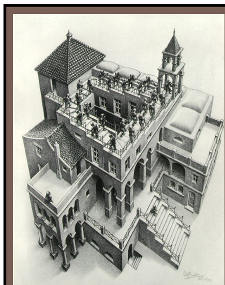

PRIORITY QUEUES AND HEAPS

Lecture 16
CS2110 Spring 2017

Announcements

2

- Next week's section: make your BugTrees hashable.

- Watch the tutorial videos on hashing:
  - http://www.cs.cornell.edu/courses/cs2110/2017sp/online/hashing/01hashing.html
  - Also linked from Recitation 07 on Lecture Notes page
  - As usual, watch videos BEFORE recitation so you can complete the assignment DURING recitation.

This lecture has a plot twist! See if you can spot it coming.

Readings and Homework

3

Read Chapter 26 "A Heap Implementation" to learn about heaps

Exercise: Salespeople often make matrices that show all the great features of their product that the competitor's product lacks. Try this for a heap versus a BST. First, try and sell someone on a BST: List some desirable properties of a BST that a heap lacks. Now be the heap salesperson: List some good things about heaps that a BST lacks. Can you think of situations where you would favor one over the other?

With ZipUltra heaps, you've got it made in the shade my friend!

Abstract vs concrete data structures

4

- Abstract data structures are **interfaces**
  - they specify only **interface** (method names and specs)
  - not **implementation** (method bodies, fields, …)
- Abstract data structures can have multiple possible **implementations.**

Abstract vs concrete data structures

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- **interface** List defines an "abstract data type".
- It has methods: add, get, remove, …
- Various **classes** implement List:

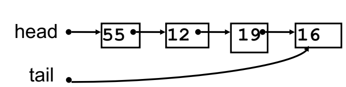
Class:	ArrayList	LinkedList
Backing storage:	array	chained nodes
add(i, val)	O(n)	O(n)
add(0, val)	O(n)	O(1)
add(n, val)	O(1)	O(1)
get(i)	O(1)	O(n)
get(0)	O(1)	O(1)
get(n)	O(1)	O(1)

Stacks and queues are restricted lists

6

- Stack (LIFO) implemented using a List
  - allows only `add(0,val), remove(0)` (push, pop)
- Queue (FIFO) implemented using a List
  - allows only `add(n,val), remove(0)` (enqueue, dequeue)
- These operations are O(1) in a LinkedList (not true in ArrayList)

Both efficiently implementable using a singly linked list with head and tail



Interface Bag (not In Java Collections)

7

```
interface Bag<E>
      implements Iterable {
   void add(E obj);
   boolean contains(E obj);
   boolean remove(E obj);
   int size();
   boolean isEmpty();
   Iterator<E> iterator()
}
```

Also called multiset

Like a set except that a value can be in it more than once. Example: a bag of coins

Refinements of Bag: Stack, Queue, PriorityQueue

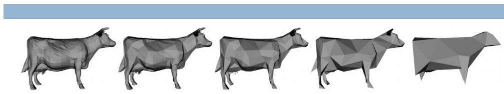
Priority queue

8

- **`Bag`** in which data items are **`Comparable`**
- Smaller elements (determined by **`compareTo()`**) have higher priority
- **`remove()`** return the element with the highest priority = least element in the **`compareTo()`** ordering
- break ties arbitrarily

Many uses of priority queues (& heaps)

9



Surface simplification [Garland and Heckbert 1997]

- Event-driven simulation: customers in a line
- Collision detection: "next time of contact" for colliding bodies
- Graph searching: Dijkstra's algorithm, Prim's algorithm
- AI Path Planning: A* search
- Statistics: maintain largest M values in a sequence
- Operating systems: load balancing, interrupt handling
- Discrete optimization: bin packing, scheduling
- College: prioritizing assignments for multiple classes.

`java.util.PriorityQueue<E>`

10

```
interface PriorityQueue<E> {                        TIME
 boolean add(E e) {...} //insert e.                 log
 void clear() {...} //remove all elems.
 E peek() {...} //return min elem.                  constant
 E poll() {...} //remove/return min elem.           log
 boolean contains(E e)                              linear
 boolean remove(E e)                                linear
 int size() {...}                                   constant
 Iterator<E> iterator()
}
```

IF implemented with a heap!

