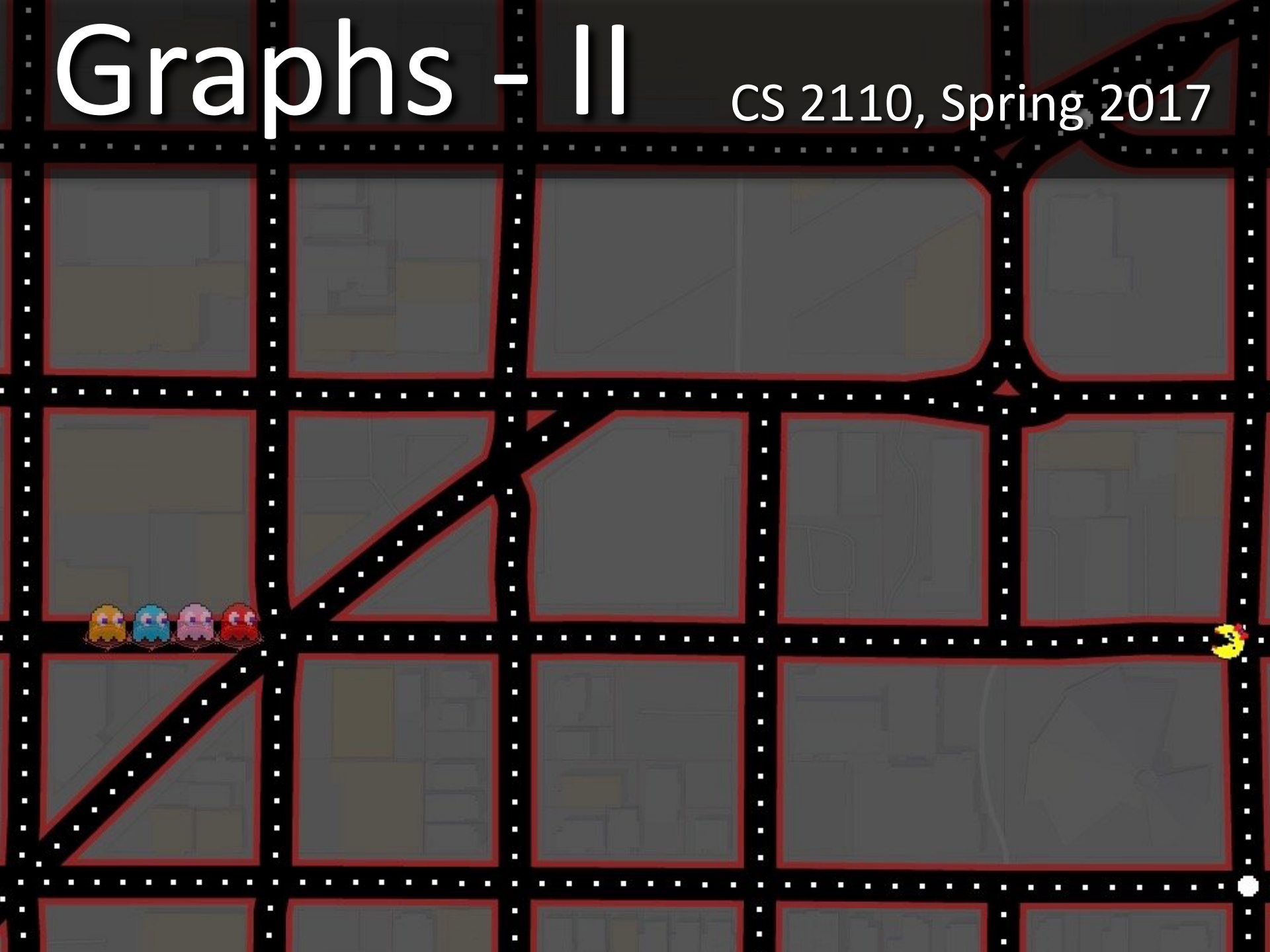
Graphs - II

CS 2110, Spring 2017



Announcements

1. **Lunch** with instructors: Still many slots open!
   - See pinned Piazza note @275

2. **Hidden Figures**: Thursday at 8:30 and 11:00!
   - If you haven't signed up on the CMS schedule do so now!
   - so far: 8:30 114/330
     11:00 147/330

Announcements

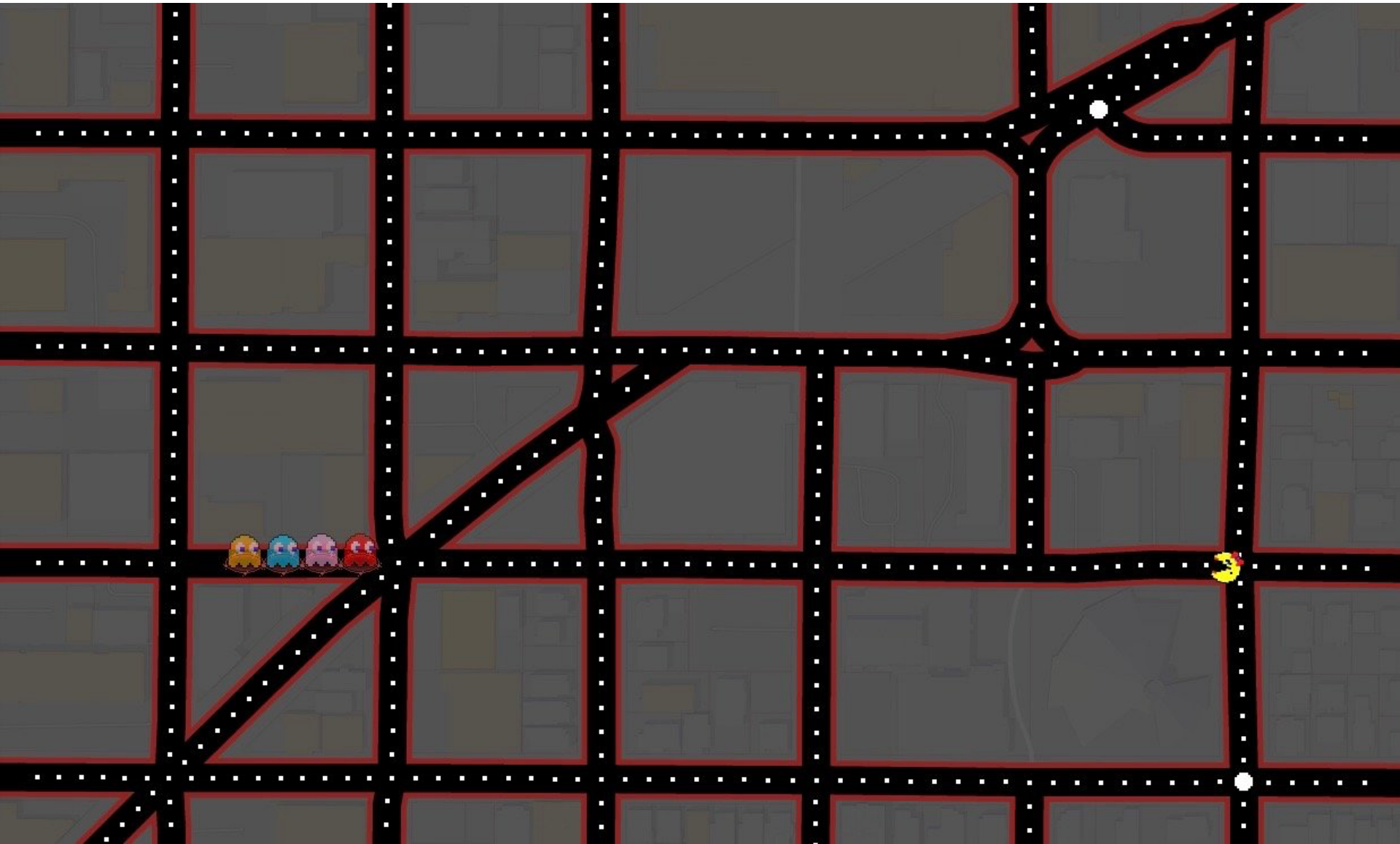
3. For next Tuesday's lecture, you **MUST** watch the tutorial on the shortest path algorithm beforehand:

http://www.cs.cornell.edu/courses/cs2110/2017sp/online/shortestPath/shortestPath.html

- Tuesday's lecture **will assume** that you understand it. Watch the tutorial once or twice and execute the algorithm on a small graph.

4. Information about **Prelim 2** is now on the Exams page of the course website and P2Conflict will be available on the CMS today (Tuesday).

Look, a graph!



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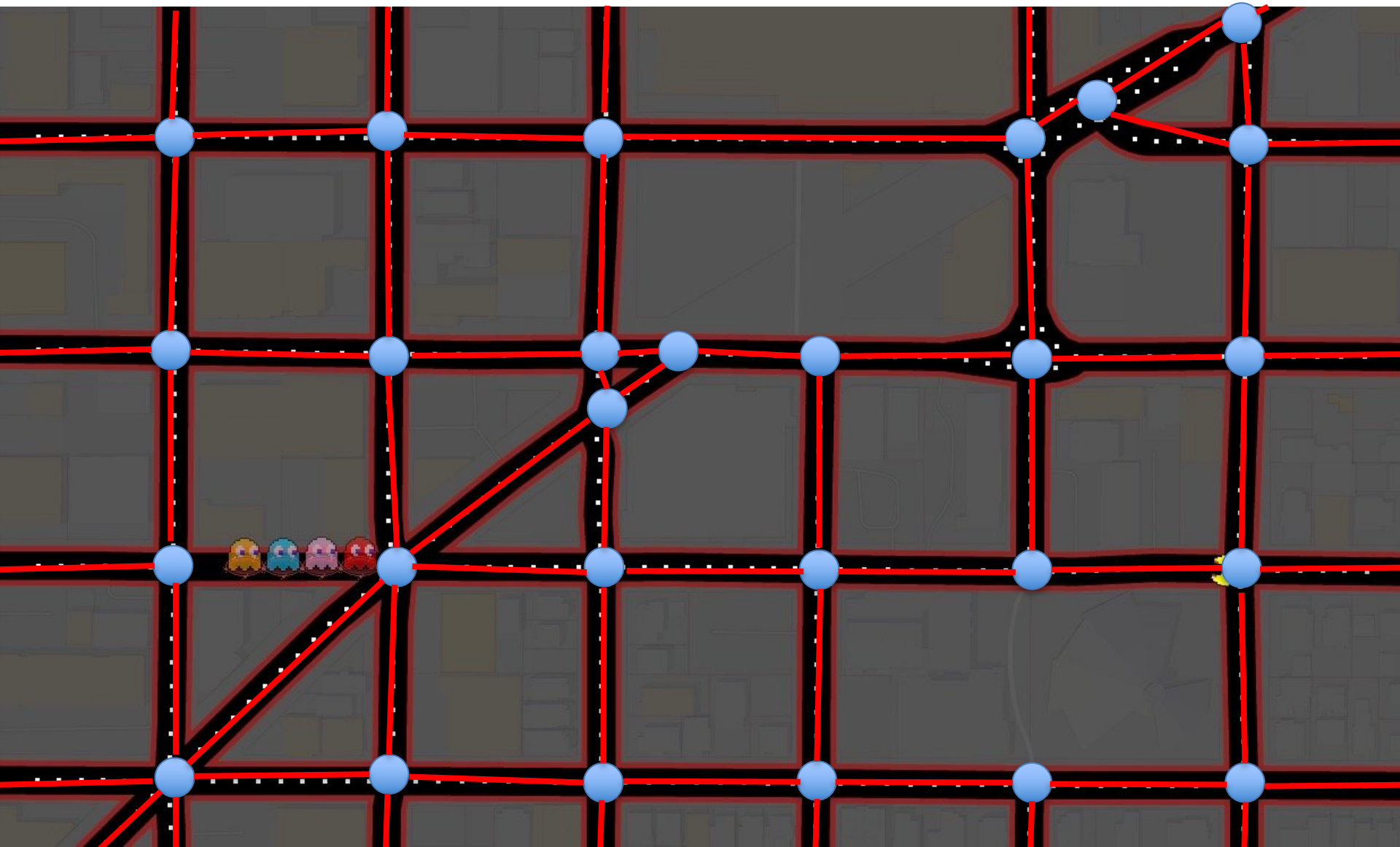

