Recitation 10

Prelim Review

Big O

See the Study Habits Note @282 on the course Piazza. There is a 2-page pdf file that says how to learn what you need to know for O-notation.

Big O definition

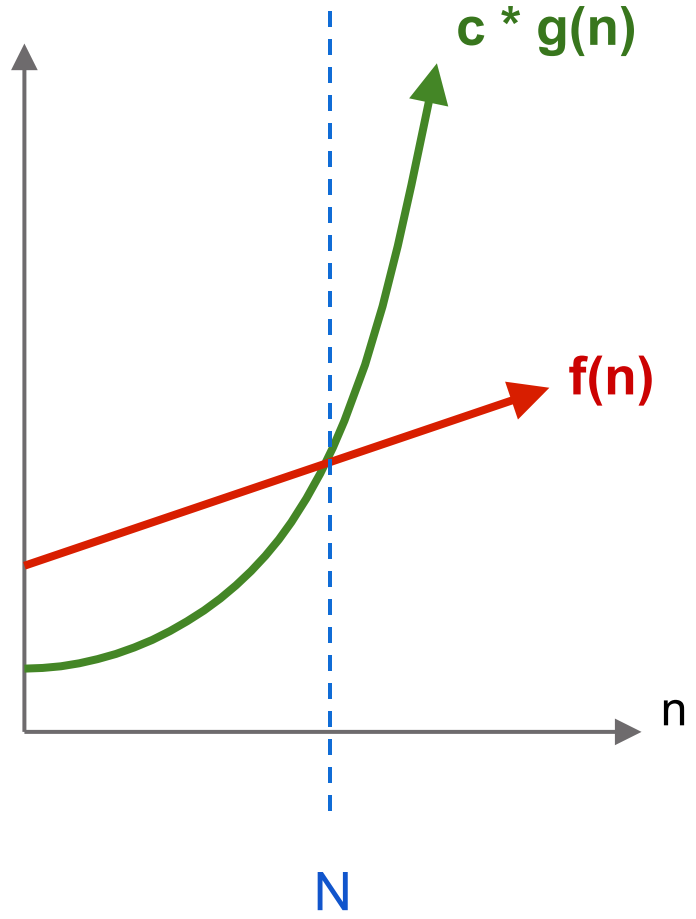
f(n) is O(g(n))

iff

There is a positive constant c

and a real number N such that:

$f(n) \leq c * g(n)$ for $n \geq N$



Is merge sort $O(n^3)$?

Yes, but not tightest upper bound

Review: Big O

Is used to classify algorithms by how they respond to changes in input size n.

Important vocabulary:

- Constant time: O(1)
- Logarithmic time: O(log n)
- Linear time: O(n)
- Quadratic time: $O(n^2)$

Let f(n) and g(n) be two functions that tell how many statements two algorithms execute when running on input of size n.

$f(n) >= 0$ and $g(n) >= 0$.

Review: Informal Big O rules

1. Usually: $O(f(n)) \times O(g(n)) = O(f(n) \times g(n))$
   - Such as if something that takes g(n) time for each of f(n) repetitions . . .

     (loop within a loop)

2. Usually: $O(f(n)) + O(g(n)) = O(\max(f(n), g(n)))$
   - "max" is whatever's dominant as n approaches infinity
   - Example: $O((n^2\text{-}n)/2) = O((1/2)n^2 + (-1/2)n) = O((1/2)n^2)$

     $= O(n^2)$

3. Why don't logarithm bases matter?
   - For constants x, y: $O(\log_x n) = O((\log_x y)(\log_y n))$
   - Since $(\log_x y)$ is a constant, $O(\log_x n) = O(\log_y n)$

Test will not require understanding such rules for logarithms

Review: Big O

1. $\log(n) + 20$	is	$O(\log(n))$	(logarithmic)
2. $n + \log(n)$	is	$O(n)$	(linear)
3. $n/2$ and $3*n$	are	$O(n)$	
4. $n * \log(n) + n$	is	$O(n * \log(n))$	
5. $n^2 + 2*n + 6$	is	$O(n^2)$	(quadratic)
6. $n^3 + n^2$	is	$O(n^3)$	(cubic)
7. $2^n + n5$	is	$O(2^n)$	(exponential)

