TREES

Lecture 12
CS2110 – Spring 2017

Important Announcements

2

- A4 is out! Due two weeks from today. Follow the timetable and enjoy a stress-free A4 experience!
- Mid-semester TA evaluations are open; please participate!
  - Your feedback can help our staff improve YOUR experience for the rest of this semester.
- Next week's recitation is canceled!
  - All Tuesday sections will be office hours instead (held in same room as recitation unless noted on Piazza)

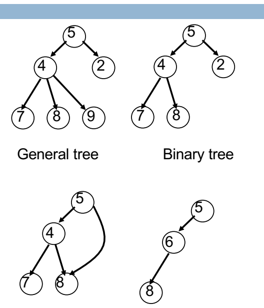
Tree Overview

3

Tree: data structure with nodes, similar to linked list

- Each node may have zero or more *successors* (children)
- Each node has exactly one *predecessor* (parent) except the *root*, which has none
- All nodes are reachable from root

Binary tree: tree in which each node can have at most two children: a left child and a right child



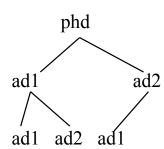
General tree   Binary tree   Not a tree   List-like tree

Binary trees were in A1!

4

You have seen a binary tree in A1.

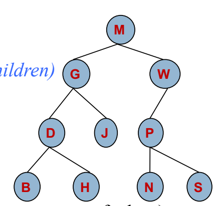
A PhD object phd has one or two advisors.
Here is an intellectual ancestral tree!



Tree terminology

5

M: root of this tree
G: *root* of the *left subtree* of M
B, H, J, N, S: *leaves (they have no children)*
N: *left child* of P; S: *right child* of P
P: *parent* of N
M and G: *ancestors* of D
P, N, S: *descendants* of W
J is at *depth* 2 (i.e. length of path from root = no. of edges)
The subtree rooted at W has *height* (i.e. length of longest path to a leaf) of 2
A collection of several trees is called a ...?



Class for binary tree node

6

```
class TreeNode<T> {
   private T datum;
   private TreeNode<T> left, right;

   /** Constructor: one-node tree with datum x */
   public TreeNode (T d) { datum= d; left= null; right= null;}

   /** Constr: Tree with root value x, left tree l, right tree r */
   public TreeNode (T d, TreeNode<T> l, TreeNode<T> r) {
      datum= d; left= l; right= r;
   }
}
```

Points to left subtree (null if empty)

Points to right subtree (null if empty)

more methods: getValue, setValue, getLeft, setLeft, etc.

Binary versus general tree

In a binary tree, each node has up to two pointers: to the left subtree and to the right subtree:

- One or both could be **null**, meaning the subtree is empty (remember, a tree is a set of nodes)

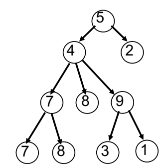
In a general tree, a node can have any number of child nodes (and they need not be ordered)

- Very useful in some situations ...
- ... one of which may be in an assignment!

Class for general tree nodes

```
class GTreeNode<T> {
   private T datum;
   private List<GTreeNode<T>> children;
   //appropriate constructors, getters,
   //setters, etc.
}
```

Parent contains a list of its children

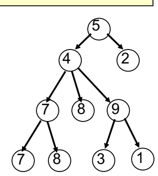


General tree

Class for general tree nodes

```
class GTreeNode<T> {
   private T datum;
   private List<GTreeNode<T>> children;
   //appropriate constructors, getters,
   //setters, etc.
}
```

Java.util.List is an interface! It defines the methods that all implementation must implement. Whoever writes this class gets to decide what implementation to use — ArrayList? LinkedList? Etc.?



General tree

Applications of Tree: Syntax Trees

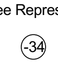
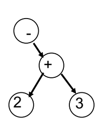
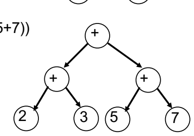
- Most languages (natural and computer) have a recursive, hierarchical structure
- This structure is *implicit* in ordinary textual representation
- Recursive structure can be made *explicit* by representing sentences in the language as trees: Abstract Syntax Trees (ASTs)
- ASTs are easier to optimize, generate code from, etc. than textual representation
- A parser converts textual representations to AST

Applications of Tree: Syntax Trees

In textual representation:
  Parentheses show hierarchical structure

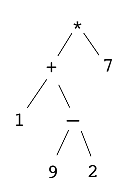
In tree representation:
  Hierarchy is explicit in the structure of the tree

We'll talk more about expressions and trees in next lecture

Text	Tree Representation
-34	
- (2 + 3)	
((2+3) + (5+7))	

Got that?