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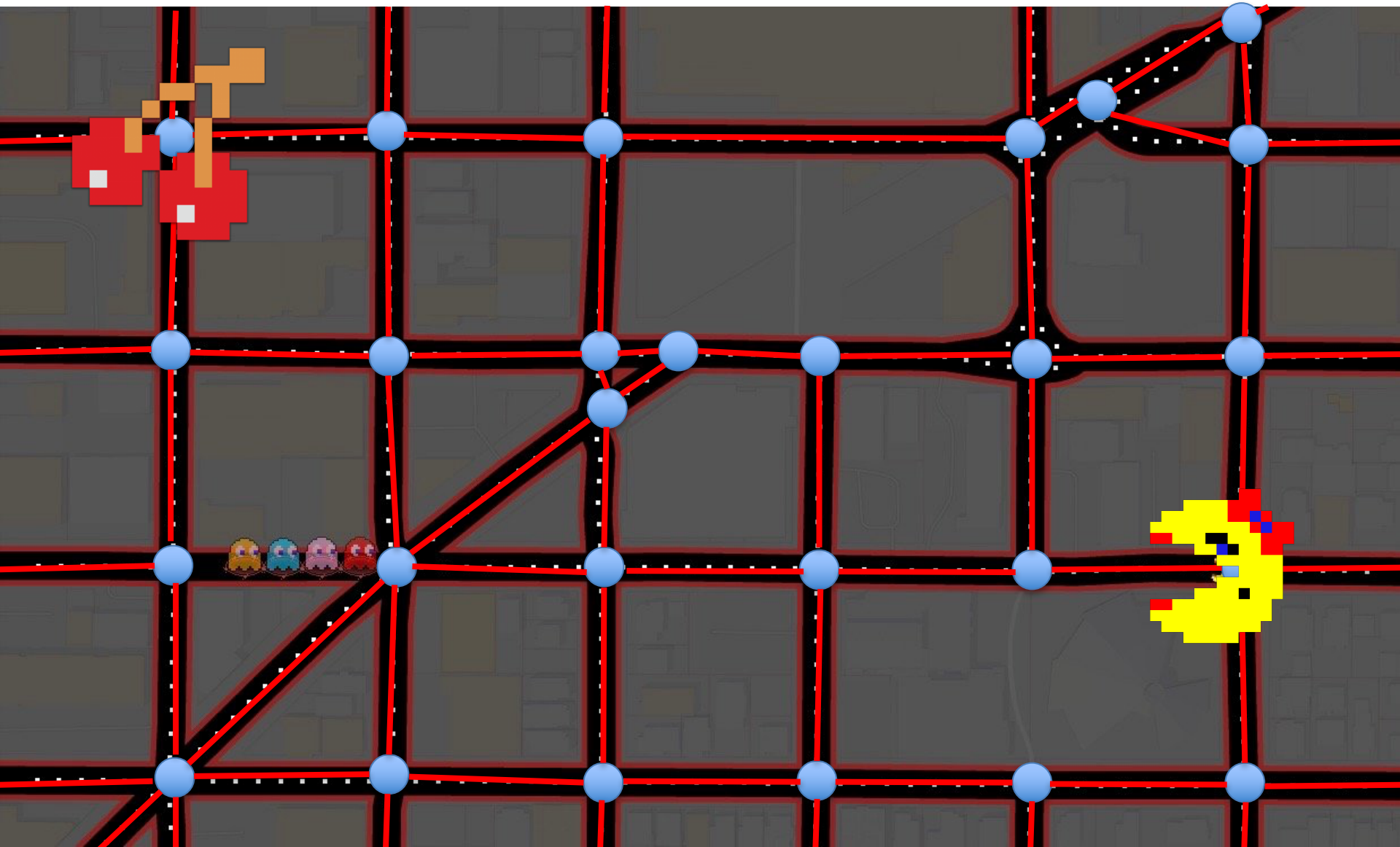
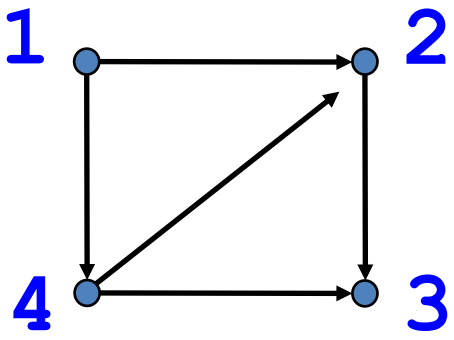


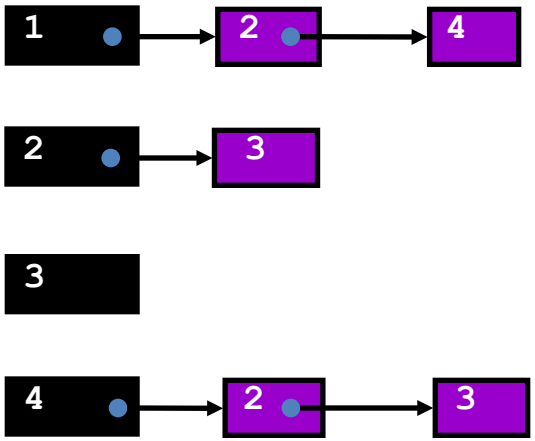
Graph Algorithms

- Search
  - Depth-first search
  - Breadth-first search
- Shortest paths
  - Dijkstra's algorithm
- Minimum spanning trees
  - Prim's algorithm
  - Kruskal's algorithm

Representations of Graphs



Adjacency List



Adjacency Matrix

	1	2	3	4
1	0	1	0	1
2	0	0	1	0
3	0	0	0	0
4	0	1	1	0

Adjacency Matrix or Adjacency List?

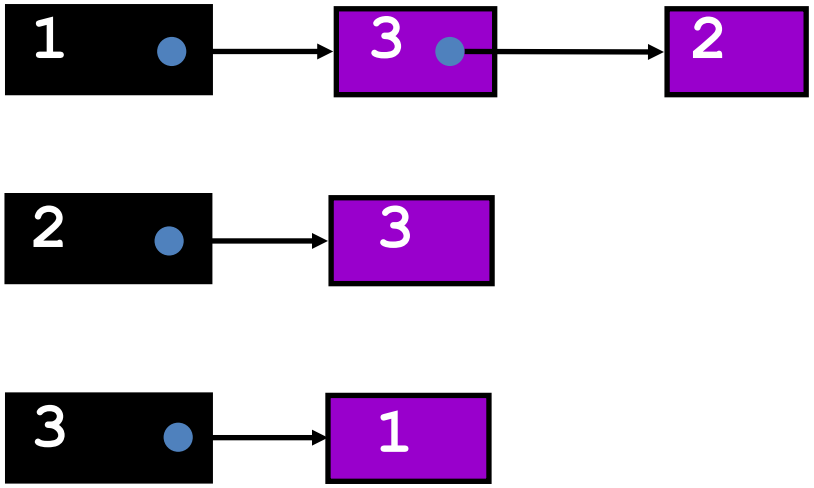
- Definitions:
  - n = number of vertices
  - e = number of edges
  - $d(u)$ = degree of u = number of edges leaving u
- Adjacency Matrix
  - Uses space $O(n^2)$
  - Can iterate over all edges in time $O(n^2)$
  - Can answer "Is there an edge from u to v?" in $O(1)$ time
  - Better for dense graphs (lots of edges)
- Adjacency List
  - Uses space $O(e + n)$
  - Can iterate over all edges in time $O(e + n)$
  - Can answer "Is there an edge from u to v?" in $O(d(u))$ time
  - Better for sparse graphs (fewer edges)

Breaking DAG

Which of the following two graphs are DAGs?

Directed Acyclic Graph

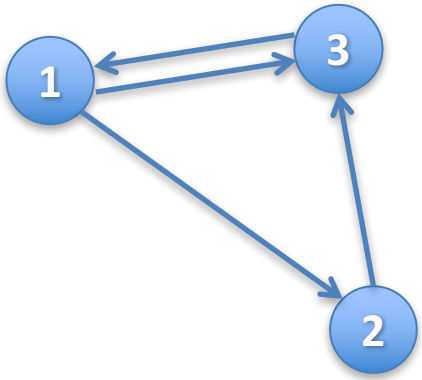
Graph 1:

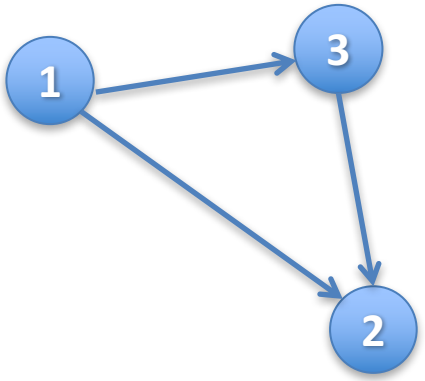


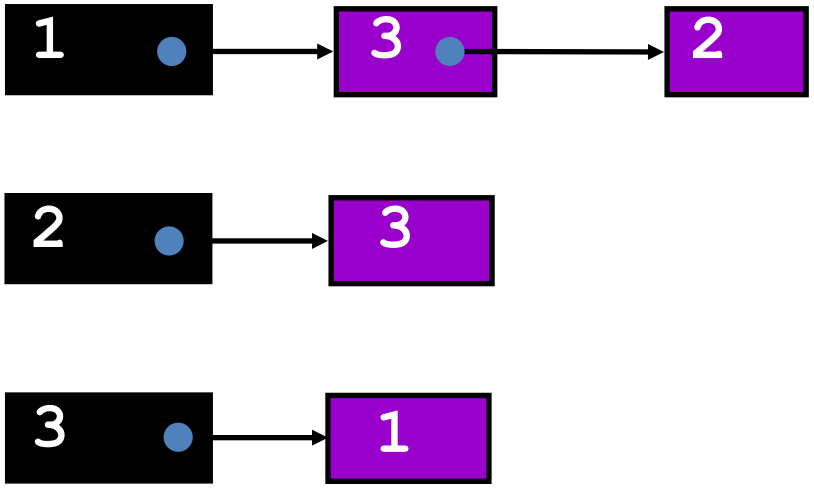
Graph 2:

