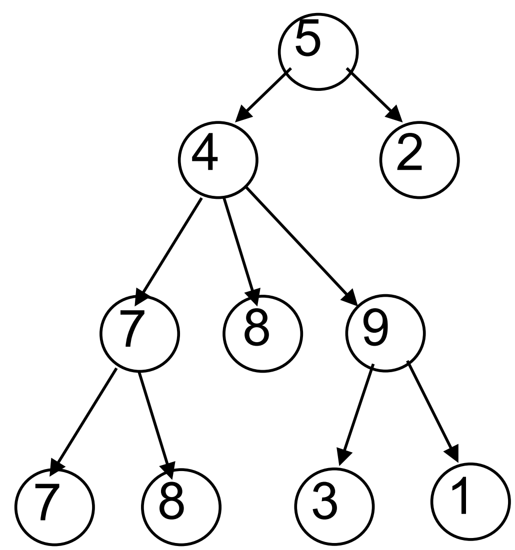
Multiple trees: a *forest*.



Class for general tree nodes

```
class GTreeNode<T> {
    private T value;
    private List<GTreeNode<T>> children;
    //appropriate constructors, getters,
    //setters, etc.
}
```

Parent contains a list of its children



General tree

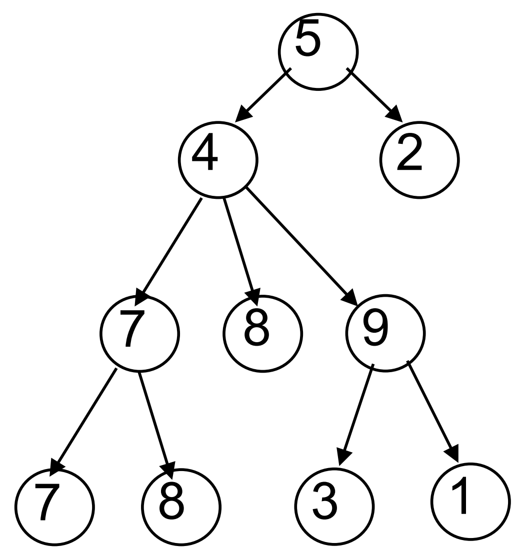
Class for general tree nodes

```
class GTreeNode<T> {
    private T value;
    private List<GTreeNode<T>> children;
    //appropriate constructors, getters,
    //setters, etc.
}
```

Java.util.List is an interface!

It defines the methods that all implementation must implement.

Whoever writes this class gets to decide what implementation to use — ArrayList? LinkedList? Etc.?

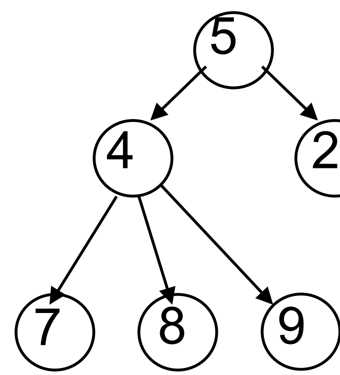


General tree

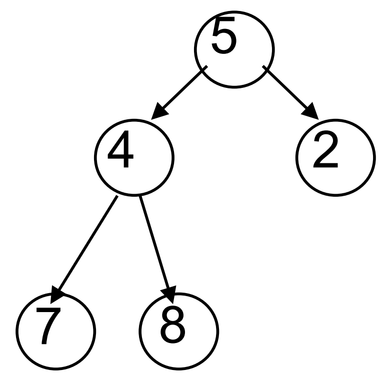
Binary Trees

A *binary tree* is a particularly important kind of tree where every node as at most two children.

In a binary tree, the two children are called the *left* and *right* children.



Not a binary tree
(a *general* tree)

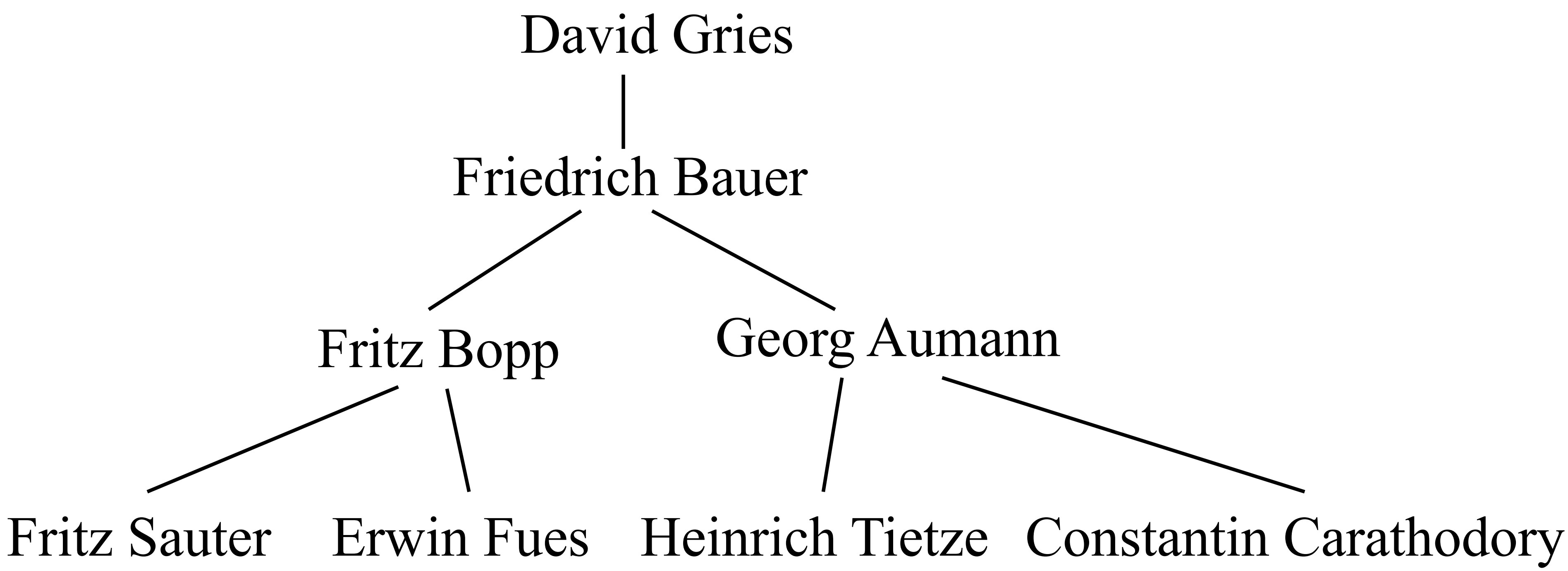


Binary tree

Binary trees were in A1!