(1 + (9 — 2)) * 7



F *, +, and 7 are ancestors of 1
T 9's parent is -
F The tree's height is 4
T 1 is a leaf node
T 9 is at depth 3
F The root is 7

Recursion on trees

13

Trees are defined recursively:

A binary tree is either

(1) empty

or

(2) a value (called the root value),

a left binary tree, and a right binary tree

Recursion on trees

14

Trees are defined recursively, so recursive methods can be written to process trees in an obvious way.

Base case

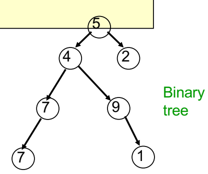
- empty tree (null)
- leaf

Recursive case

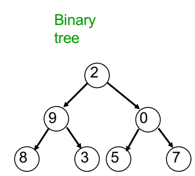
- solve problem on each subtree
- put solutions together to get solution for full tree

Class for binary tree nodes

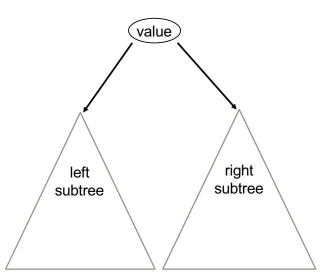
```
class BinTreeNode<T> {
    private T datum;
    private BinTreeNode<T> left;
    private BinTreeNode<T> right;
    //appropriate constructors, getters,
    //setters, etc.
}
```



Looking at trees recursively



Looking at trees recursively

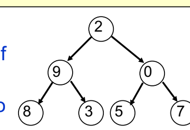


Searching in a Binary Tree

18

```
/** Return true iff x is the datum in a node of tree  t*/
public static boolean treeSearch(T x, TreeNode<T> t) {
   if (t == null) return false;
   if (x.equals(t.datum)) return true;
   return treeSearch(x, t.left) || treeSearch(x, t.right);
}
```

- Analog of linear search in lists: given tree and an object, find out if object is stored in tree
- Easy to write recursively, harder to write iteratively



Searching in a Binary Tree

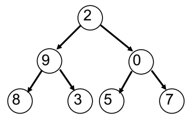
19

```
/** Return true iff x is the datum in a node of tree  t*/
public static boolean treeSearch(T x, TreeNode<T> t) {
   if (t == null) return false;
   if (x.equals(t.datum)) return true;
   return treeSearch(x, t.left) || treeSearch(x, t.right);
}
```

VERY IMPORTANT!

We sometimes talk of t as the root of the tree.

But we also use t to denote the whole tree.



Some useful methods – what do they do?

20

```
/** Method A ??? */
public static boolean A(Node n) {
   return n != null  &&  n.left == null  &&  n.right == null;
}
/** Method B ???  */
public static int B(Node n) {
   if (n== null) return -1;
   return 1 + Math.max(B(n.left), B(n.right));
}
/** Method C ???  */
public static int C(Node n) {
   if (n== null) return 0;
   return 1 + C(n.left) + C(n.right);
}
```

Some useful methods

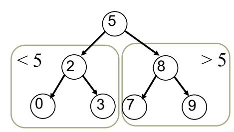
21

```
/** Return true iff node n is a leaf */
public static boolean isLeaf(Node n) {
   return n != null  &&  n.left == null  &&  n.right == null;
}
/** Return height of node n (postorder traversal) */
public static int height(Node n) {
   if (n== null) return -1; //empty tree
   return 1 + Math.max(height(n.left), height(n.right));
}
/** Return number of nodes in n (postorder traversal) */
public static int numNodes(Node n) {
   if (n== null) return 0;
   return 1 + numNodes(n.left) + numNodes(n.right);
}
```

Binary Search Tree (BST)