	1	2	3
1	0	1	1
2	0	0	0
3	0	1	0

Breaking DAG







	1	2	3
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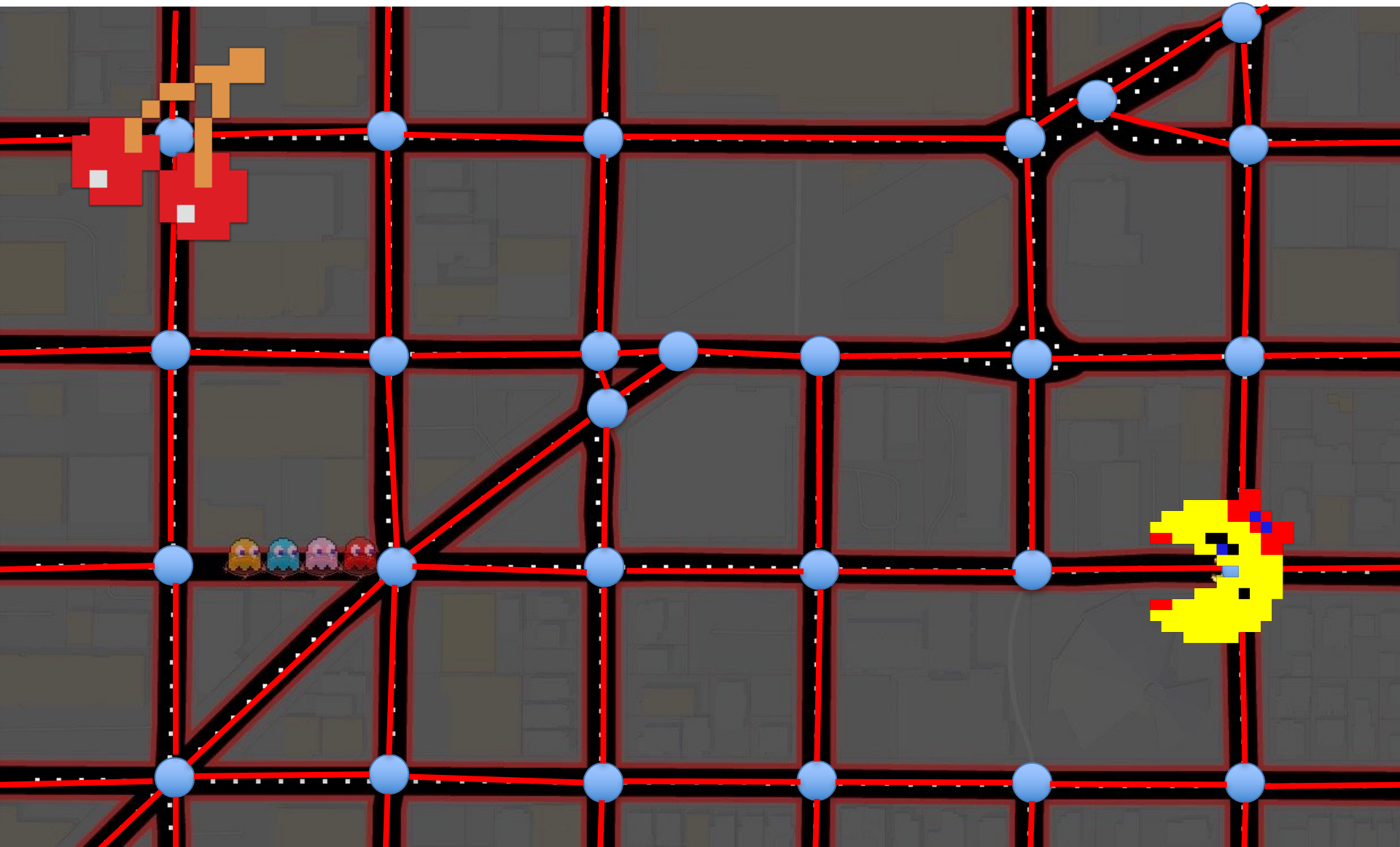
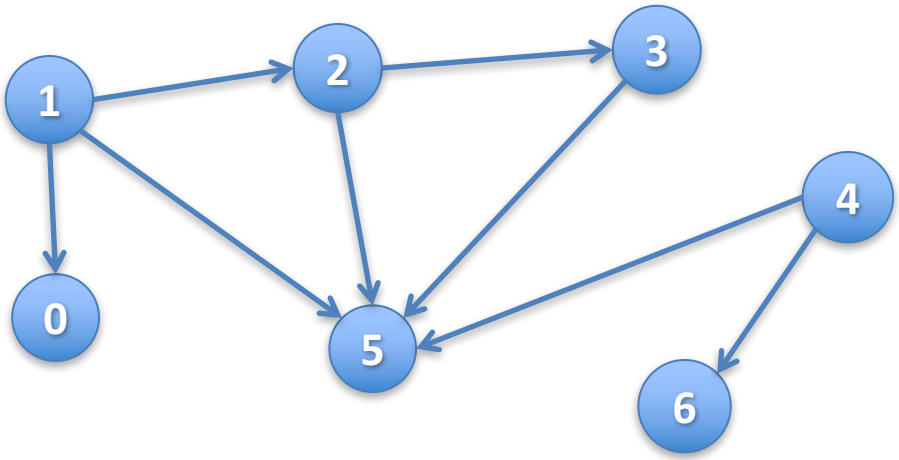
Back to Important Things:



Depth-First Search

- Given a graph and one of its nodes u

  (say node 1 below)



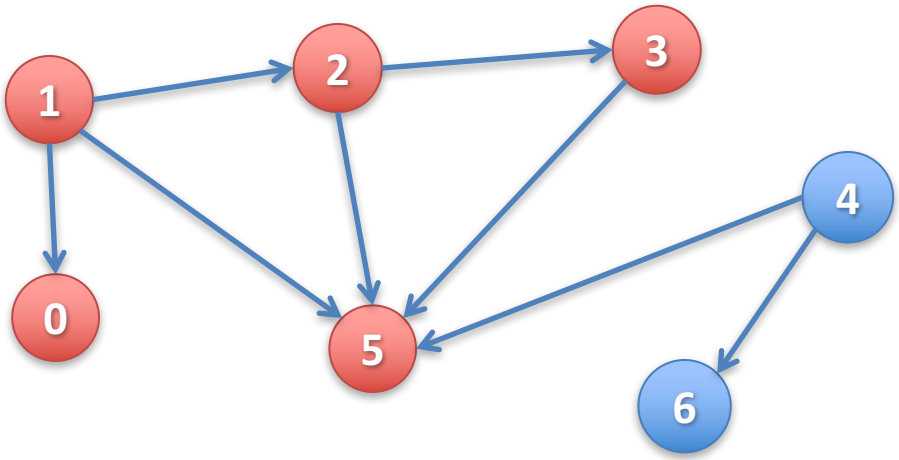
Depth-First Search

- Given a graph and one of its nodes u

  (say node 1 below)

- We want to "visit" each node reachable from u

  (nodes 1, 0, 2, 3, 5)



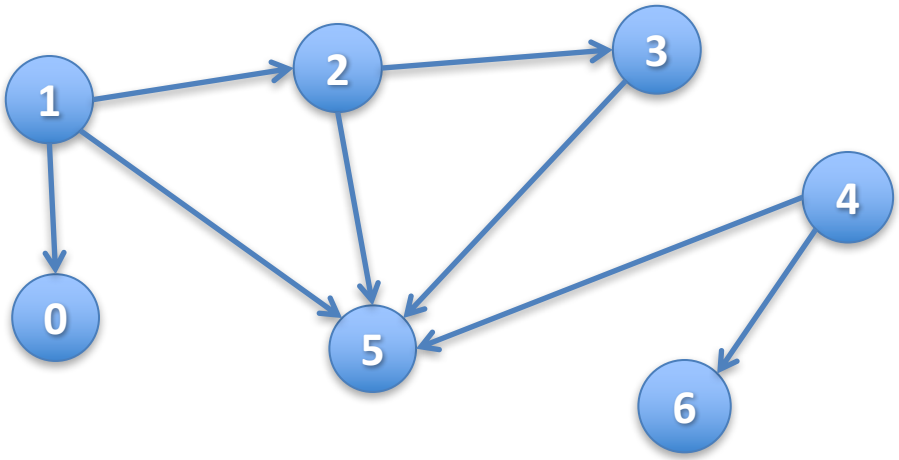
There are many paths to some nodes.

How do we visit all nodes efficiently, without doing extra work?

Depth-First Search

boolean[ ] visited;

- Node u is visited means: visited[u] is true
- To visit u means to: set visited[u] to true
- v is REACHABLE* from u if there is a path (u, …, v)
  *in which all nodes of the path are unvisited.



Suppose all nodes are unvisited.

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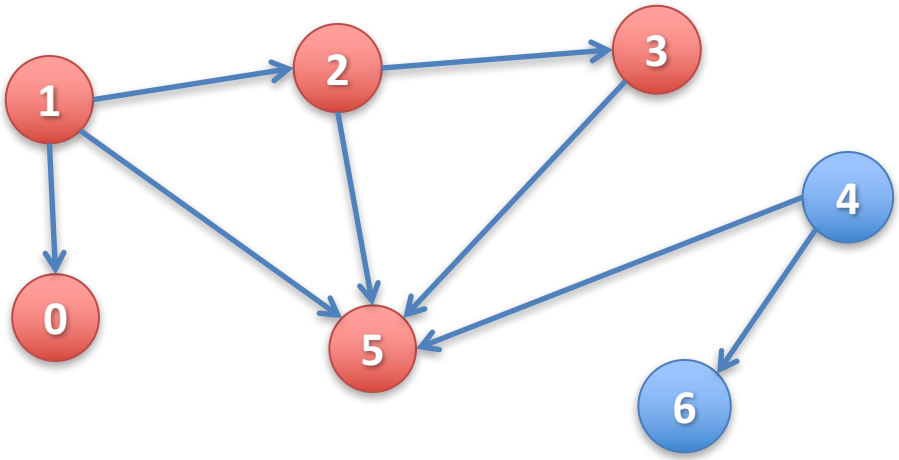

Suppose all nodes are unvisited.