Review: Big O examples

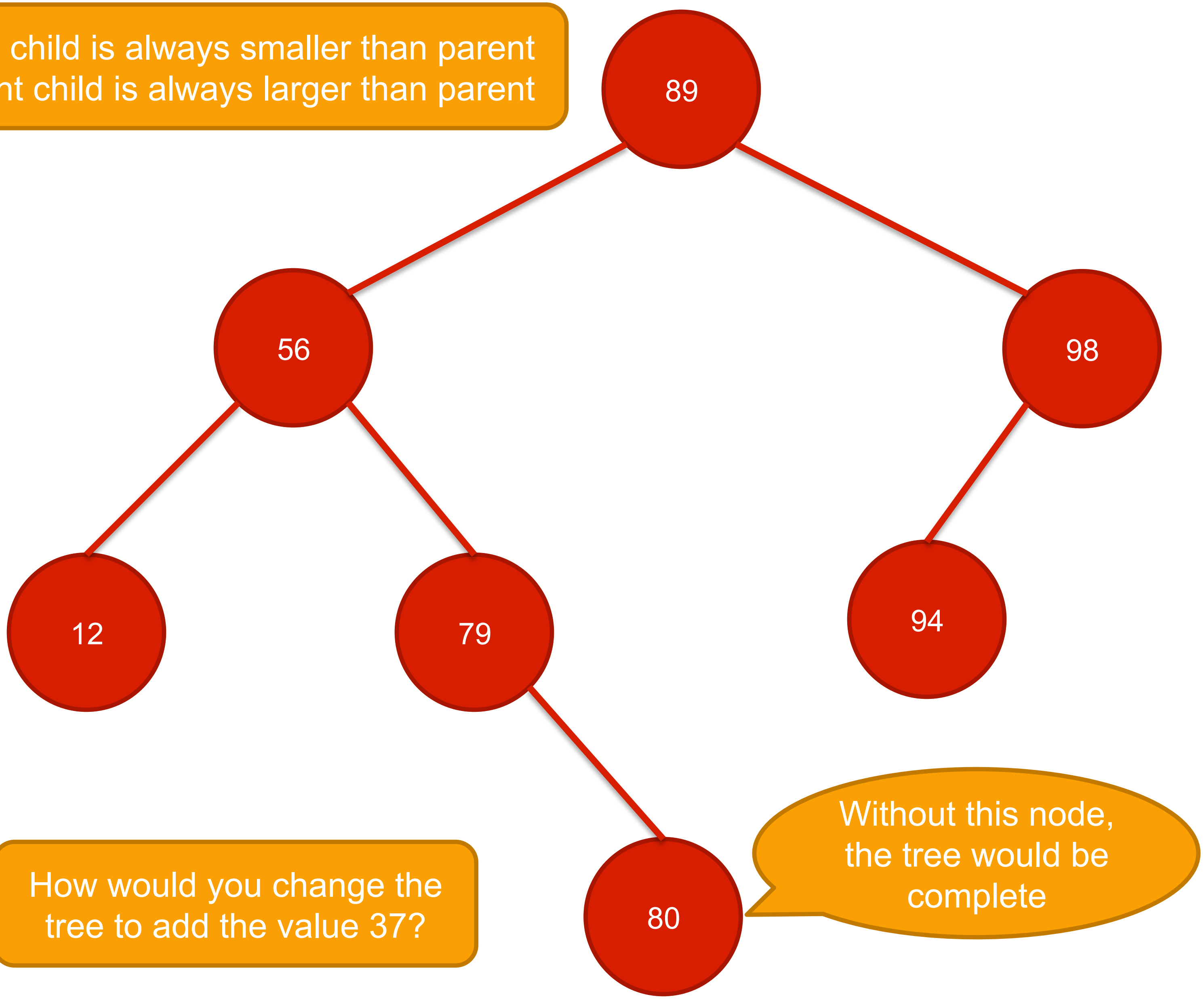
1. What is the runtime of an algorithm that runs insertion sort on an array $O(n^2)$ and then runs binary search $O(\log n)$ on that now sorted array?