You have seen a binary tree in A1.

A PhD object has one or two advisors. (Confusingly, the advisors are the "children.")



Class for binary tree node


```
class TreeNode<T> {
  private T value;
  private TreeNode<T> left, right;

  /** Constructor: one-node tree with datum x */
  public TreeNode (T d) { datum= d; left= null; right= null;}

  /** Constr: Tree with root value x, left tree l, right tree r */
  public TreeNode (T d, TreeNode<T> l, TreeNode<T> r) {
     datum= d; left= l; right= r;
  }
}
```

Either might be null if the subtree is empty.

more methods: getValue, setValue, getLeft, setLeft, etc.

Binary versus general tree

In a binary tree, each node has up to two pointers: to the left subtree and to the right subtree:

- One or both could be **null**, meaning the subtree is empty (remember, a tree is a set of nodes)

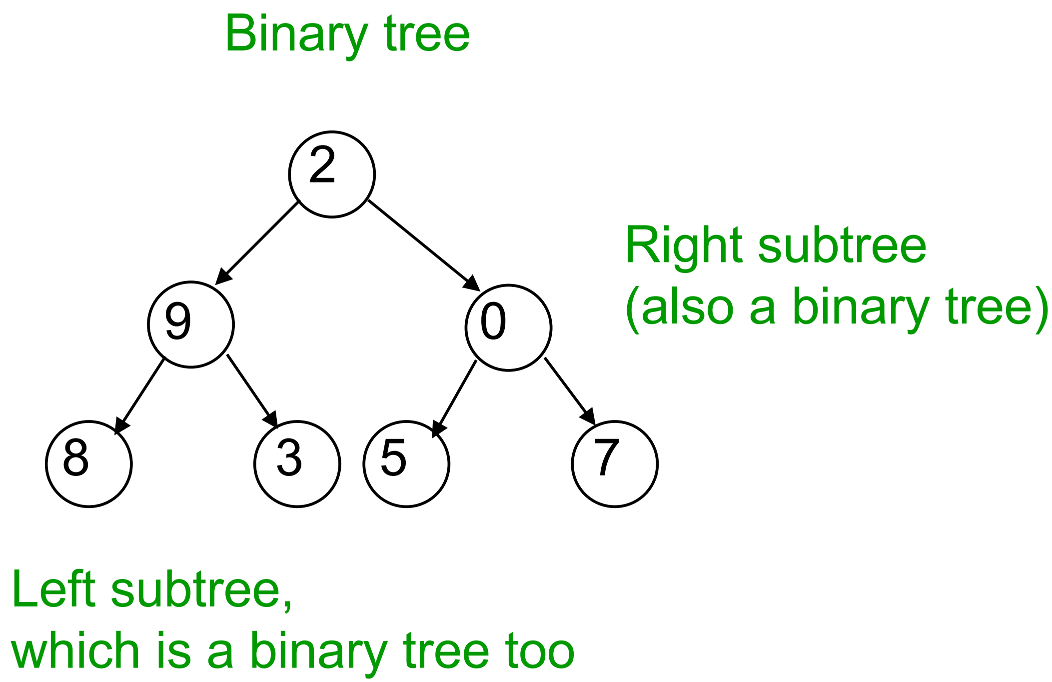
In a general tree, a node can have any number of child nodes (and they need not be ordered)

- Very useful in some situations ...
- ... one of which may be in an assignment!

