Graphs - II

CS 2110, Spring 2016

Where did David leave that book?

http://www.geahvet.com

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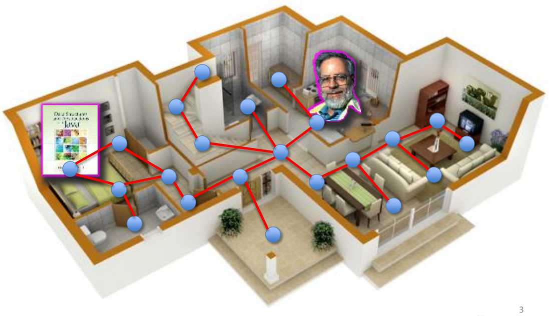
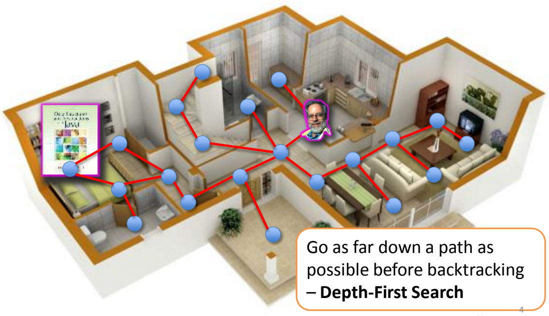


http://www.geahvet.com

Where did David leave that book?



Go as far down a path as possible before backtracking – **Depth-First Search**

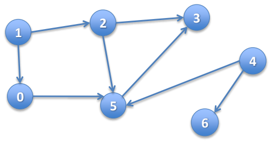
http://www.geahvet.com

Graph Algorithms

- Search
  - Depth-first search
  - Breadth-first search
- Shortest paths
  - Dijkstra's algorithm
- Minimum spanning trees
  - Prim's algorithm
  - Kruskal's algorithm

Reachability

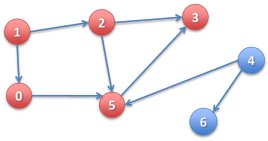
Node v is reachable from node u if there is a path from u to v.



Which nodes are reachable from node **1**?

Reachability

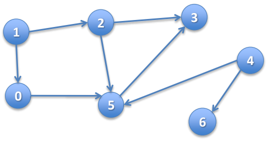
Node v is reachable from node u if there is a path from u to v.



Which nodes are reachable from node **1**?
0, 1, 2, 3, 5

Reachability

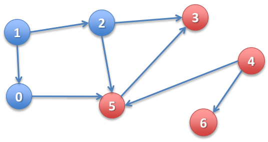
Node v is reachable from node u if there is a path from u to v.



Which nodes are reachable from node **4**?

Reachability

Node v is reachable from node u if there is a path from u to v.



Which nodes are reachable from node **4**?
3, 4, 5, 6

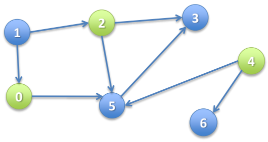
Reachability

We need an invariant!

How to determine reachability efficiently?

Reachability

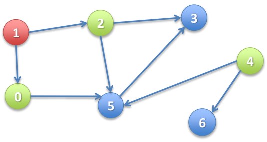
Node v is reachable from node u without green nodes if there is a path from u to v without green nodes.



Which nodes are reachable from node **1** without green nodes?

Reachability

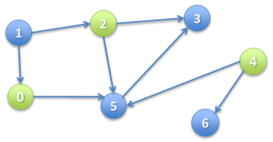
Node v is reachable from node u without green nodes if there is a path from u to v without green nodes.



Which nodes are reachable from node **1** without green nodes?
1

Reachability

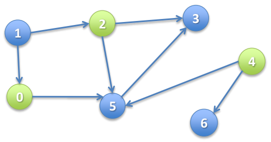
Node v is reachable from node u without green nodes if there is a path from u to v without green nodes.



Which nodes are reachable from node **4** without green nodes?

Reachability

Node v is reachable from node u without green nodes if there is a path from u to v without green nodes.

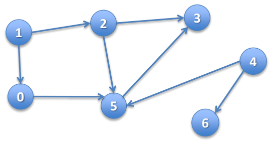


Which nodes are reachable from node **4** without green nodes?
None!

Node **4** is green, so all paths from node **4** contain a green node!

Depth-First Search

- Keep pushing the search forward
- Mark nodes as “visited” (green) as you go
- Backtrack only when you can’t go any further



Which nodes are reachable from node **1**?