Priority queues as lists

11

- Maintain as unordered list
  - **`add()`** put new element at front – O(1)
  - **poll()** must search the list – O(n)
  - **peek()** must search the list – O(n)
- Maintain as ordered list
  - **`add()`** must search the list – O(n)
  - **poll()** min element at front – O(1)
  - **peek()** O(1)

Can we do better?

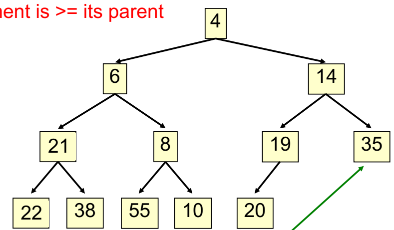
Heap: binary tree with certain properties

12

- A *heap* is a concrete data structure that can be used to implement priority queues
- Gives better complexity than either ordered or unordered list implementation:
  - **`add()`**: O(log n) (n is the size of the heap)
  - **`poll()`**: O(log n)
- O(n log n) to process n elements
- Do not confuse with *heap memory*, where the Java virtual machine allocates space for objects – different usage of the word *heap*

Heap: first property

Every element is >= its parent

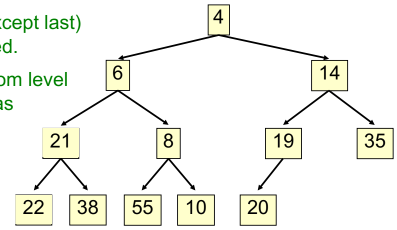


Note: 19, 20 < 35: Smaller elements can be deeper in the tree!

Heap: second property: is complete, has no holes

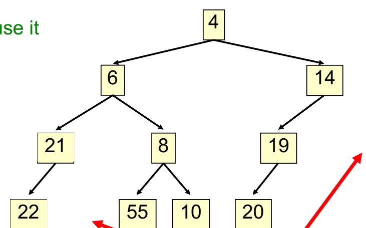
Every level (except last) completely filled.

Nodes on bottom level are as far left as possible.



Heap: Second property: has no "holes"

Not a heap because it has two holes



missing nodes

Not a heap because:

- missing a node on level 2
- bottom level nodes are not as far left as possible

Heap

- Binary tree with data at each node
- Satisfies the *Heap Order Invariant*:

1. Every element is ≥ its parent.

- Binary tree is **complete** (no holes)

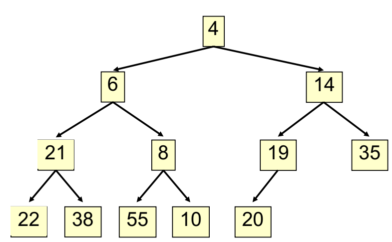
2. Every level (except last) completely filled. Nodes on bottom level are as far left as possible.

Heap Quiz 1: Heap it real.

Which of the following are valid heaps?

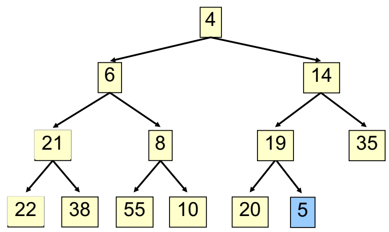
```
   (A)              (B)        (C)              (D)
    5                5          5               -5
   / \              /          / \             /   \
  12    15        12          12   15         12     15
 / \             / \          /    / \       / \     /
13  11          13  14       13   15  16    18  13  15
```

add(e)



add(e)

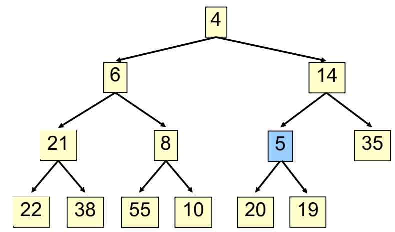
19



1. Put in the new element in a new node

add()

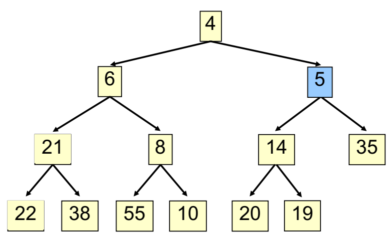
20



2. Bubble new element up if less than parent

add()