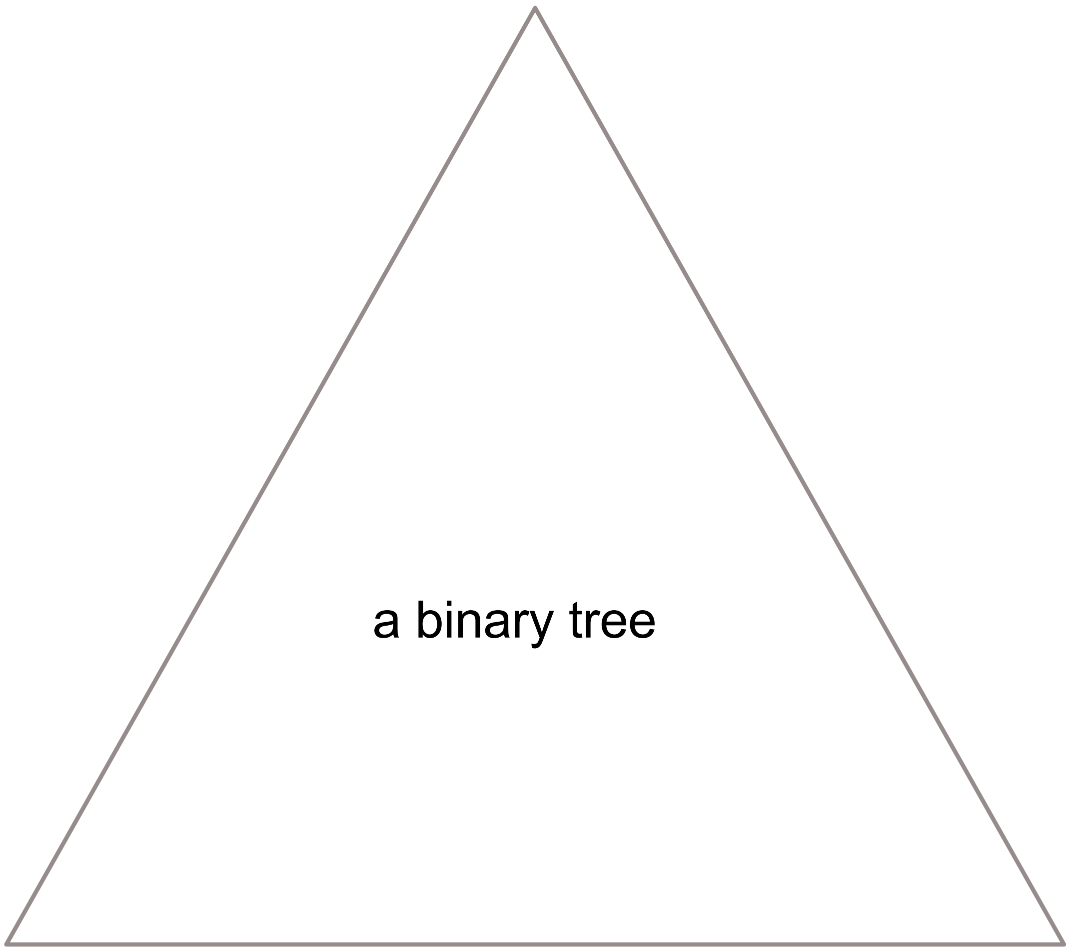
A Tree is a Recursive Thing

A binary tree is either `null` or an object consisting of a value, a left binary tree, and a right binary tree.