2. What is the runtime of finding and removing the fifth element from a linked list? What if in the middle of that remove operation we swapped two integers exactly 100000 times, what is the runtime now?

3. What is the runtime of running merge sort 4 times? *n* times?

Binary Search Trees

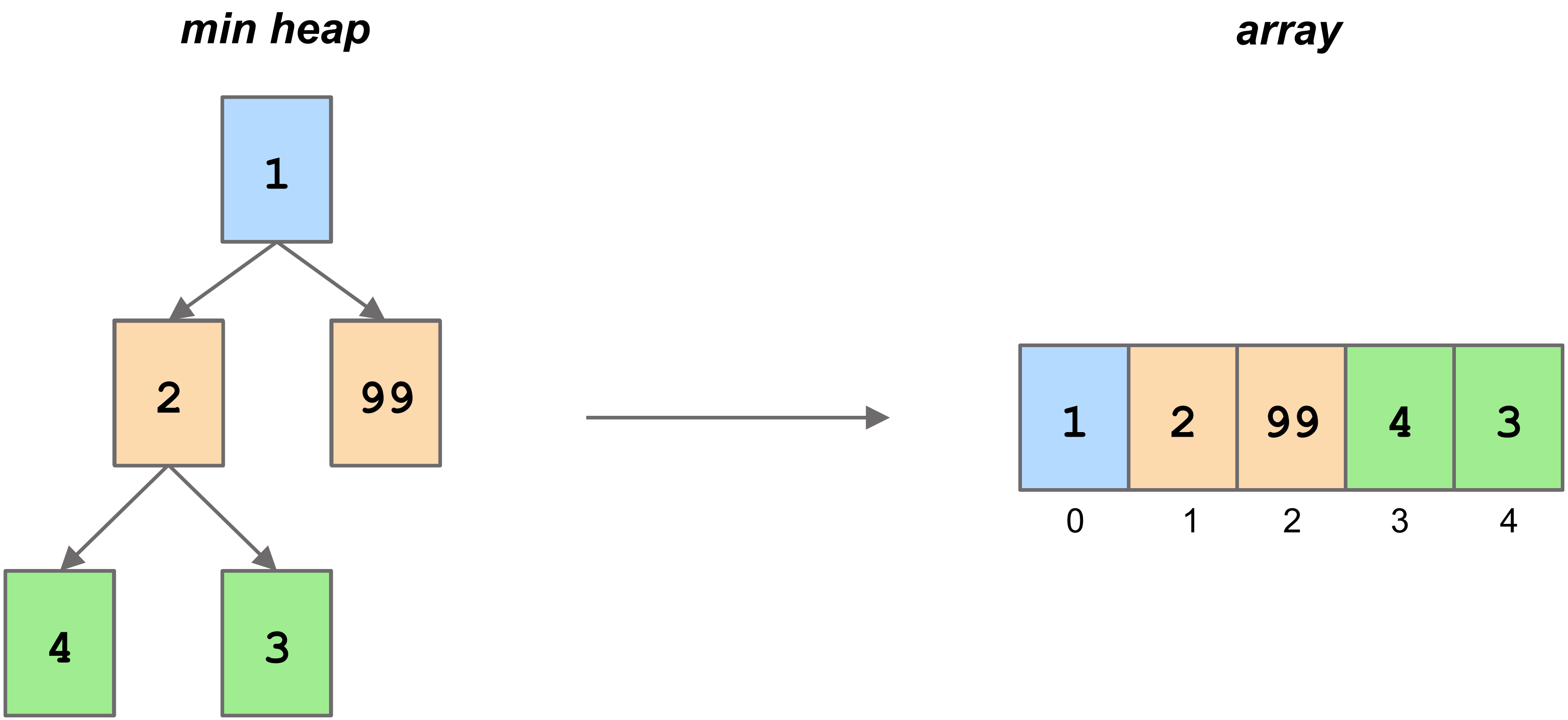
Left child is always smaller than parent
Right child is always larger than parent



How would you change the tree to add the value 37?