Nodes REACHABLE* from node **1**:
{1, 0, 2, 3, 5}

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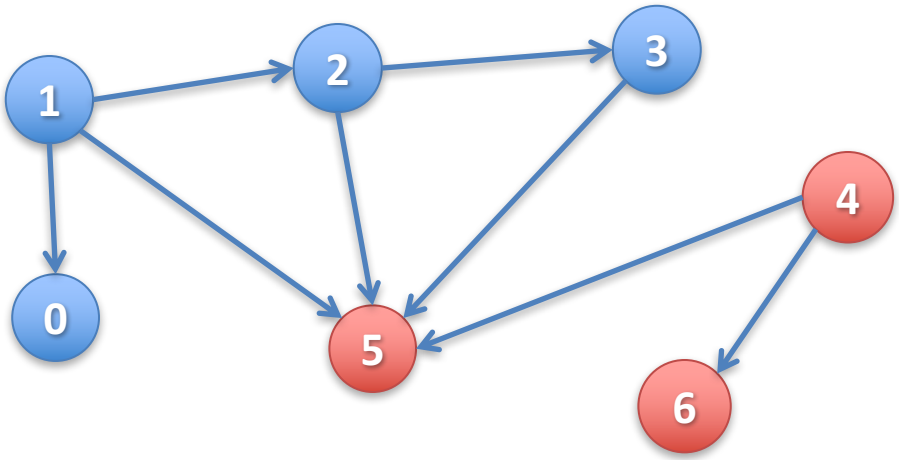

Suppose all nodes are unvisited.

Nodes REACHABLE* from node **1**:
{1, 0, 2, 3, 5}

Nodes REACHABLE* from **4**: {4, 5, 6}

Depth-First Search

boolean[ ] visited;

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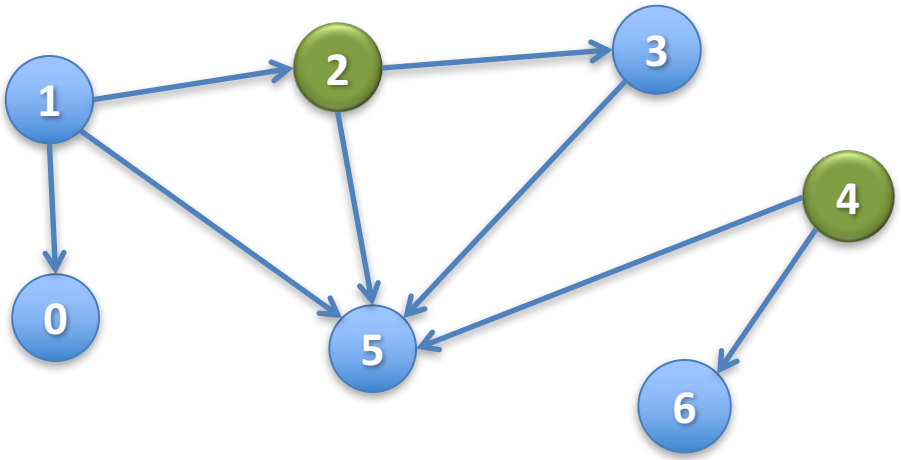

Green: visited
Blue: unvisited

Depth-First Search

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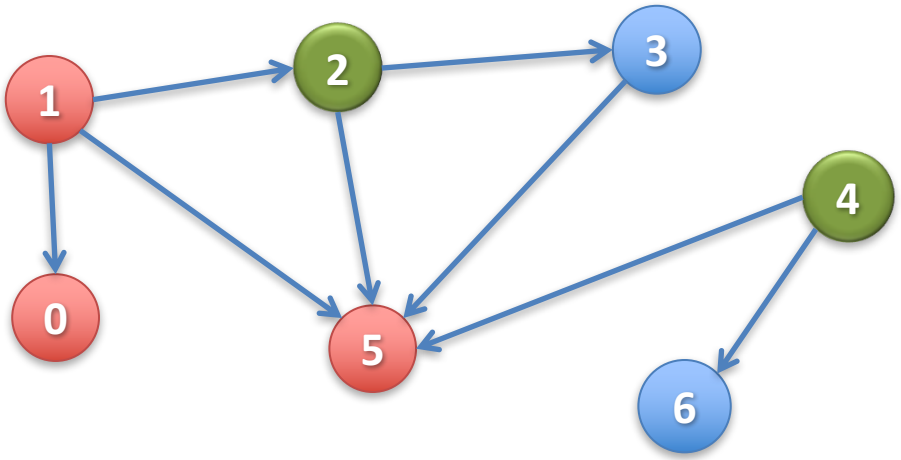

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Nodes REACHABLE* from node **1**:
{1, 0, 5}

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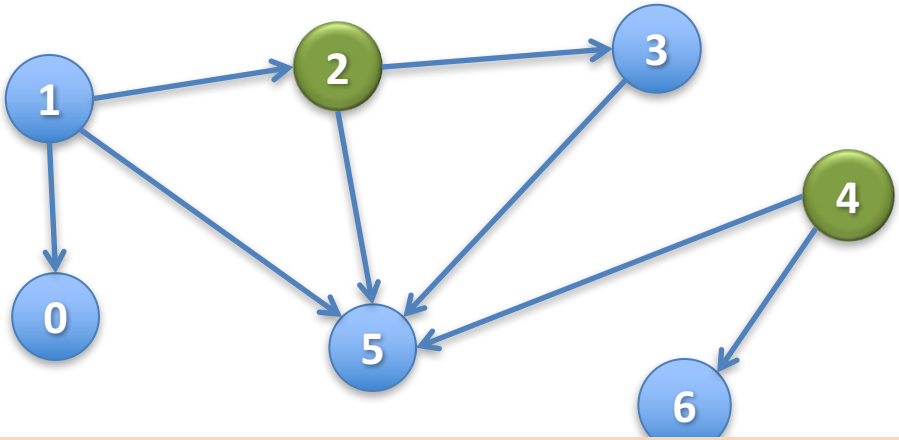


Green: visited
Blue: unvisited

Nodes REACHABLE * from node **1**:
{1, 0, 5}

Nodes REACHABLE * from **4**: none

Not even **4** itself, because it's already been visited!

Depth-First Search

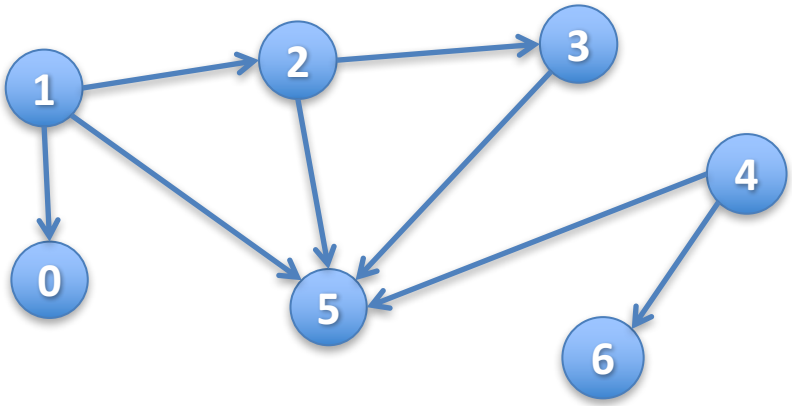
```
/** Visit all nodes that are REACHABLE*
from u. Precondition: u is unvisited. */
public static void dfs(int u) {

}
```

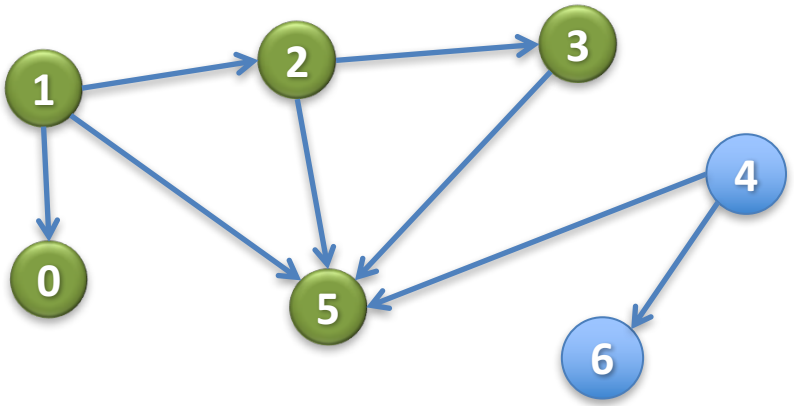
Let u be **1**

The nodes REACHABLE* from 1 are 1, 0, 2, 3, 5

Start



End



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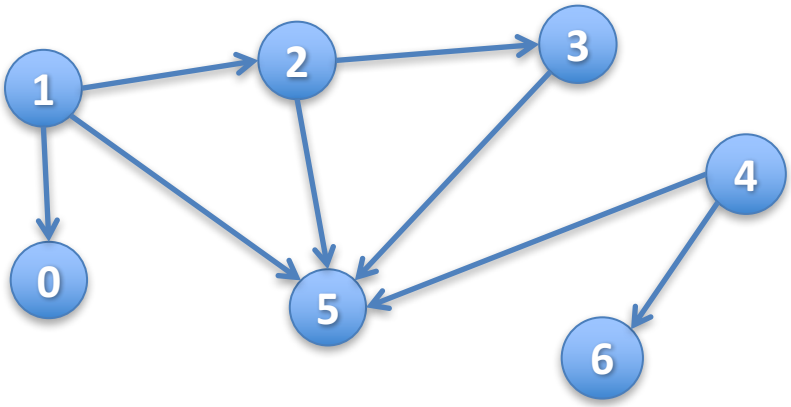

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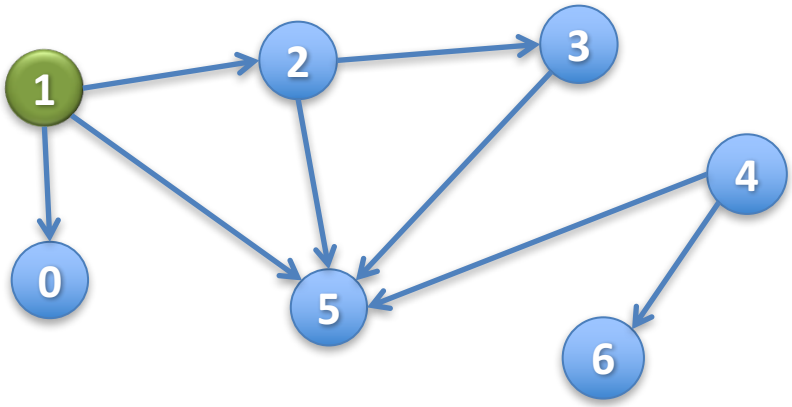

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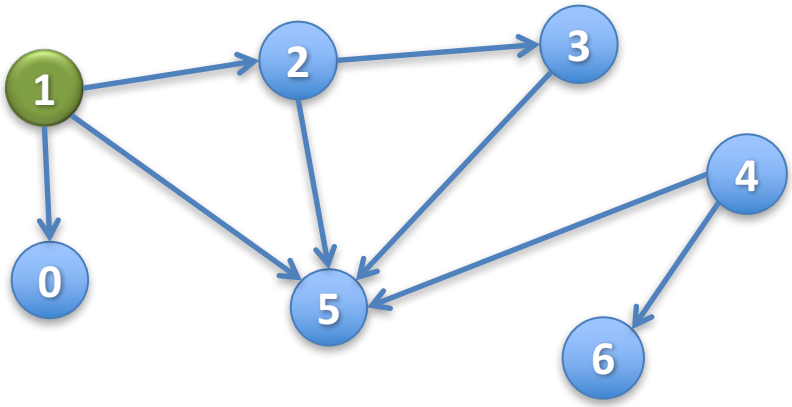
Let u be **1** (visited)