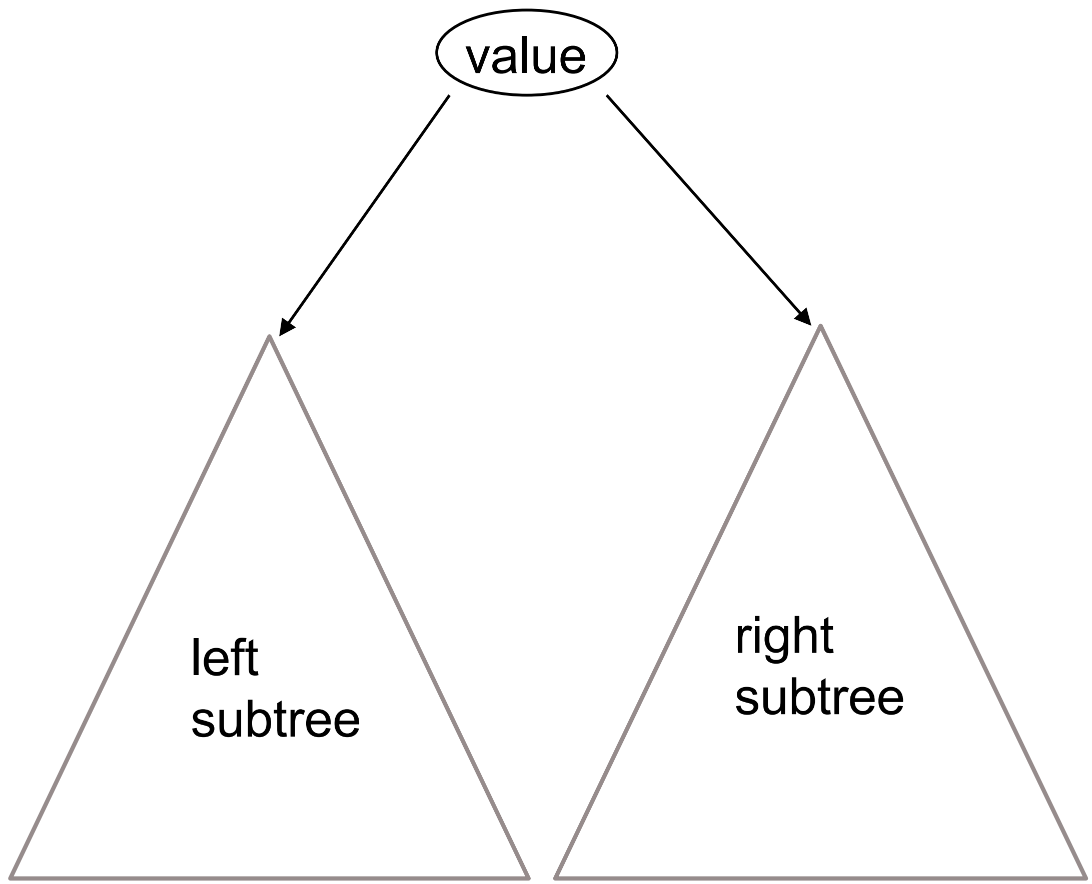
Looking at trees recursively



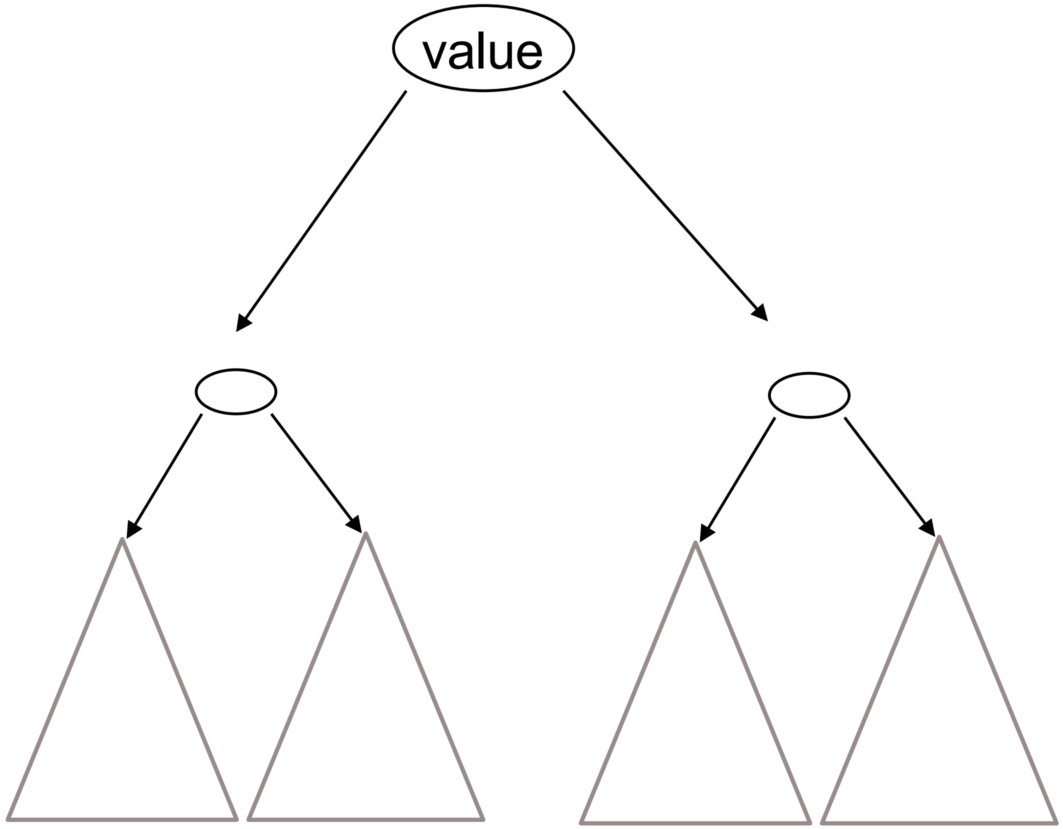
Looking at trees recursively



Looking at trees recursively



Looking at trees recursively



A Recipe for Recursive Functions

Base case:

If the input is “easy,” just solve the problem directly.

Recursive case:

Get a smaller part of the input (or several parts).
Call the function on the smaller value(s).
Use the recursive result to build a solution for the full input.

Recursive Functions on Binary Trees

Base case:

empty tree (null)
or, possibly, a leaf

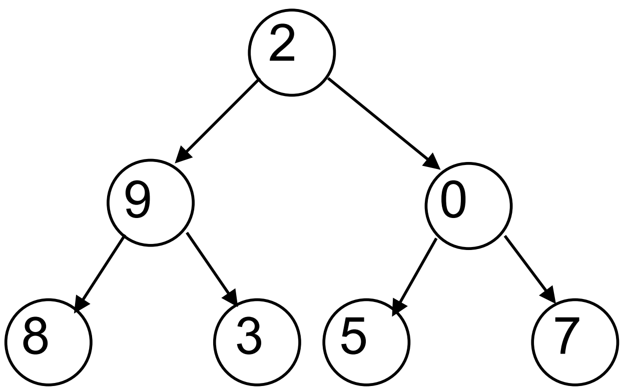
Recursive case:

Call the function on each subtree.
Use the recursive result to build a solution for the full input.

Searching in a Binary Tree


```
/** Return true iff x is the datum in a node of tree  t*/
public static boolean treeSearch(T x, TreeNode<T> t) {
   if (t == null) return false;
   if (x.equals(t.datum)) return true;
   return treeSearch(x, t.left) || treeSearch(x, t.right);
}
```

- Analog of linear search in lists: given tree and an object, find out if object is stored in tree
- Easy to write recursively, harder to write iteratively