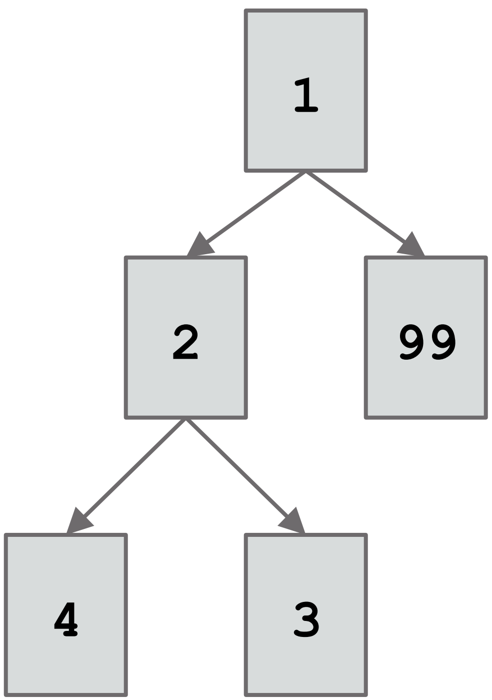
Heaps

Array Representation of Binary Heap



Review: Binary heap

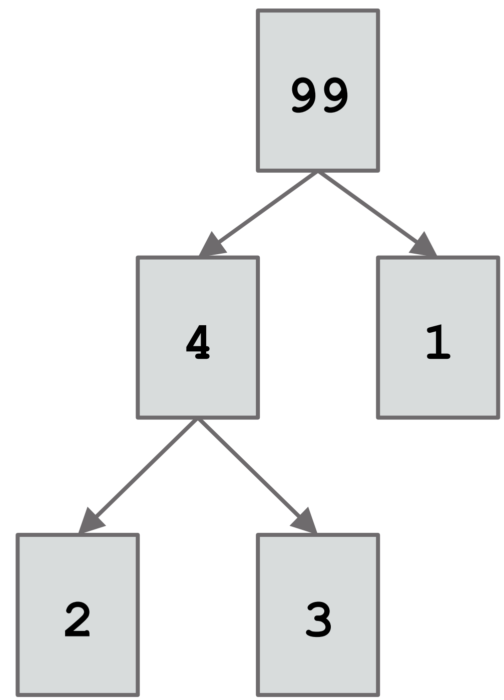
min heap



`PriorityQueue`

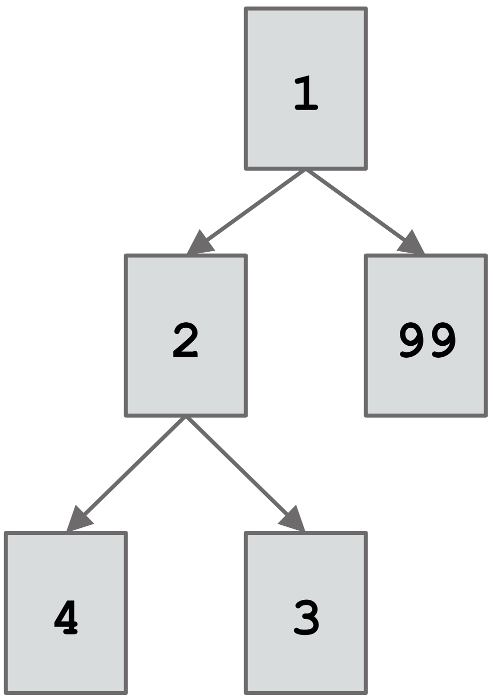
- Maintains max or min of collection (no duplicates)
- Follows *heap order invariant* at every level
- Always balanced!
- **worst case**:
  O(log n) insert
  O(log n) update
  O(1) peek
  O(log n) removal

max heap



Review: Binary heap

min heap



How do we insert element 0 into the min heap?

After we remove the root node, what is the resulting heap?

How are heaps usually represented? If we want the right child of index i, how do we access it?