Depth-First Search

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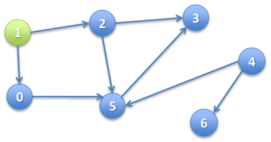

Which nodes are reachable from node **1**?

- Start at node **1**

Depth-First Search

- Keep pushing the search forward
- Mark nodes as “visited” (green) as you go
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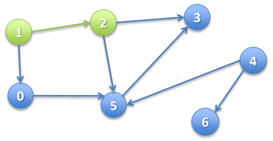

Which nodes are reachable from node **1**?

- Extend path to some child

Depth-First Search

- Keep pushing the search forward
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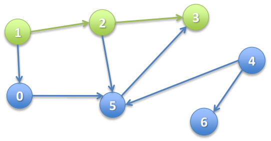

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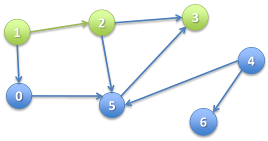

Which nodes are reachable from node **1**?

- No new way to extend path, so backtrack

Depth-First Search

- Keep pushing the search forward
- Mark nodes as “visited” (green) as you go
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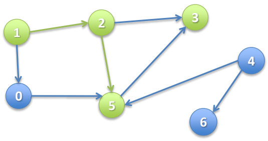

Which nodes are reachable from node **1**?

- Extend path to a different child

Depth-First Search

- Keep pushing the search forward
- Mark nodes as “visited” (green) as you go
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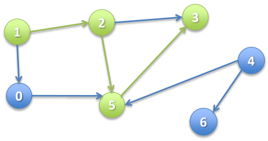

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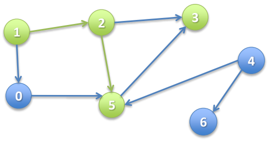

Which nodes are reachable from node **1**?

- Already visited, so backtrack

Depth-First Search

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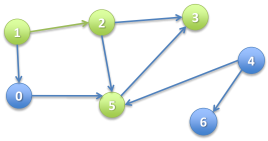

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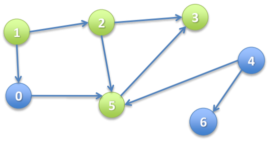

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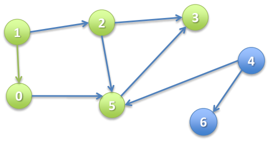

Which nodes are reachable from node **1**?

- Extend path to a different child

Depth-First Search

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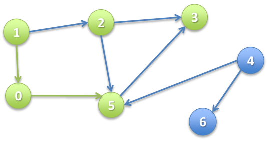

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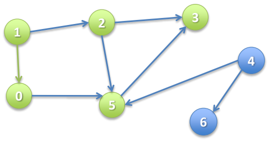

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- Already visited, so backtrack

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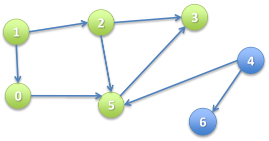

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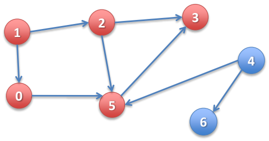


Which nodes are reachable from node **1**?

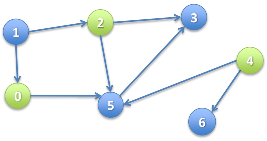
- Nothing to backtrack, so all done!

Depth-First Search using Recursion

```
/** Visit all nodes reachable from u without visited nodes */
void dfs(Node u) {
    if (u.hasBeenVisited()) return;



}
```

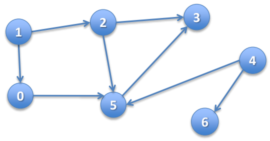


Which nodes are reachable from node **4** without green nodes? None!

Depth-First Search using Recursion

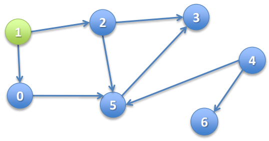
```
/** Visit all nodes reachable from u without visited nodes */
void dfs(Node u) {
    if (u.hasBeenVisited()) return;


}
```



Depth-First Search using Recursion

```
/** Visit all nodes reachable from u without visited nodes */
void dfs(Node u) {
    if (u.hasBeenVisited()) return;
    u.visit();
    for (Node v with edge from u to v) dfs(v);
}
```



Depth-First Search using Recursion

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void dfs(Node u) {
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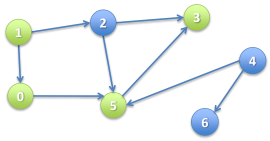