Searching in a Binary Tree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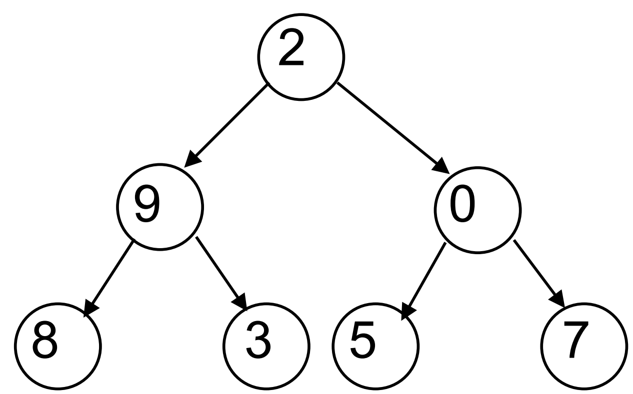
public static boolean treeSearch(T x, TreeNode<T> t) {
   if (t == null) return false;
   if (x.equals(t.datum)) return true;
   return treeSearch(x, t.left) || treeSearch(x, t.right);
}
```

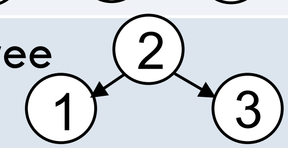
VERY IMPORTANT!

We sometimes talk of t as the root of the tree.

But we also use t to denote the whole tree.



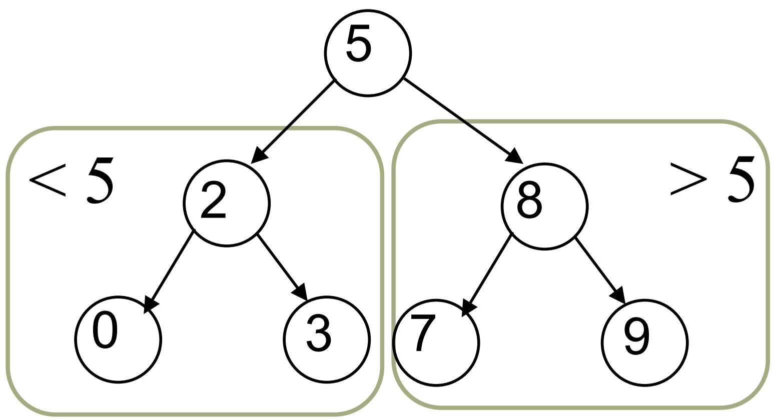
Comparing Data Structures

Data Structure	add(val x)	lookup(int i)	search(val x)
Array 2 1 3 0	$O(n)$	$O(1)$	$O(n)$
Linked List 2 → 1 → 3 → 0	$O(1)$	$O(n)$	$O(n)$
Binary Tree 	$O(1)$	$O(n)$	$O(n)$

Binary Search Tree (BST)

A *binary search tree* is a binary tree that is **ordered** and **has no duplicate values.** In other words, for *every* node:

- All nodes in the left subtree have values that are less than the value in that node, and
- All values in the right subtree are greater.



A BST is the key to making search way faster.