Hashing

Review: Hashing

`HashSet<String>`

MA			NY	CA	⊗
0	1	2	3	4	5

Method	Expected Runtime	Worst Case
`add`	*O(1)*	*O(n)*
`contains`	*O(1)*	*O(n)*
`remove`	*O(1)*	*O(n)*

load factor, for open addressing:

$$\frac{\text{number of non-null entries}}{\text{size of array}}$$

load factor, for chaining:

$$\frac{\text{size of set}}{\text{size of array}}$$

If load factor becomes > 1/2, create an array twice the size and rehash every element of the set into it, use new array

Review: Hashing

HashSet<String>

MA			NY	CA	⊗
0	1	2	3	4	5

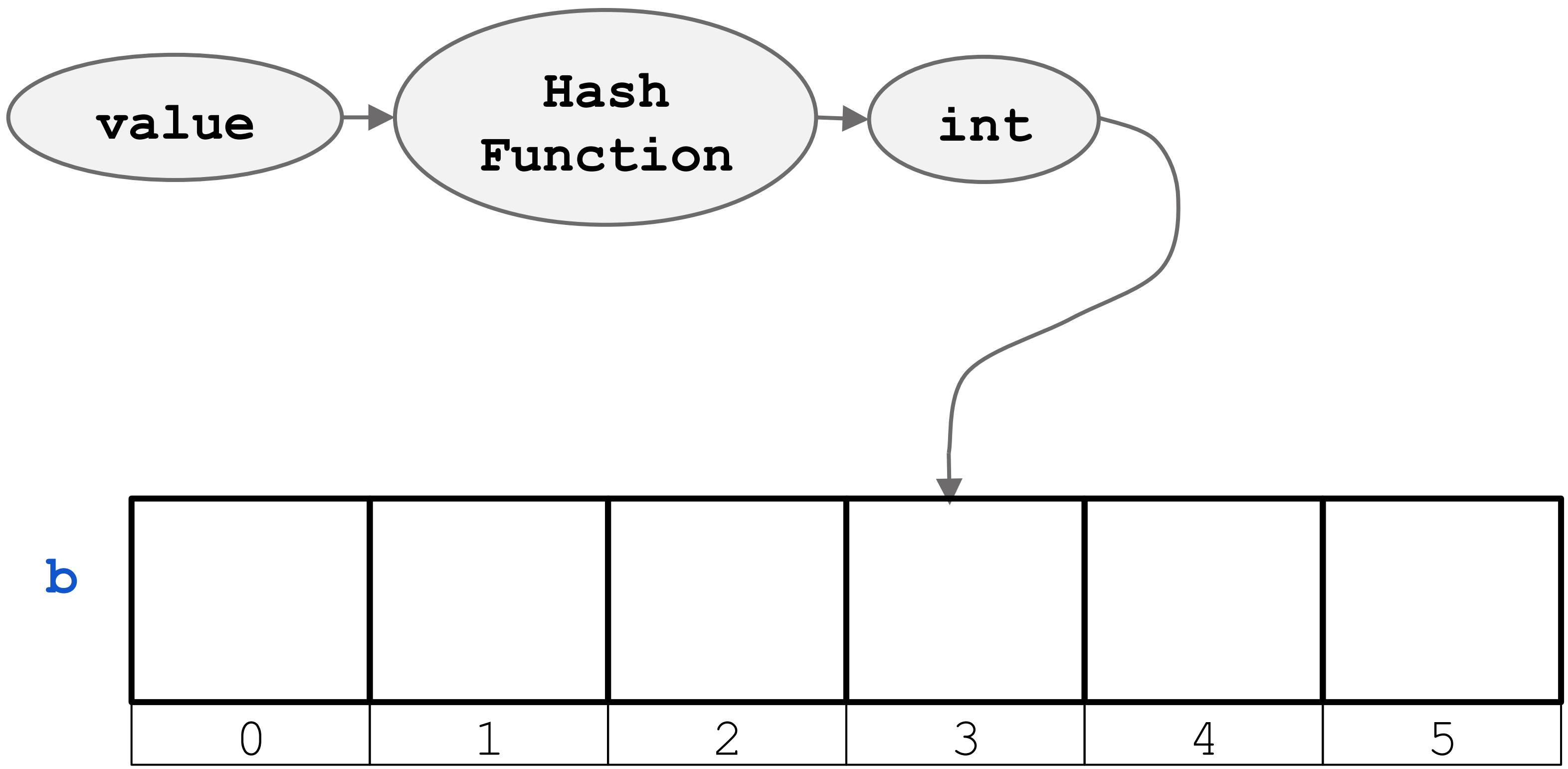
Method	Expected Runtime	Worst Case
`add`	*O(1)*	*O(n)*
`contains`	*O(1)*	*O(n)*
`remove`	*O(1)*	*O(n)*