The nodes to be visited are 0, 2, 3, 5



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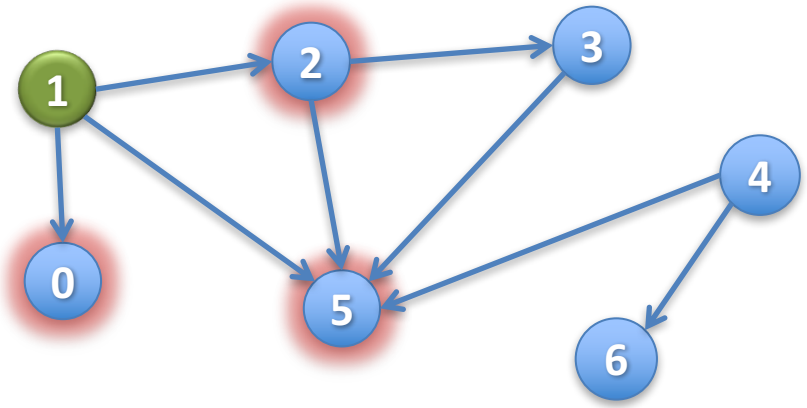
Let u be **1** (visited)

The nodes to be visited are 0, 2, 3, 5

Have to do DFS on all unvisited neighbors of u!



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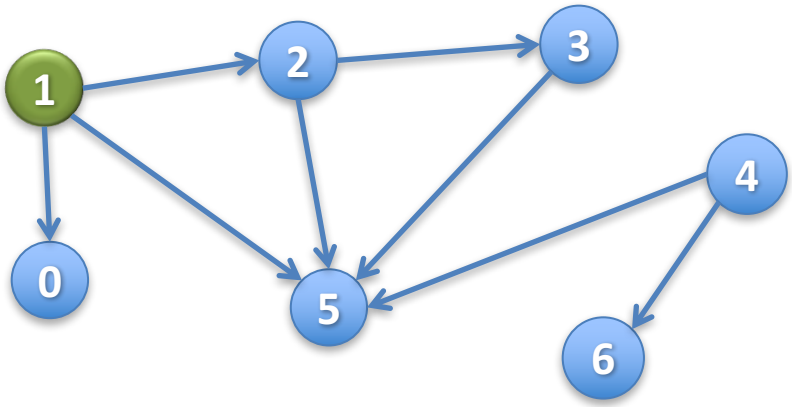
```

Suppose the **for** loop visits neighbors in numerical order. Then **dfs(1)** visits the nodes in this order: 1 …



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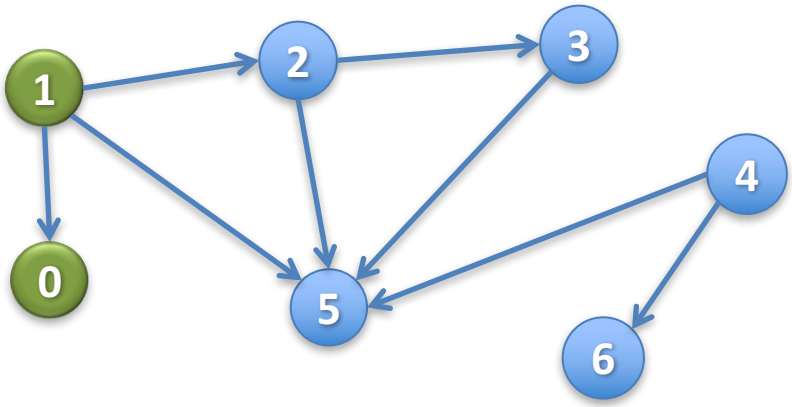

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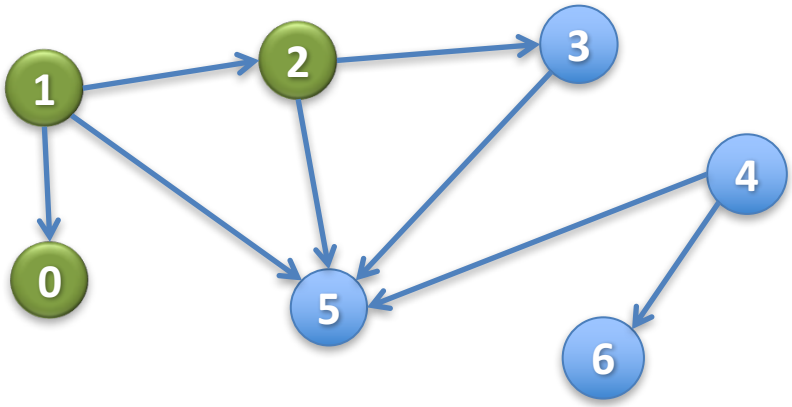

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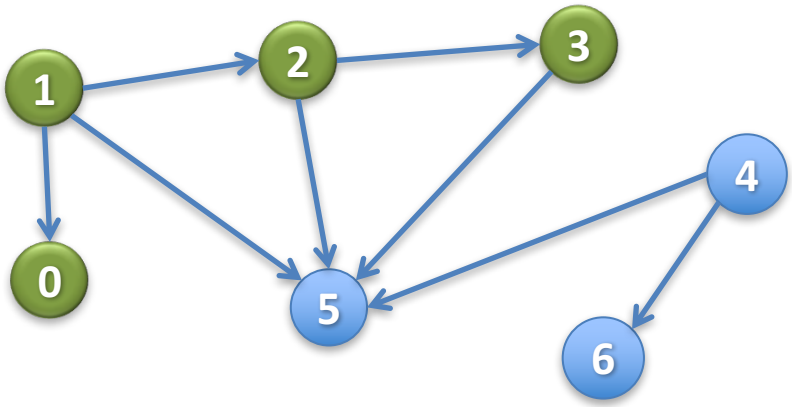

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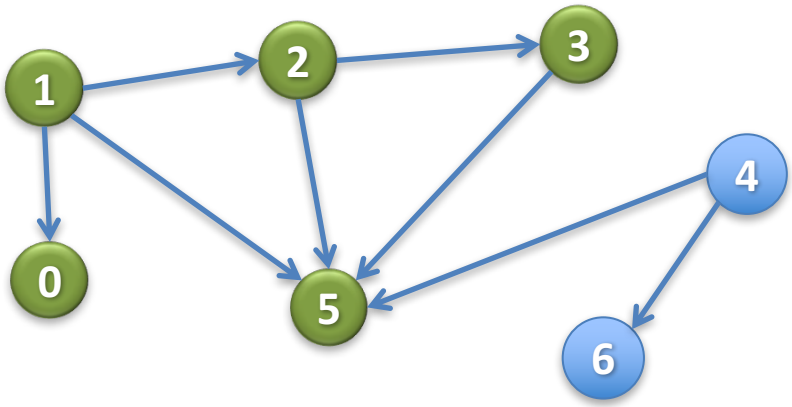

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```

Suppose n nodes are REACHABLE* along e edges (in total). What is

- Worst-case runtime? O(n+e)
- Worst-case space? O(n)

Depth-First Search

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}
```

That's all there is to basic DFS. You may have to change it to fit a particular situation.

If you don't have this spec and you do something different, it's probably wrong.

Example: Use different way (other than array visited) to know whether a node has been visited

Example: We really haven't said what data structures are used to implement the graph

Depth-First Search in OO fashion

```
public class Node {
    boolean visited;
    List<Node> neighbors;

    /** Visit all nodes that are REACHABLE*
     * from u. Precondition: u is unvisited */
    public void dfs() {
        visited= true;
        for (Node n: neighbors) {
            if (!n.visited) n.dfs();
        }
    }
}
```

Each node of the graph is an object of type Node

No need for a parameter. The object is the node.