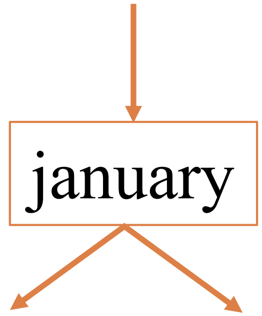
Building a BST

- To insert a new item:
  - Pretend to look for the item
  - Put the new node in the place where you fall off the tree

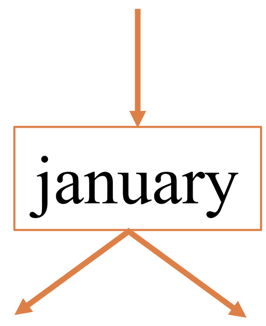
Building a BST

january

Building a BST

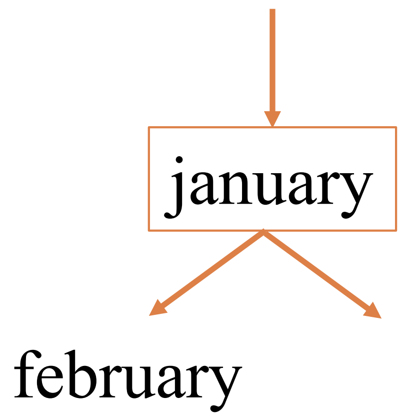


Building a BST

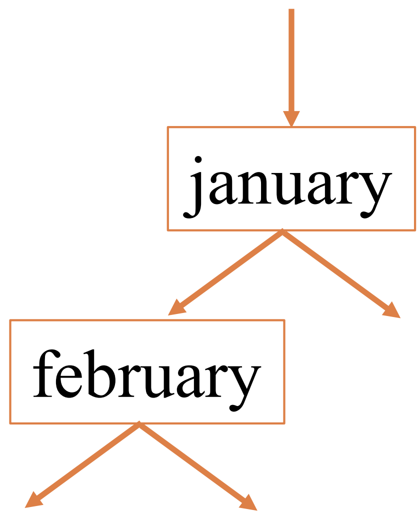


february

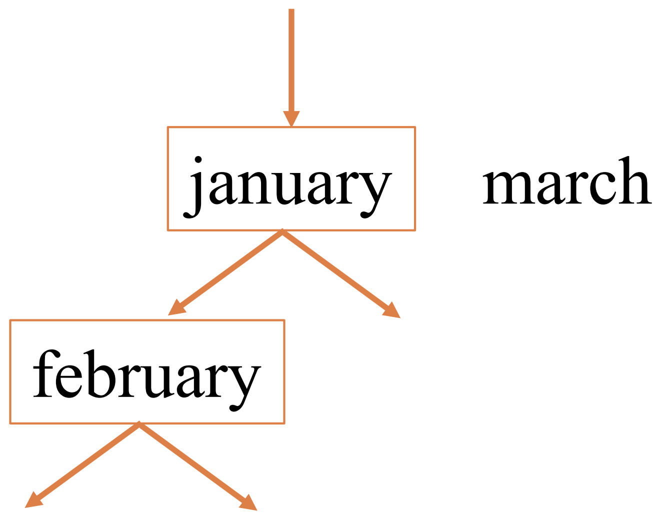
Building a BST



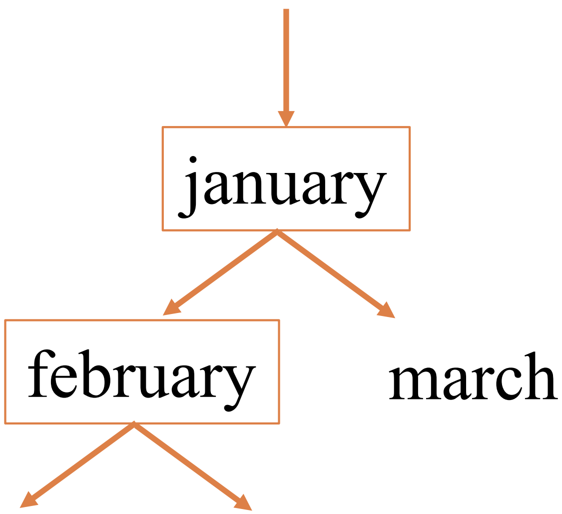
Building a BST



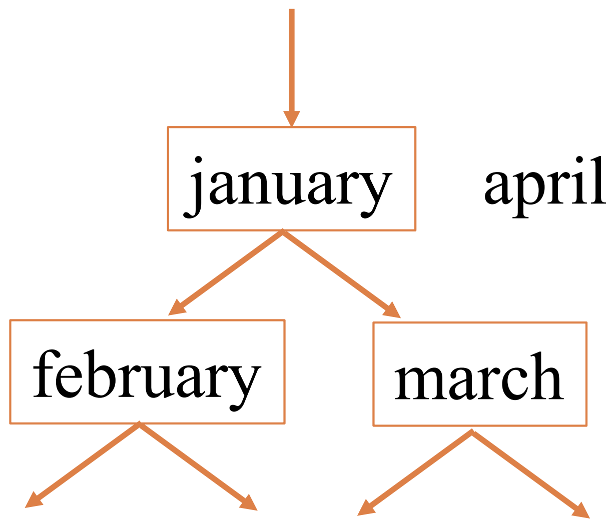
Building a BST



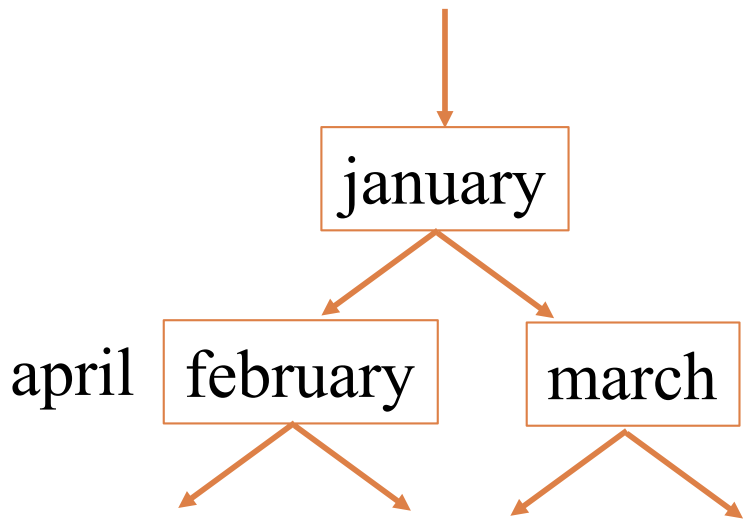
Building a BST



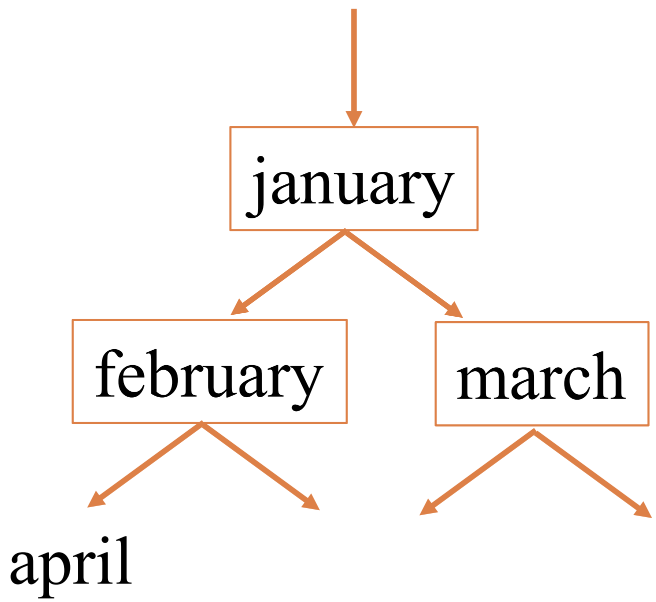
Building a BST



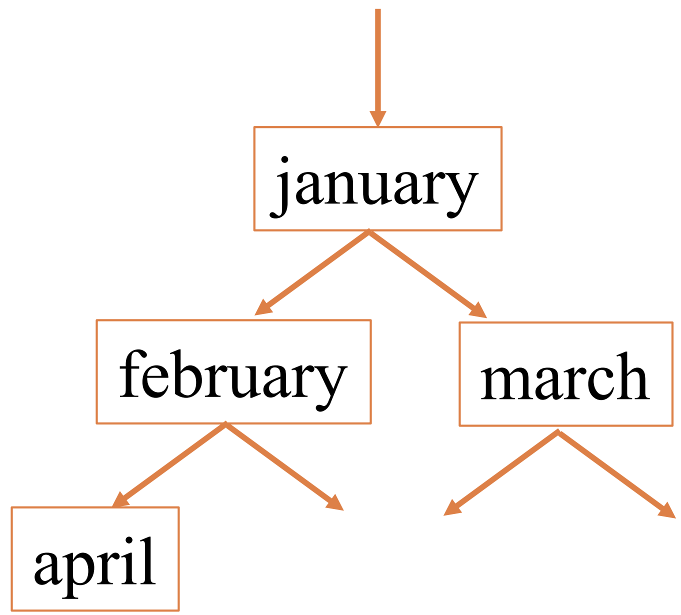
Building a BST



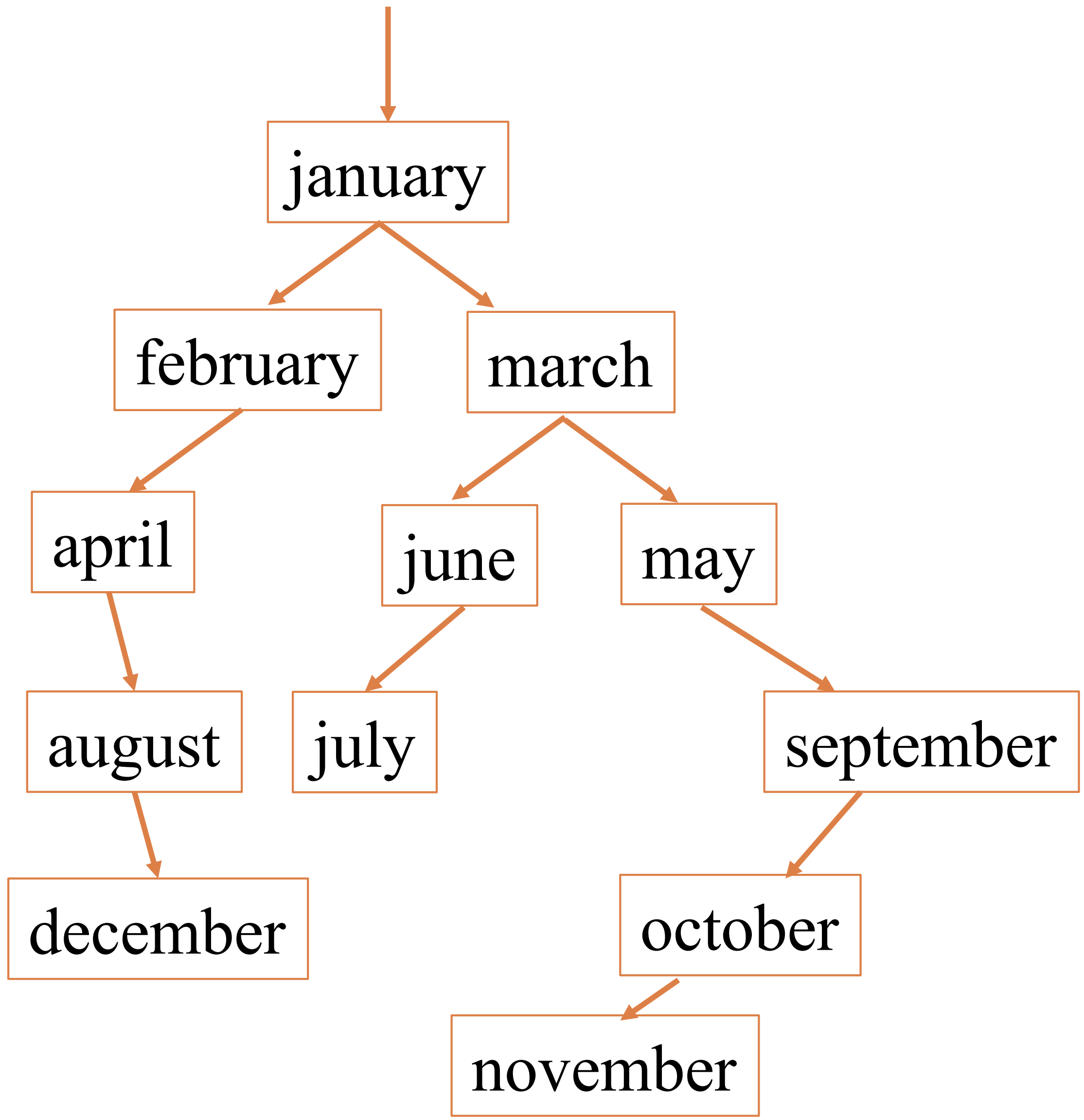
Building a BST



Building a BST



Building a BST



Printing contents of BST

Because of ordering rules for a BST, it's easy to print the items in alphabetical order

- Recursively print left subtree
- Print the node
- Recursively print right subtree