Depth-First Search using Recursion

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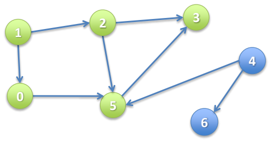
/** Visit all nodes reachable from u without visited nodes */
void dfs(Node u) {
    if (u.hasBeenVisited()) return;
    u.visit();
    for (Node v with edge from u to v) dfs(v);
}
```



OO-style Recursive Depth-First Search

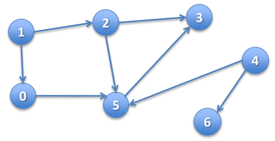
```
class Node {
    final List<Node> targets; // edges go from this to targets
    boolean visited= false; // has this node been visited?
    Node(Node... targets) { this.targets= Arrays.asList(targets); }
    /*Visit all nodes reachable from this without visited nodes*/
    void dfs() {
        if (visited) return;
        visited= true;
        for (Node v : targets) v.dfs();
    }
}
```

Depth-First Search using Iteration

```
/** Visit all nodes reachable from u without visited nodes */
void dfs(Node u) {
    Collection<Node> work= new Stack<Node>();
    work.add(u);
    // inv: all nodes that have to be visited are
    //        reachable (without visited nodes) from some node in work
    while (!work.isEmpty()) {
        Node u= work.pop();    // Remove first node and put it in u
        if (!u.hasBeenVisited()) {
            u.visit();
            for (Node v with edge from u to v)
                work.add(v); // Stack adds nodes to front
        }
    }
}
```

Breadth-First Search

- Mark closest nodes as “visited” (green) first
- Then push search out further



Which nodes are reachable from node **1**?

Breadth-First Search

- Mark closest nodes as “visited” (green) first
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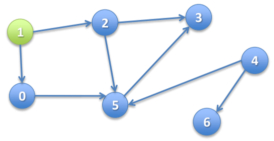
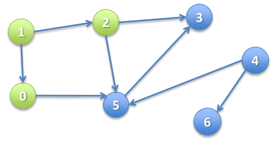


Which nodes are reachable from node **1**?

- Visit nodes distance 0 from node **1**

Breadth-First Search

- Mark closest nodes as “visited” (green) first
- Then push search out further

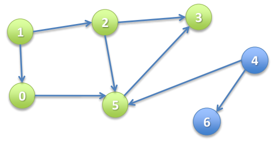


Which nodes are reachable from node **1**?

- Visit nodes distance 1 from node **1**

Breadth-First Search

- Mark closest nodes as “visited” (green) first
- Then push search out further

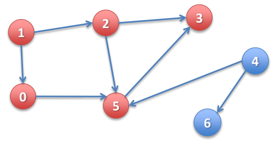


Which nodes are reachable from node **1**?

- Visit nodes distance 2 from node **1**

Breadth-First Search

- Mark closest nodes as “visited” (green) first
- Then push search out further

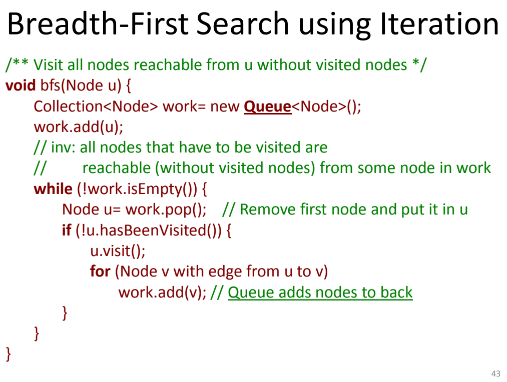


Which nodes are reachable from node **1**?

- No nodes at distance 3, so all done!

Depth-First Search using Iteration

```
/** Visit all nodes reachable from u without visited nodes */
void dfs(Node u) {
    Collection<Node> work= new Stack<Node>();
    work.add(u);
    // inv: all nodes that have to be visited are
    //       reachable (without visited nodes) from some node in work
    while (!work.isEmpty()) {
        Node u= work.pop();    // Remove first node and put it in u
        if (!u.hasBeenVisited()) {
            u.visit();
            for (Node v with edge from u to v)
                work.add(v); // Stack adds nodes to front
        }
    }
}
```



Breadth-First Search using Iteration

```
/** Visit all nodes reachable from u without visited nodes */
void bfs(Node u) {
    Collection<Node> work= new Queue<Node>();
    work.add(u);
    // inv: all nodes that have to be visited are
    //        reachable (without visited nodes) from some node in work
    while (!work.isEmpty()) {
        Node u= work.pop();    // Remove first node and put it in u
        if (!u.hasBeenVisited()) {
            u.visit();
            for (Node v with edge from u to v)
                work.add(v); // Queue adds nodes to back
        }
    }
}
```