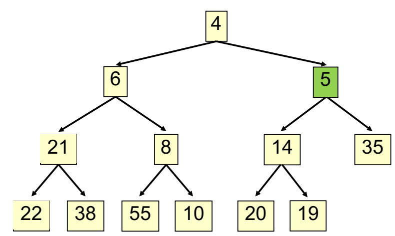
21



2. Bubble new element up if less than parent

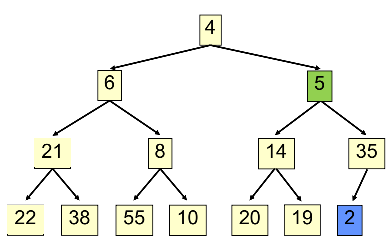
add()

22



add()

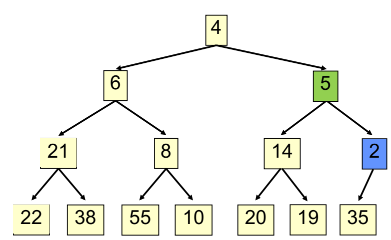
23



1. Put in the new element in a new node

add()

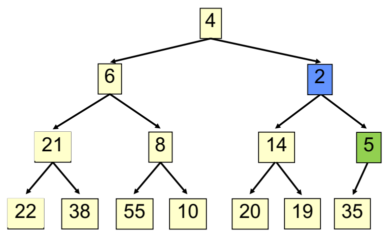
24



2. Bubble new element up if less than parent

add()

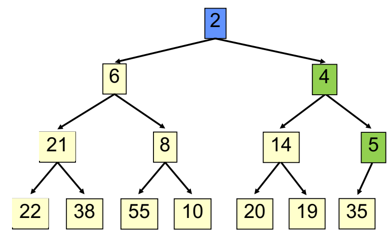
25



2. Bubble new element up if less than parent

add()

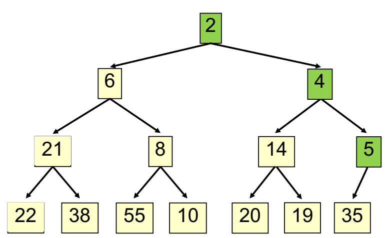
26



2. Bubble new element up if less than parent

add()

27



add(e)

28

- Add e at the leftmost empty leaf
- Bubble e up until it no longer violates heap order
- The heap invariant is maintained!

add() to a tree of size n

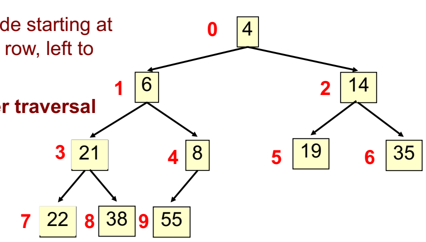
29

- Time is O(log n), since the tree is balanced
  - size of tree is exponential as a function of depth
  - depth of tree is logarithmic as a function of size

Numbering the nodes in a heap

Number node starting at root row by row, left to right

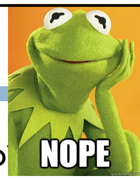
Level-order traversal

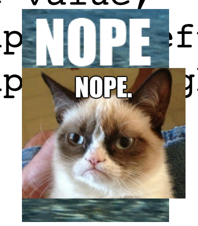


Children of node k are nodes 2k+1 and 2k+2
Parent of node k is node (k-1)/2

Implementing Heaps

31






```
public class Heap
  private int value;
  private Heap      eft;
  private Heap      ght;
  ...
}
```

Implementing Heaps

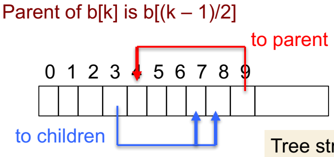
32

```
public class HeapNode {
  private int[] heap;
  ...
}
```

Store a heap in an array (or ArrayList) b!

33

- Heap nodes in b in order, going across each level from left to right, top to bottom
- Children of b[k] are b[2k + 1] and b[2k + 2]
- Parent of b[k] is b[(k – 1)/2]



Tree structure is implicit.
No need for explicit links!

add() --assuming there is space

34

```
/** An instance of a heap */
class Heap<E> {
  E[] b= new E[50];  // heap is b[0..n-1]
  int n= 0;          // heap invariant is true

  /** Add e to the heap */
  public void add(E e) {
    b[n]= e;
    n= n + 1;
    bubbleUp(n - 1); // given on next slide
  }
}
```

add(). Remember, heap is in b[0..n-1]

35

```
class Heap<E> {
  /** Bubble element #k up to its position.
    * Pre: heap inv holds except maybe for k */
  private void bubbleUp(int k) {
    int p= (k-1)/2;
    // inv: p is parent of k and every elmnt
    // except perhaps k is >= its parent
    while (k > 0  &&  b[k].compareTo(b[p]) < 0) {
       swap(b[k], b[p]);
       k= p;
       p= (k-1)/2;
    }
}
```

Heap Quiz 2: Pile it on!