```
/** Print BST t in alpha order */
private static void print(TreeNode<T> t) {
   if (t== null) return;
   print(t.left);
   System.out.print(t.value);
   print(t.right);
}
```

Tree traversals

"Walking" over the whole tree is a tree traversal

- Done often enough that there are standard names

Previous example:
in-order traversal

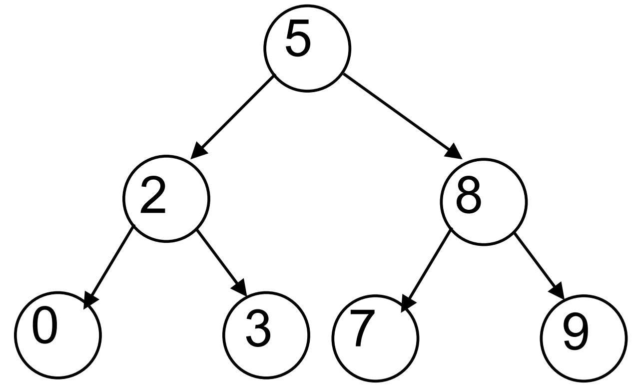
- Process left subtree
- Process root
- Process right subtree

Note: Can do other processing besides printing

Other standard kinds of traversals

- preorder traversal
  - Process root
  - Process left subtree
  - Process right subtree
- postorder traversal
  - Process left subtree
  - Process right subtree
  - Process root
- level-order traversal
  - Not recursive: uses a queue (we'll cover this later)

Binary Search Tree (BST)



Compare binary tree to binary search tree:

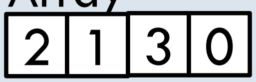
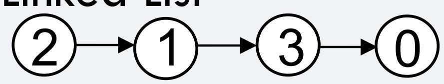
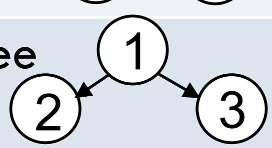
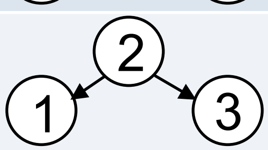
```
boolean searchBT(n, v):
  if n==null, return false
  if n.v == v, return true
  return searchBST(n.left, v)
       || searchBST(n.right, v)
```

2 recursive calls