HashMap<String,Integer>

to	2
be	2
or	1
not	1
that	1
is	1
the	1
question	1

Review: Hashing

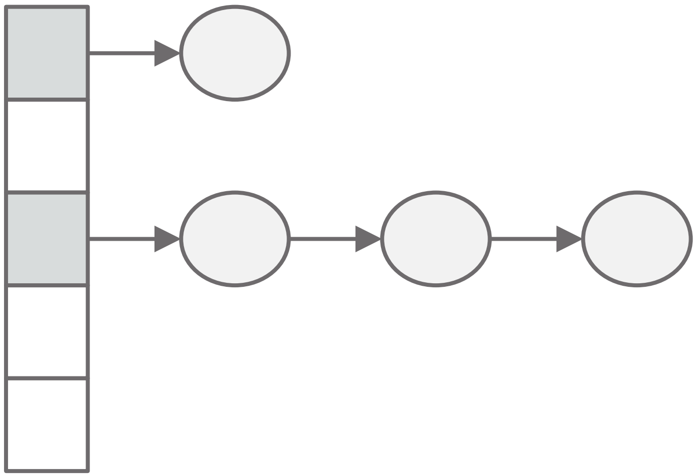
Idea: finding an element in an array takes constant time when you know which index it is stored in



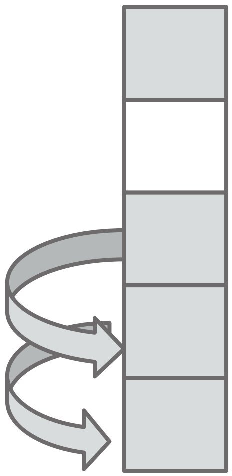
Collision resolution

Two ways of handling collisions:

1. Chaining



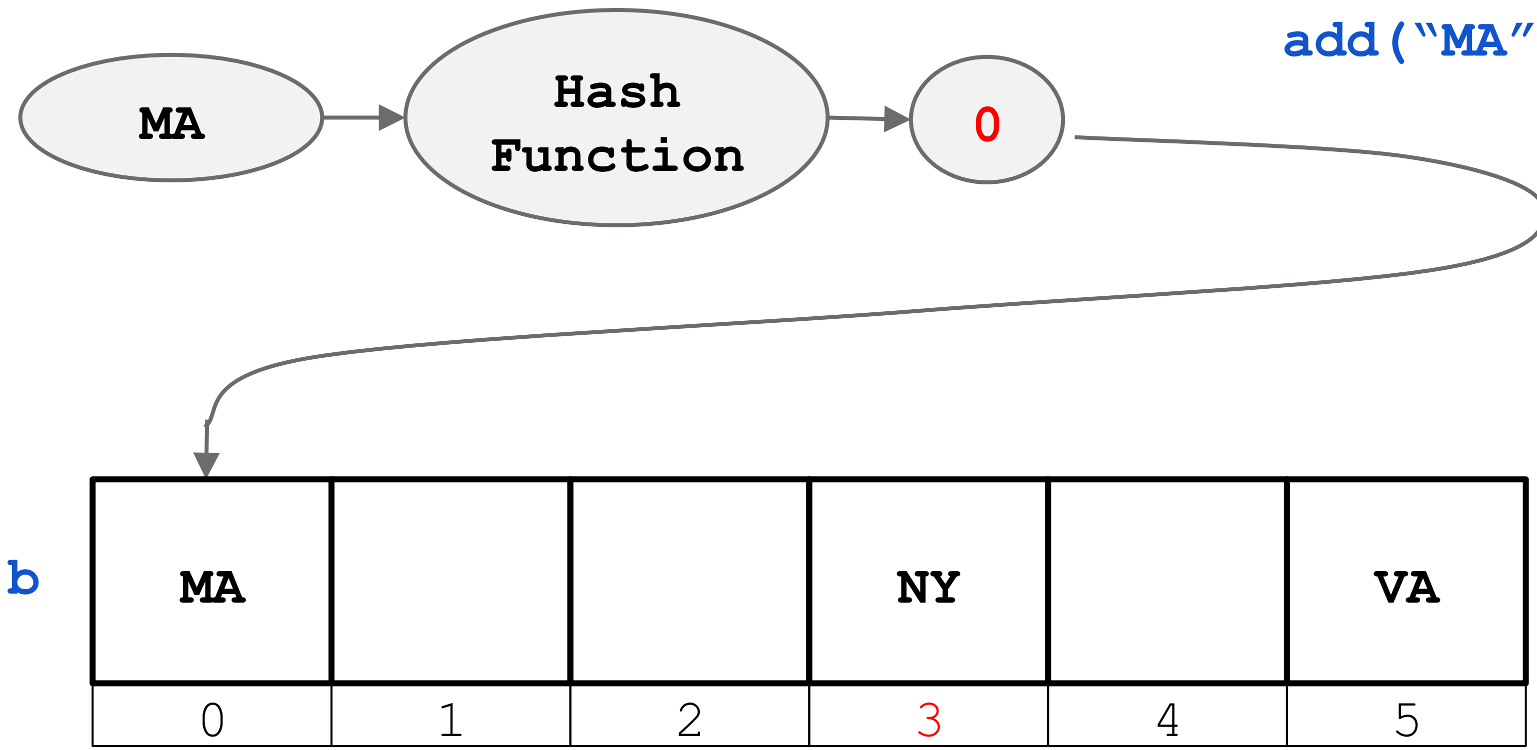
2. Open Addressing



Load factor: b's saturation

Load factor: $\lambda = \dfrac{\#\ of\ entries}{length\ of\ array} = \dfrac{3}{6}$

add("MA")



Question: Hashing

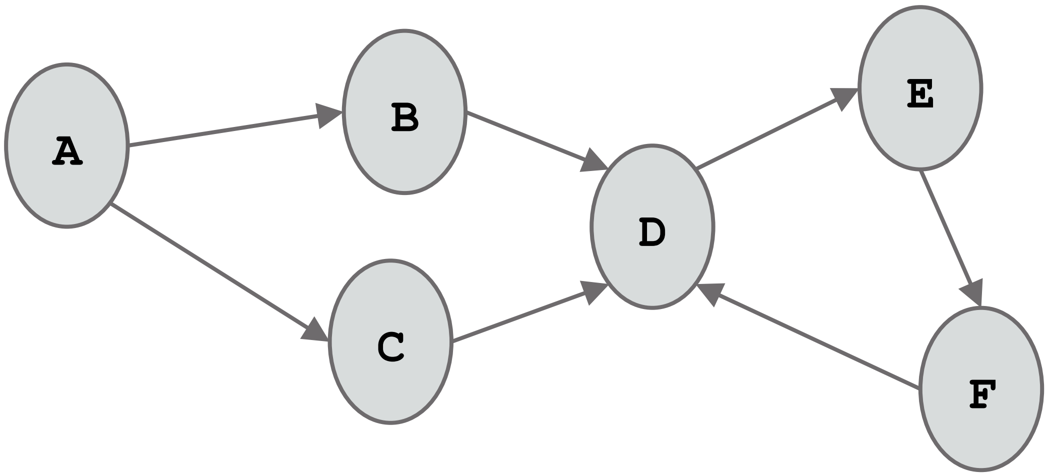
Using linear probing to resolve collisions,