36

Here's a heap, stored in an array:

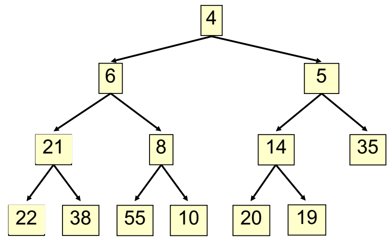
[1 5 7 6 7 10]

Write the array after execution of add(4)? Assume the existing array is large enough to store the additional element.

A. [1 5 7 6 7 10 4]
B. [1 4 5 6 7 10 7]
C. [1 5 4 6 7 10 7]
D. [1 4 56 7 6 7 10]

poll()

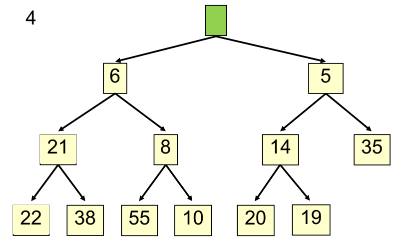
37



poll()

38

4

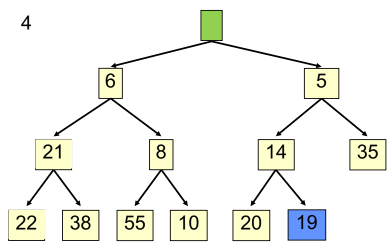


1. Save top element in a local variable

poll()

39

4

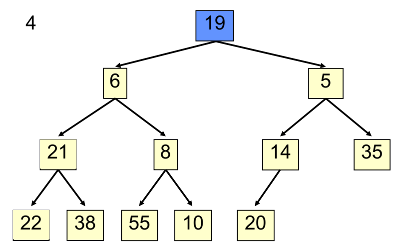


2. Assign last value to the root, delete last value from heap

poll()

40

4

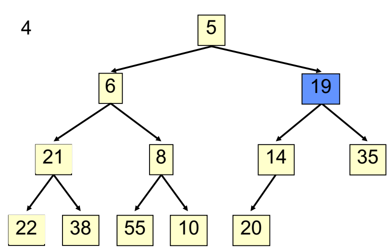


3. Bubble root value down

poll()

41

4

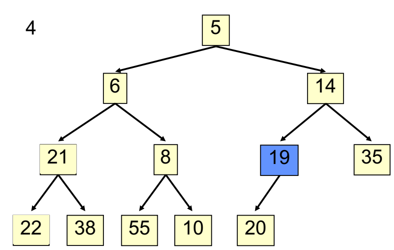


3. Bubble root value down

poll()

42

4

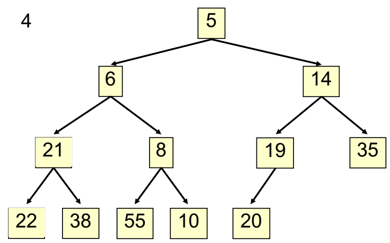


3. Bubble root value down

poll()

43

4

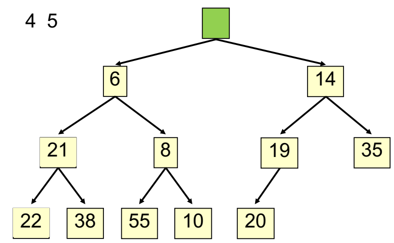


1. Save top element in a local variable

poll()

44

4 5

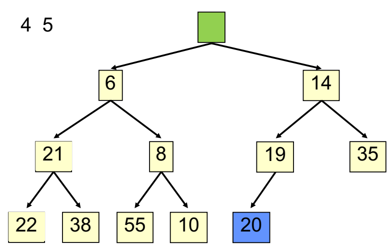


2. Assign last value to the root, delete last value from heap

poll()