22

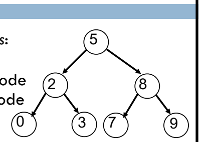
If the tree data is *ordered and has no duplicate values:*
in every subtree,
All *left* descendents of a node come *before* the node
All *right* descendents of a node come *after* the node
Search can be made MUCH faster



Binary Search Tree (BST)

23

If the tree data is *ordered and has no duplicate values:*
in every subtree,
All *left* descendents of a node come *before* the node
All *right* descendents of a node come *after* the node
Search can be made MUCH faster



Compare binary tree to binary search tree:

```
boolean searchBT(n, v):
  if n==null, return false
  if n.v == v, return true
  return searchBST(n.left, v)
      || searchBST(n.right, v)
```

2 recursive calls

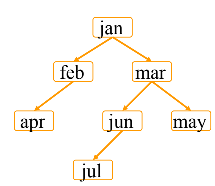
```
boolean searchBST(n, v):
  if n==null, return false
  if n.v == v, return true
  if v < n.v
    return searchBST(n.left, v)
  else
    return searchBST(n.right, v)
```

1 recursive call

Building a BST

24

- To insert a new item
  - Pretend to look for the item
  - Put the new node in the place where you fall off the tree
- This can be done using either recursion or iteration
- Example
  - Tree uses *alphabetical order*
  - Months appear for insertion in *calendar order*



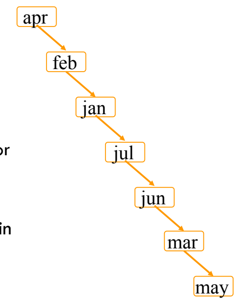
What can go wrong?

25

A BST makes searches very fast, *unless…*

- Nodes are inserted in increasing order
- In this case, we're basically building a linked list (with some extra wasted space for the `left` fields, which aren't being used)

BST works great if data arrives in random order



Printing contents of BST

26

Because of ordering rules for a BST, it's easy to print the items in alphabetical order

- Recursively print left subtree
- Print the node
- Recursively print right subtree

```
/** Print BST t in alpha order */
private static void print(TreeNode<T> t) {
   if (t== null) return;
   print(t.left);
   System.out.print(t.datum);
   print(t.right);
}
```

Tree traversals

27

"Walking" over the whole tree is a tree traversal

- Done often enough that there are standard names

Previous example: in-order traversal

- Process left subtree
- Process root
- Process right subtree

Note: Can do other processing besides printing

Other standard kinds of traversals

- preorder traversal
  - Process root
  - Process left subtree
  - Process right subtree
- postorder traversal
  - Process left subtree
  - Process right subtree
  - Process root
- level-order traversal
  - Not recursive: uses a queue (we'll cover this later)

Useful facts about binary trees

28

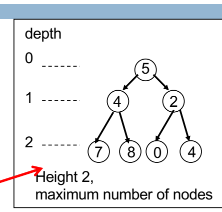
Max # of nodes at depth d: 2^d

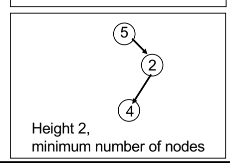
If height of tree is h

- min # of nodes: $h + 1$
- max #of nodes in tree:
- $2^0 + \ldots + 2^h = 2^{h+1} - 1$

Complete binary tree

- All levels of tree down to a certain depth are completely filled





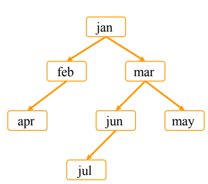
Things to think about

29

What if we want to *delete* data from a BST?

A BST works great as long as it's *balanced*

How can we keep it balanced? *This turns out to be hard enough to motivate us to create other kinds of trees*



Tree Summary

30

- A *tree* is a recursive data structure
  - Each node has 0 or more successors (*children*)
  - Each node except the *root* has exactly one predecessor (*parent*)
  - All node are reachable from the *root*
  - A node with no children (or empty children) is called a *leaf*
- Special case: *binary tree*
  - Binary tree nodes have a left and a right child
  - Either or both children can be empty (null)
- Trees are useful in many situations, including exposing the recursive structure of natural language and computer programs