Depth-First Search written iteratively

```
/** Visit all nodes REACHABLE* from u. Pre: u is unvisited. */
public static void dfs(int u) {
    Stack s= (u);    // Not Java!
    // inv: all nodes that have to be visited are
    //       REACHABLE* from some node in s
    while ( s is not empty ) {
        u= s.pop();    // Remove top stack node, put in u
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

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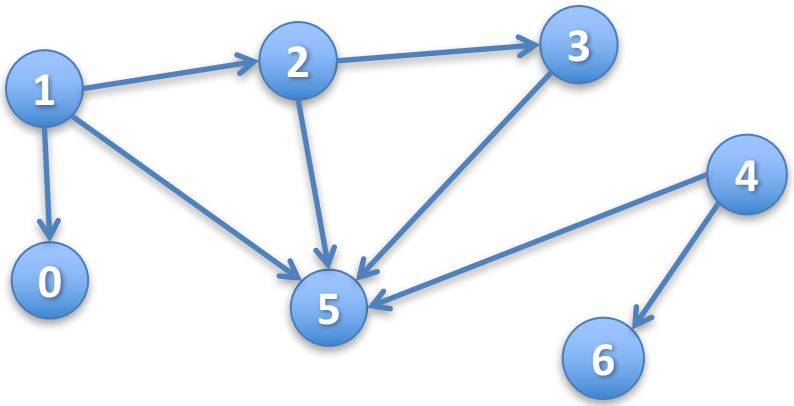
Call dfs(1)



1

Stack s

Depth-First Search written iteratively

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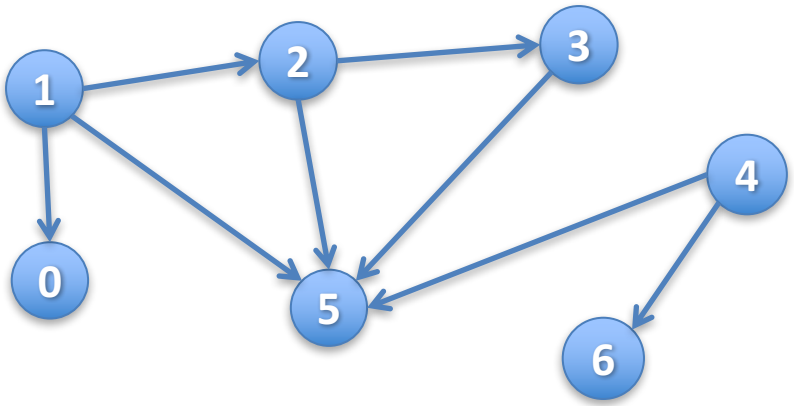
Iteration 0



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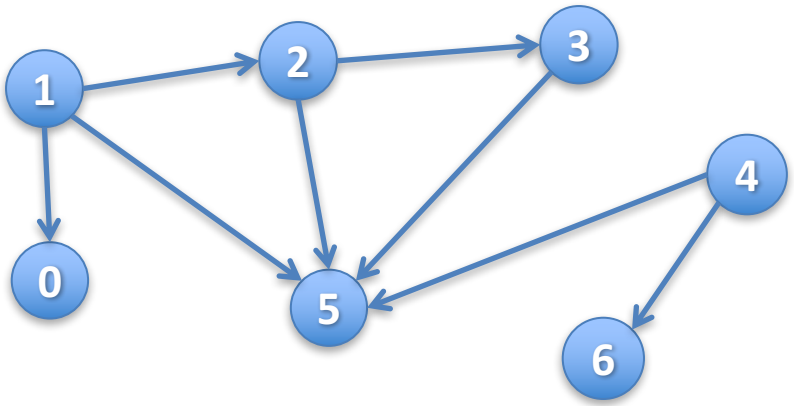

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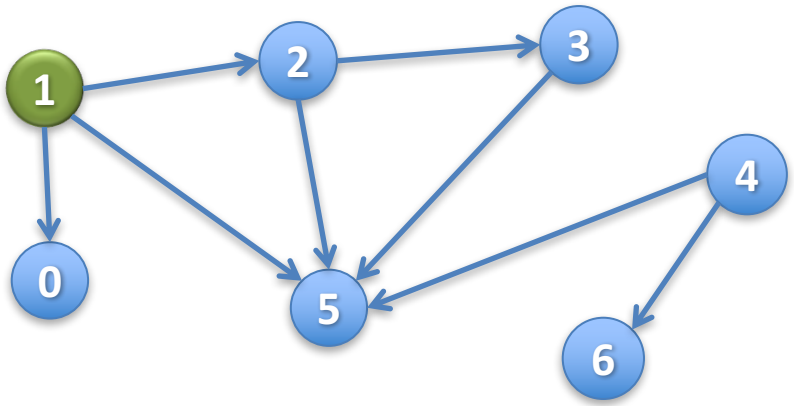
Call dfs(1)

Iteration 0



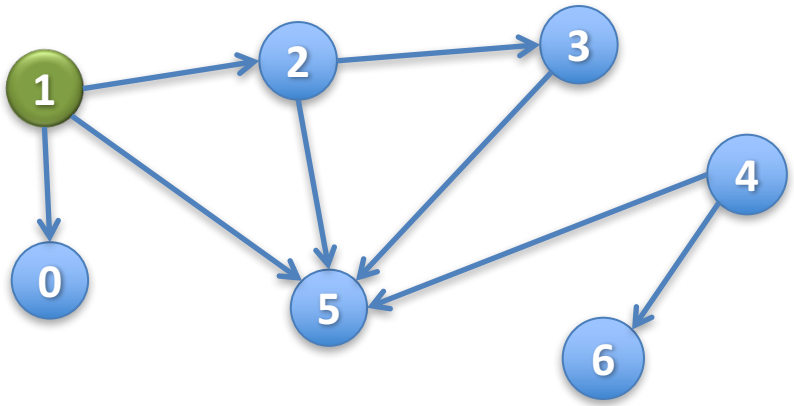
Stack s

Depth-First Search written iteratively

```
/** Visit all nodes REACHABLE* from u. Pre: u is unvisited.  */
public static void dfs(int u) {
      Stack s= (u);
      while (s is not empty) {
            u= s.pop();
            if (u has not been visited) {
                  visit u;
                  for each edge (u, v) leaving u:
                        s.push(v);
            }
      }
}
```

Call dfs(1)

Iteration 0



0

2

5

Stack s

Depth-First Search written iteratively

```
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public static void dfs(int u) {
      Stack s= (u);
      while (s is not empty) {
            u= s.pop();
            if (u has not been visited) {
                  visit u;
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                        s.push(v);
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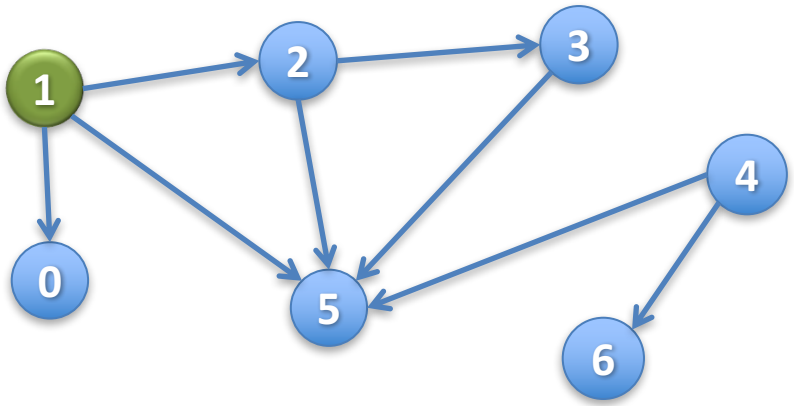
      }
}
```

Call dfs(1)

Iteration 1



0
2
5

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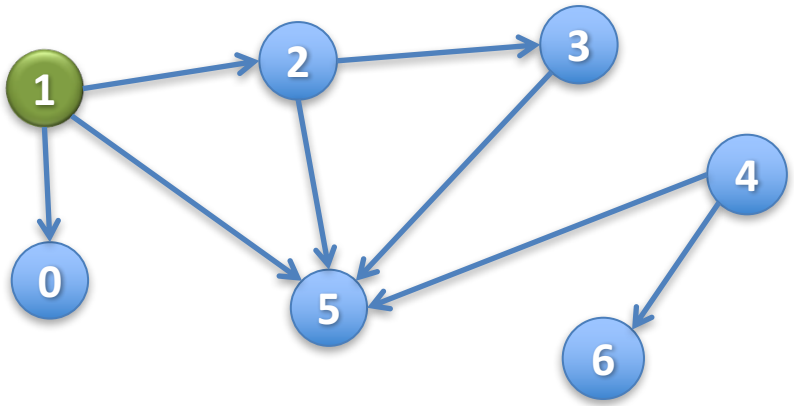
Call dfs(1)

Iteration 1



2

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Depth-First Search written iteratively

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      }
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```
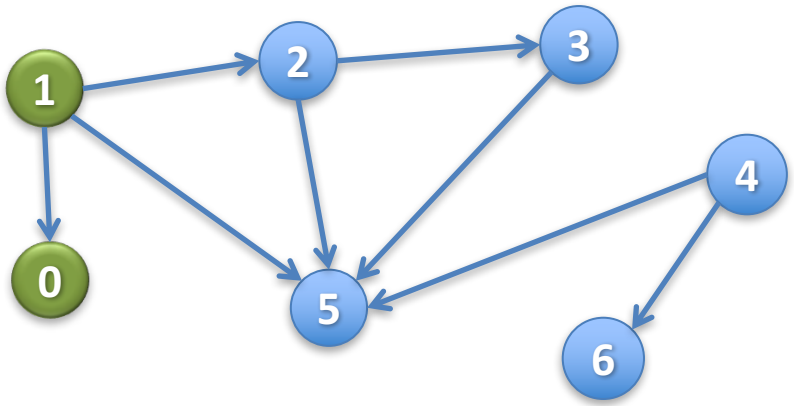

Call dfs(1)

Iteration 1



2

5

Stack s

Depth-First Search written iteratively

```
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                  for each edge (u, v) leaving u:
                        s.push(v);
            }
      }
}
```
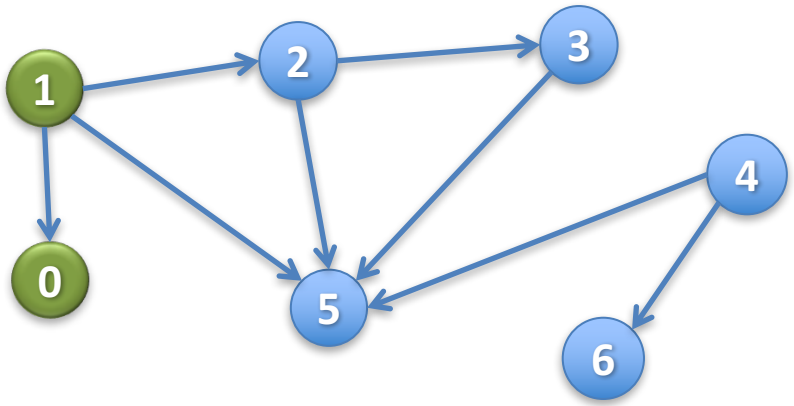

Call dfs(1)

Iteration 2



2

5

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Depth-First Search written iteratively

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                  for each edge (u, v) leaving u:
                        s.push(v);
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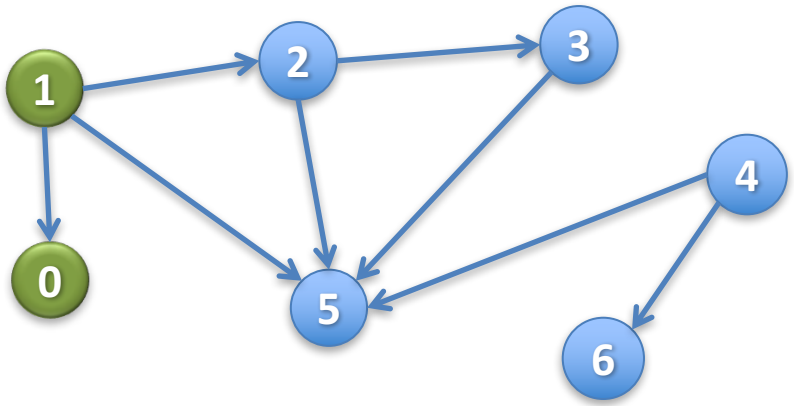
```

Call dfs(1)

Iteration 2



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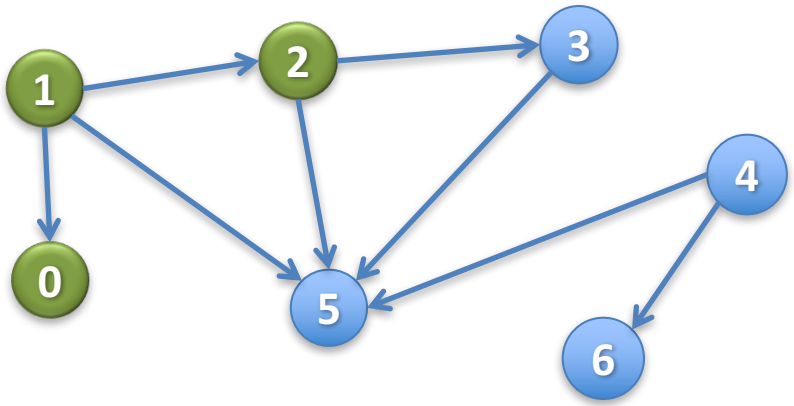
Call dfs(1)