45

4 5

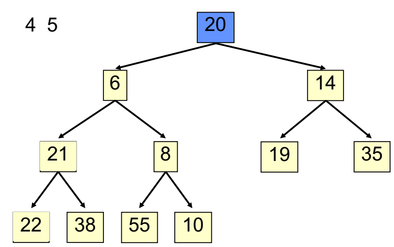


2. Assign last value to the root, delete last value from heap

poll()

46

4 5

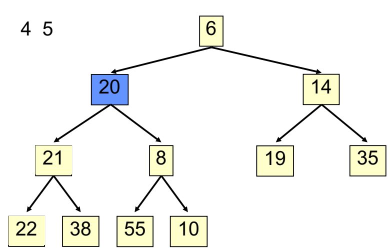


3. Bubble root value down

poll()

47

4 5

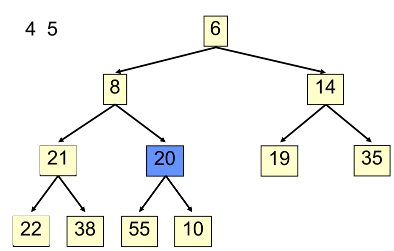


3. Bubble root value down

poll()

48

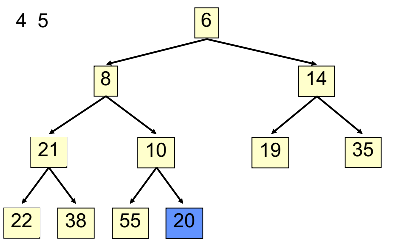
4 5



3. Bubble root value down

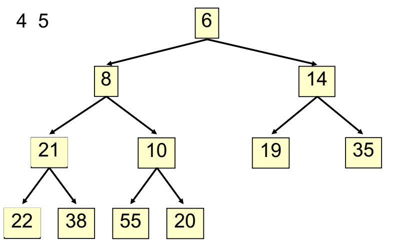
poll()

49



poll()

50



3. Bubble root value down

poll()

51

- Save the least element (the root)
- Assign last element of the heap to the root.
- Remove last element of the heap.
- Bubble element down –always with smaller child, until heap invariant is true again.

  The heap invariant is maintained!
- Return the saved element

  Time is O(log n), since the tree is balanced

poll(). Remember, heap is in b[0..n-1]

52

```
/** Remove and return the smallest element
  * (return null if list is empty) */
public E poll() {
    if (n == 0) return null;
    E v=  b[0];   // smallest value at root.
    n= n – 1;     // move last
    b[0]= b[n];   // element to root
    bubbleDown(0);
    return v;
}
```

c's smaller child

53

```
/** Tree has n node.
 *  Return index of smaller child of node k
    (2k+2 if k >= n) */
public int smallerChild(int k, int n) {
   int c= 2*k + 2;     // k's right child
   if (c >= n || b[c-1].compareTo(b[c]) < 0)
      c= c-1;
   return c;
}
```

54

```
/** Bubble root down to its heap position.
    Pre: b[0..n-1] is a heap except maybe b[0] */
private void bubbleDown() {
   int k= 0;
   int c= smallerChild(k, n);
   // inv: b[0..n-1] is a heap except maybe b[k] AND
   //      b[c] is b[k]'s smallest child
   while ( c < n &&  b[k].compareTo(b[c]) > 0) {

      swap(b[k], b[c]);
      k= c;
      c= smallerChild(k, n);
   }
}
```

Change heap behaviour a bit

55

Separate priority from value and do this:

add(e, p); //add element e with priority p (a double)

THIS IS EASY!

Be able to change priority

change(e, p); //change priority of e to p

THIS IS HARD!

Big question: How do we find e in the heap?
Searching heap takes time proportional to its size! No good!
Once found, change priority and bubble up or down. OKAY

Assignment A6: implement this heap! Use a second data structure to make change-priority expected log n time

HeapSort(b, n) —Sort b[0..n-1]

56

Whet your appetite –use heap to get exactly n log n in-place sorting algorithm. 2 steps, each is O(n log n)

1. Make b[0..n-1] into a max-heap (in place)

1. for (k= n-1; k > 0; k= k-1) {
   b[k]= poll –i.e. take max element out of heap.
   }

This algorithm is on course website

A max-heap has max value at root