```
boolean searchBST(n, v):
  if n==null, return false
  if n.v == v, return true
  if v < n.v
    return searchBST(n.left, v)
  else
    return searchBST(n.right, v)
```

1 recursive call

Comparing Data Structures

Data Structure	add(val x)	lookup(int i)	search(val x)
Array 	$O(n)$	$O(1)$	$O(n)$
Linked List 	$O(1)$	$O(n)$	$O(n)$
Binary Tree 	$O(1)$	$O(n)$	$O(n)$
BST 	$O(depth)$	$O(depth)$	$O(depth)$

Inserting in Alphabetical Order

april

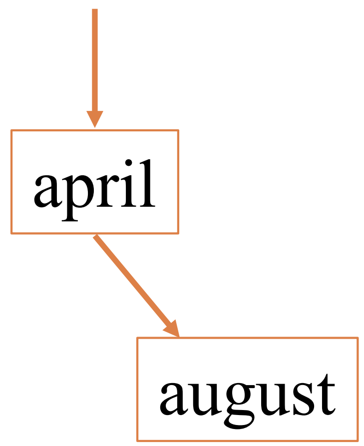
Inserting in Alphabetical Order

april

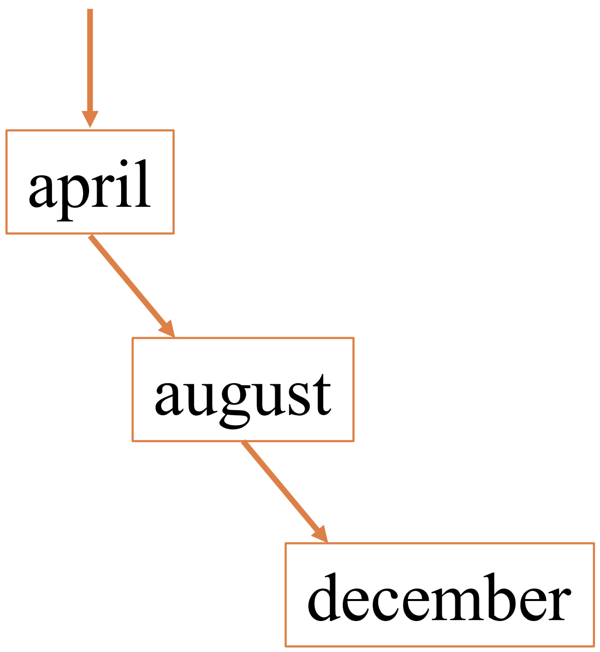
Inserting in Alphabetical Order

april august

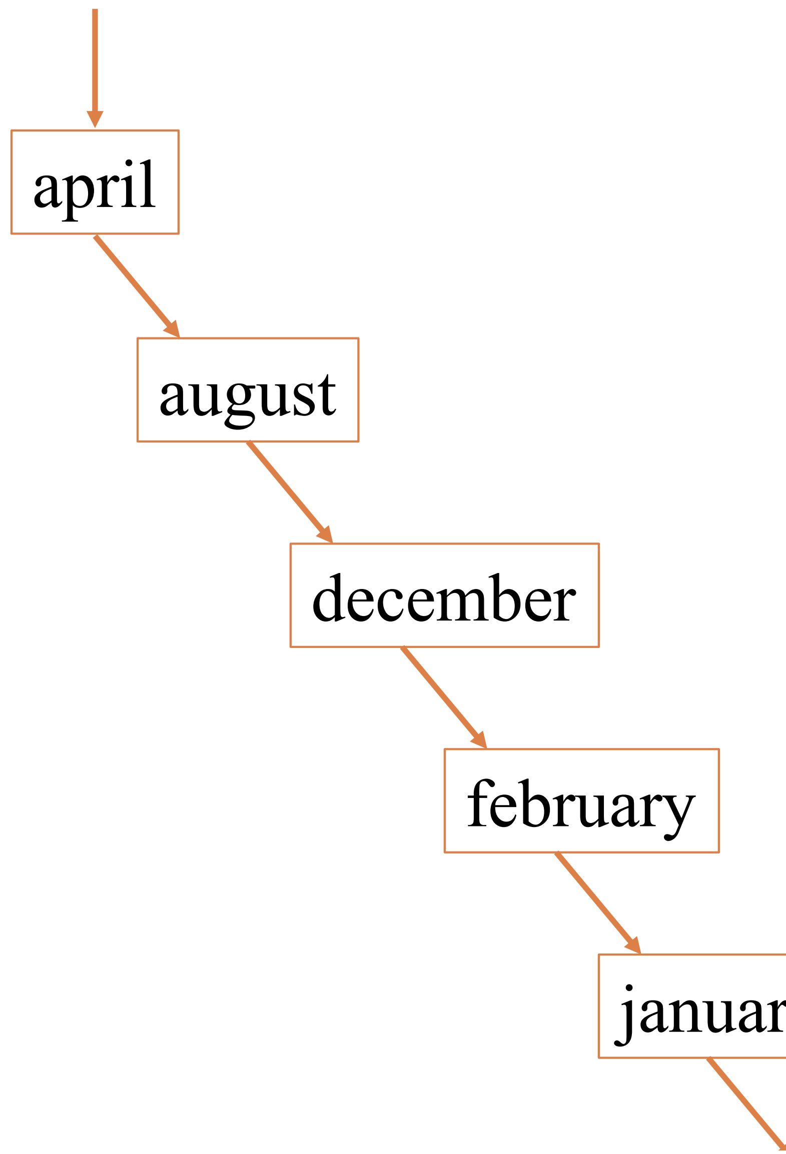
Inserting in Alphabetical Order



Inserting in Alphabetical Order



Inserting in Alphabetical Order

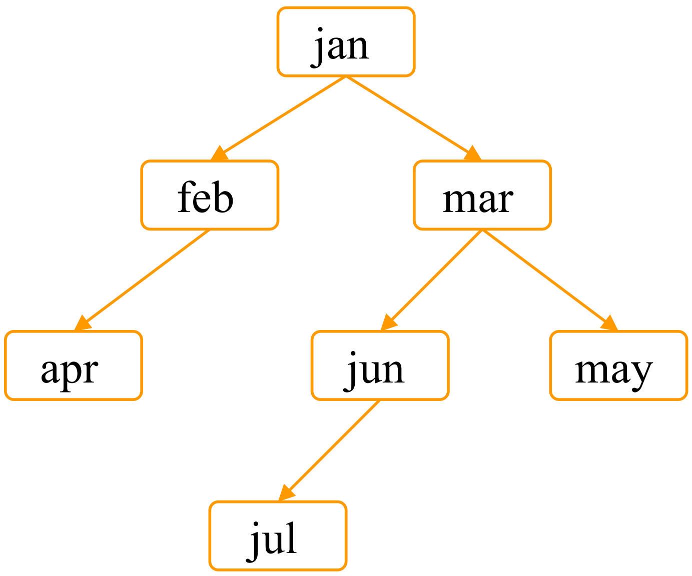


Insertion Order Matters

- A *balanced* binary tree is one where the two subtrees of any node are about the same size.
- Searching a binary search tree takes O(h) time, where h is the height of the tree.
- In a balanced binary search tree, this is O(log n).
- But if you insert data in sorted order, the tree becomes imbalanced, so searching is O(n).

Things to think about

What if we want to *delete* data from a BST?



A BST works great as long as it's *balanced*.

There are kinds of trees that can *automatically* keep themselves balanced as you insert things!

Useful facts about binary trees

Max # of nodes at depth d: 2^d

If height of tree is h

- min # of nodes: $h + 1$
- max #of nodes in tree:
- $2^0 + \ldots + 2^h = 2^{h+1} - 1$

Complete binary tree

- All levels of tree down to a certain depth are completely filled