Iteration 2



5

Stack s

Depth-First Search written iteratively

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/** Visit all nodes REACHABLE* from u. Pre: u is unvisited. . */
public static void dfs(int u) {
      Stack s= (u);
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                  for each edge (u, v) leaving u:
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            }
      }
}
```
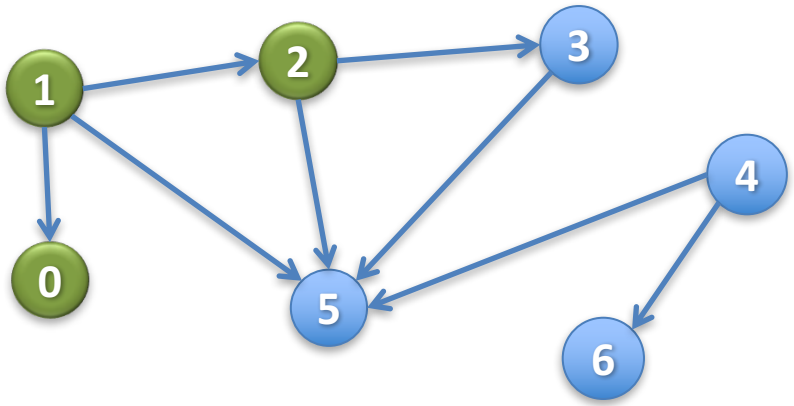

Call dfs(1)

Iteration 2

Yes, 5 is put on the stack twice, once for each edge to it. It will be visited only once.



3
5
5

Stack s

Depth-First Search written iteratively

```
/** Visit all nodes REACHABLE* from u. Pre: u is unvisited. . */
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

That's DFS!

```
/** Visit all nodes REACHABLE* from u. Pre: u is unvisited. */
public static void dfs(int u) {
    Stack s= (u);    // Not Java!
    // inv: all nodes that have to be visited are
    //       REACHABLE* from some node in s
    while ( s is not empty ) {
        u= s.pop();    // Remove top stack node, put in u
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

Want to see a magic trick?

Depth-First Search

```
/** Visit all nodes REACHABLE* from u. Pre: u is unvisited. */
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                s.push(v);
        }
    }
}
```

Breadth-First Search

```
/** Visit all nodes REACHABLE* from u. Pre: u is unvisited. */
public static void bfs(int u) {
    Queue q= (u);    // Not Java!
    // inv: all nodes that have to be visited are
    //       REACHABLE* from some node in s
    while ( q is not empty ) {
        u= q.popFirst();    // Remove first node in queue, put in u
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);    // Add to end of queue
        }
    }
}
```

Breadth-First Search

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/** Visit all nodes REACHABLE* from u. Pre: u is unvisited. */
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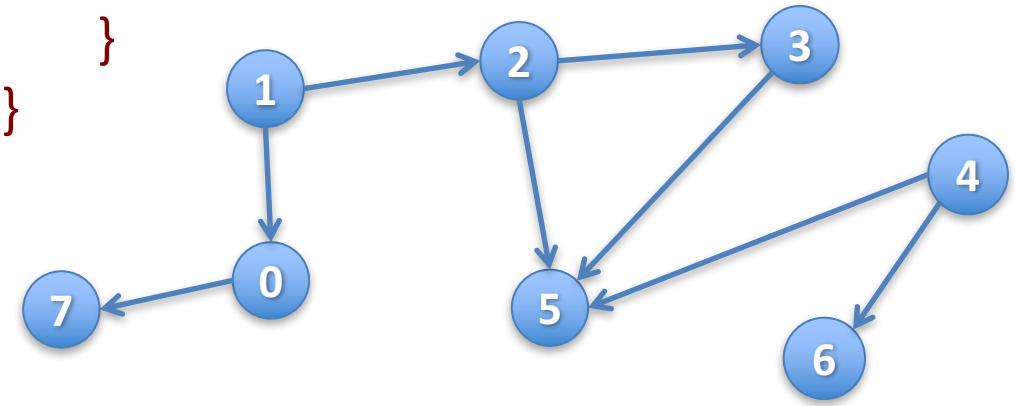
```

Call bfs(1)



1

Queue q

Breadth-First Search

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public static void bfs(int u) {
      Queue q= (u);
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                  visit u;
                  for each edge (u, v) leaving u:
                        q.append(v);
            }
      }
}
```
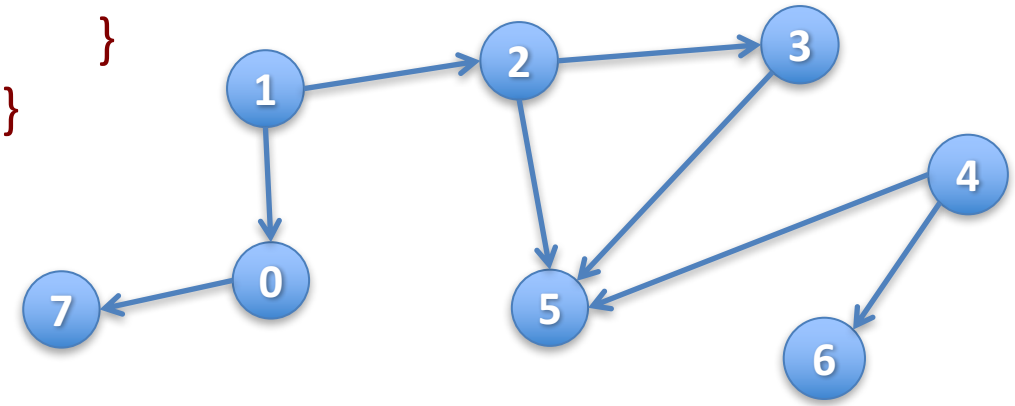

Call bfs(1)

Iteration 0



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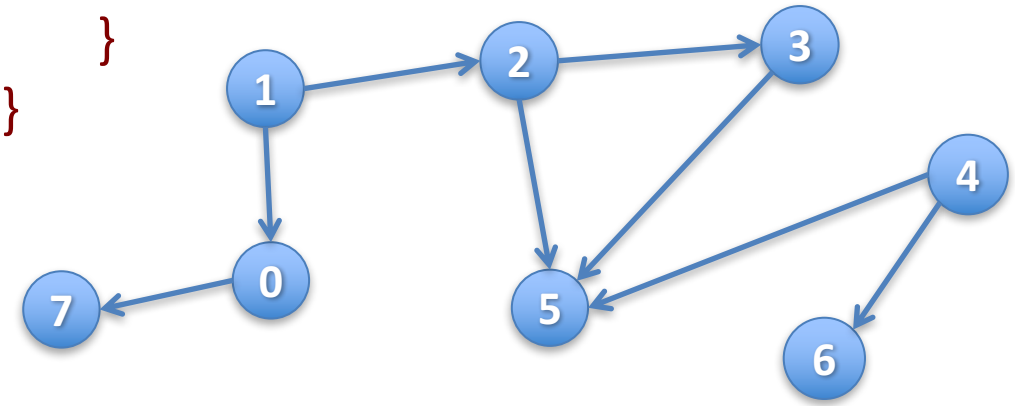
Call bfs(1)

Iteration 0



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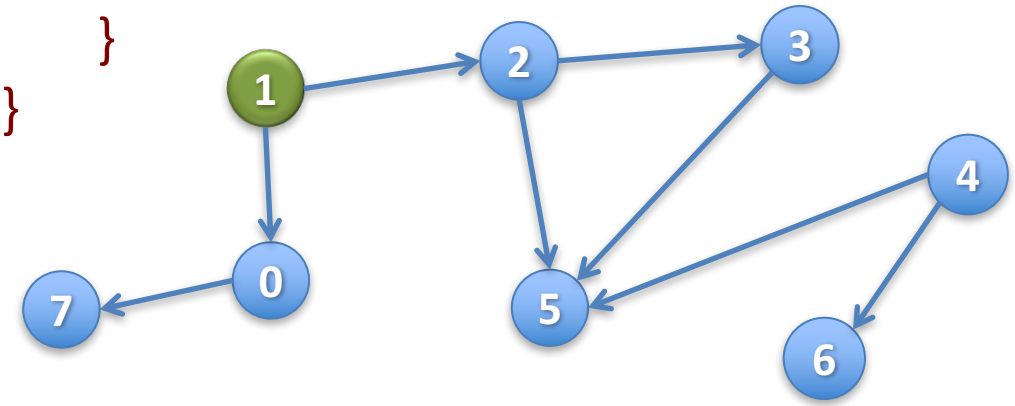

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Breadth-First Search

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                  for each edge (u, v) leaving u:
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      }
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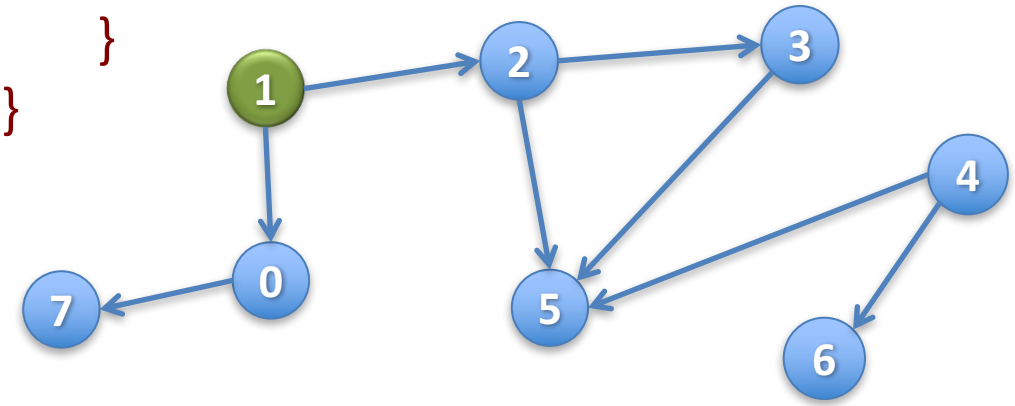
Call bfs(1)

Iteration 0



0 2

Queue q

Breadth-First Search

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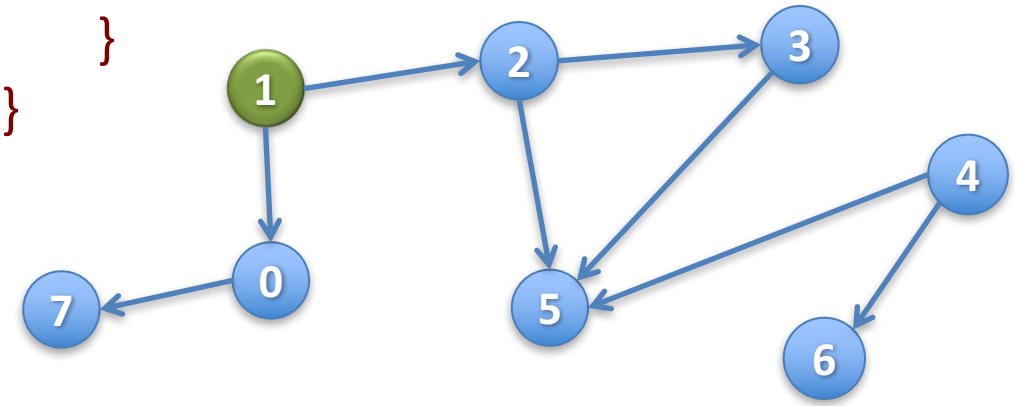
```