1. Add element SC (hashes to 9).
2. Remove VA (hashes to 3).
3. Check to see if MA (hashes to 21) is in the set.
4. What should we do if we override equals()?

b	**MA**			**NY**		**VA**
	0	1	2	3	4	5

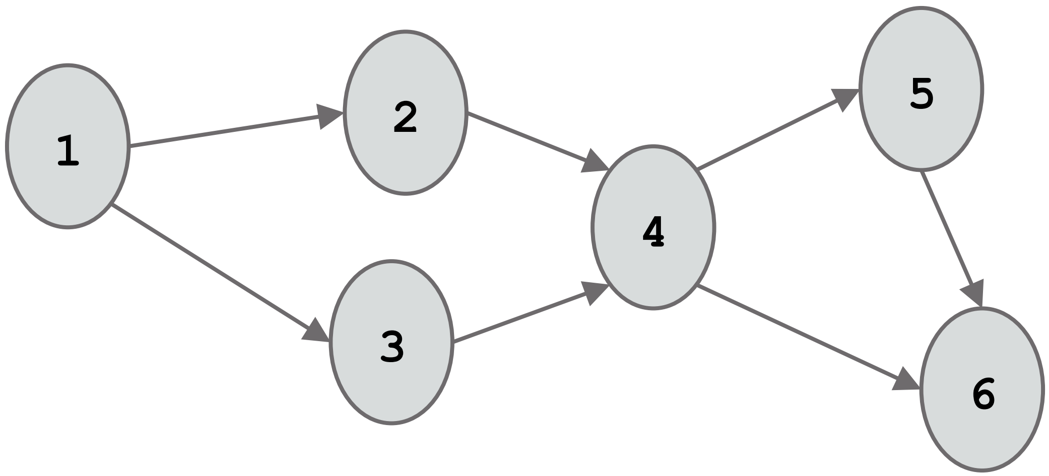
Graphs

Question: What is BFS and DFS?



1. Starting from node A, run BFS and DFS to find node Z. What is the order that the nodes were processed in? Visit neighbors in alphabetical order.
2. What is the difference between DFS and BFS?
3. What algorithm would be better to use if our graph were near infinite and a node was nearby?
4. Is Dijkstra's more like DFS or BFS? Why?
5. Can you run topological sort on this graph?

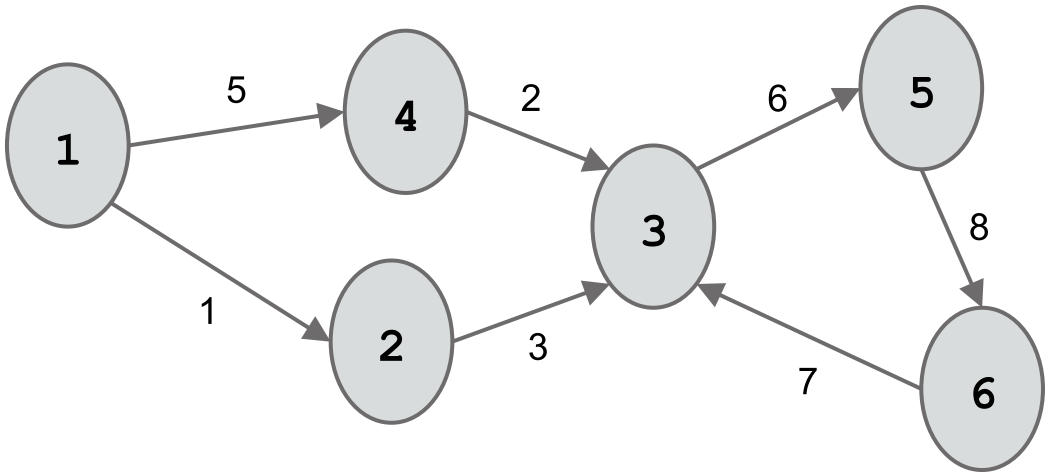
Topological ordering



All edges go from a smaller-numbered node to a larger-numbered node.

How can this be useful?

Dijkstra's Algorithm



The nodes are numbered in the order they are visited if we start at 1.

Why are they visited in this order?