Call bfs(1)

Iteration 1



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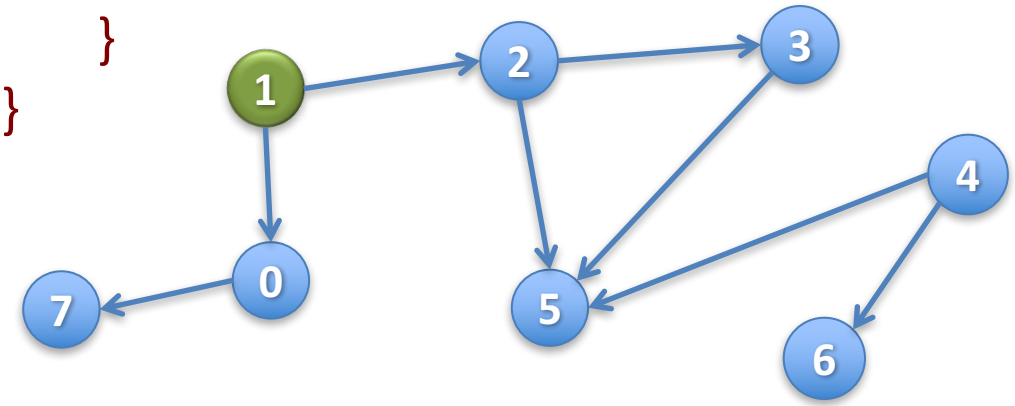
Call bfs(1)

Iteration 1



2

Queue q

Breadth-First Search

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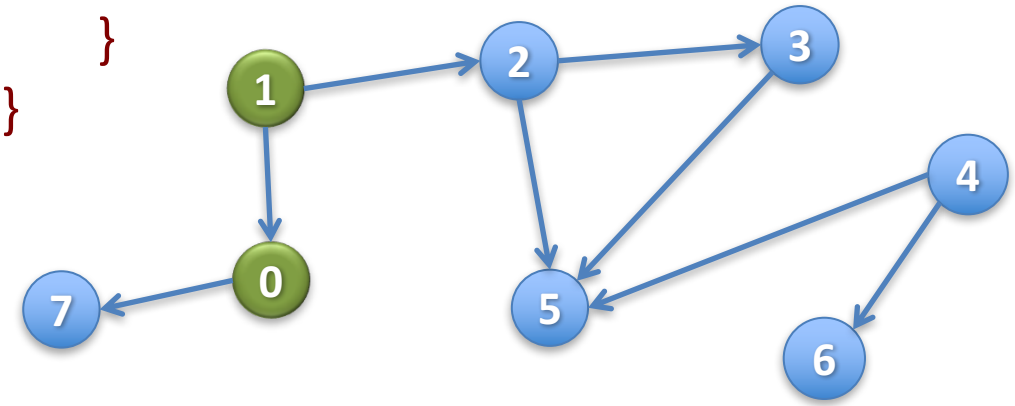
Call bfs(1)

Iteration 1



2

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Breadth-First Search

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```
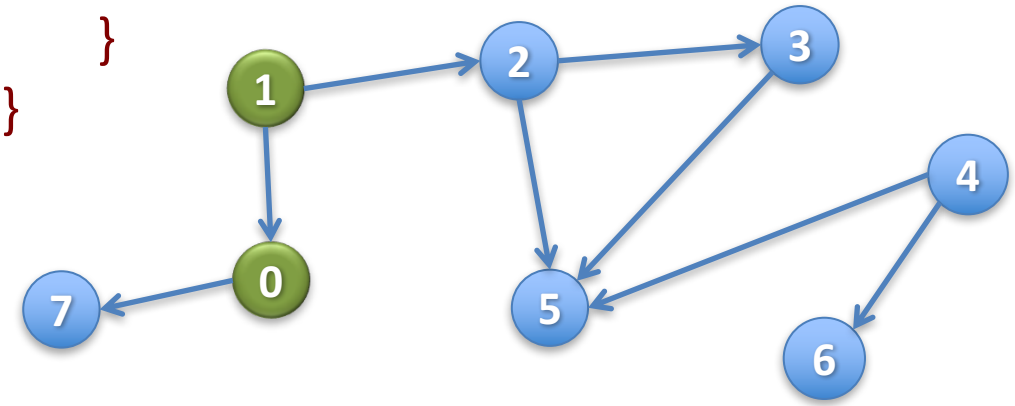

Call bfs(1)

Iteration 1



2 7

Queue q

Breadth-First Search

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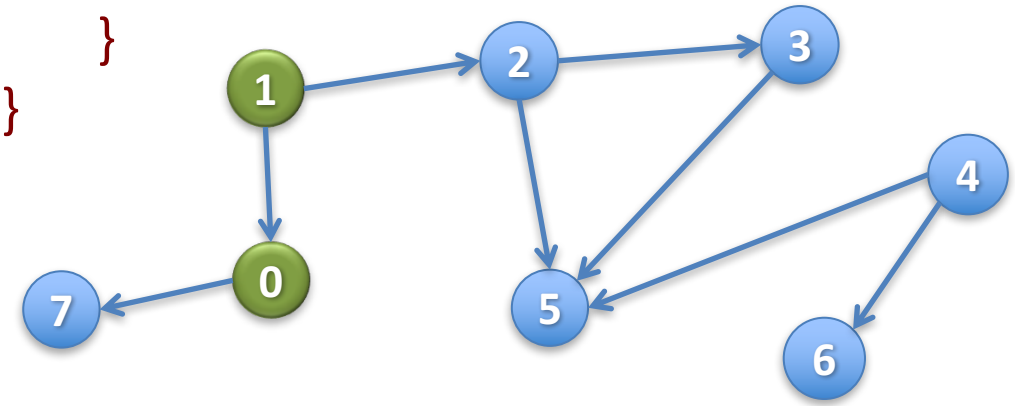
            }
      }
}
```

Call bfs(1)

Iteration 2



2 7

Queue q

Breadth-First Search

```
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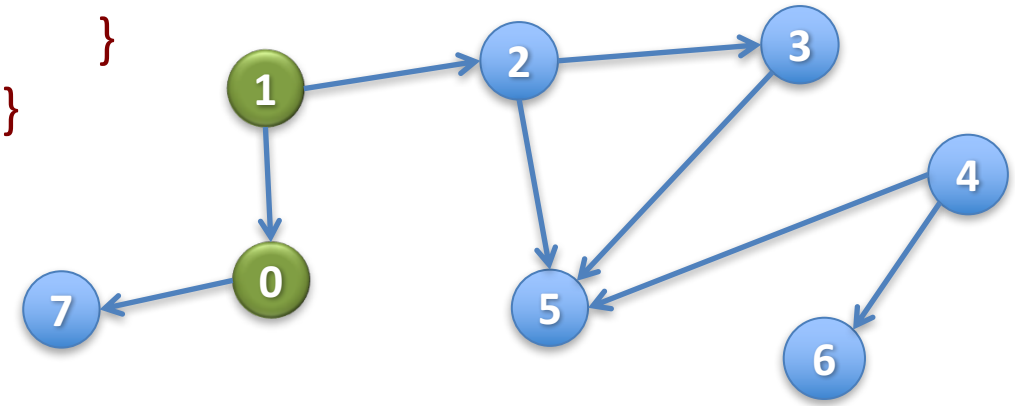
```

Call bfs(1)

Iteration 2



7

Queue q

Breadth-First Search

```
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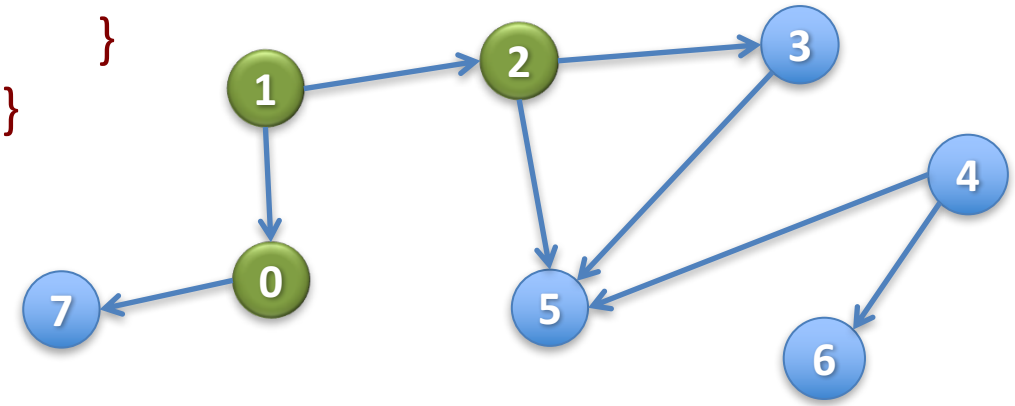
            }
      }
}
```

Call bfs(1)

Iteration 2



7

Queue q

Breadth-First Search

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                  for each edge (u, v) leaving u:
                        q.append(v);
            }
      }
}
```
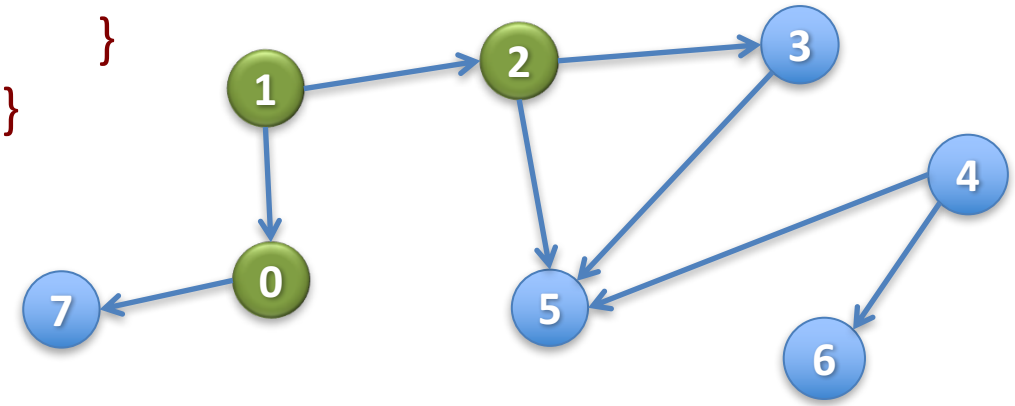

Call bfs(1)

Iteration 2

Breadth first:
(1) Node u
(2) All nodes 1 edge from u
(3) All nodes 2 edges from u
(4) All nodes 3 edges from u
…



7 3 5

Queue q

Some food for thought:

- BFS(root) on a tree corresponds to which tree traversal?
- Write out the order nodes are visited in this undirected graph, when calling:
  - BFS(5)
  - DFS(5)
  - DFS(0)

(if there are ties, visit the lower # first)