Graphs - II
CS 2110, Spring 2015

Where did I leave that book?

http://www.geahvet.com

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http://www.geahvet.com

Where did I leave that book?

Go as far down a path as possible before backtracking –
Depth-First Search

http://www.geahvet.com

Graph Algorithms

- Search
  - Depth-first search
  - Breadth-first search
- Shortest paths
  - Dijkstra's algorithm
- Minimum spanning trees
  - Prim's algorithm
  - Kruskal's algorithm

Representations of Graphs

Adjacency List   Adjacency Matrix

	1	2	3	4
1	0	1	0	1
2	0	0	1	0
3	0	0	0	0
4	0	1	1	0

Adjacency Matrix or Adjacency List?

- Definitions:
  - n = number of vertices
  - m = number of edges
  - $d(u)$ = degree of u = number of edges leaving u
- Adjacency Matrix
  - Uses space $O(n^2)$
  - Can iterate over all edges in time $O(n^2)$
  - Can answer "Is there an edge from u to v?" in $O(1)$ time
  - Better for dense graphs (lots of edges)
- Adjacency List
  - Uses space $O(m + n)$
  - Can iterate over all edges in time $O(m + n)$
  - Can answer "Is there an edge from u to v?" in $O(d(u))$ time
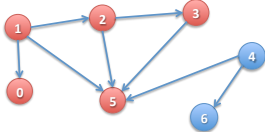 - Better for sparse graphs (fewer edges)

Depth-First Search

- Given a graph and one of its nodes u
  (say node 1 below)
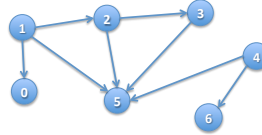
Depth-First Search

- Given a graph and one of its nodes u
  (say node 1 below)
- We want to "visit" each node reachable from u
  (nodes 1, 0, 2, 3, 5)
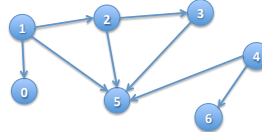
There are many paths to some nodes.

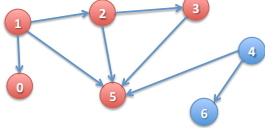How do we visit all nodes efficiently, without doing extra work?

Depth-First Search

boolean[ ] visited;

- Node u is visited means: visited[u] is true
- To visit u means to: set visited[u] to true
- Node v is REACHABLE from node u if there is a path (u, ..., v) in which all nodes of the path are unvisited.

Suppose all nodes are unvisited.

Depth-First Search

boolean[ ] visited;

- Node u is visited means: visited[u] is true
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- Node v is REACHABLE from node u if there is a path (u, ..., v) in which all nodes of the path are unvisited.

Suppose all nodes are unvisited.

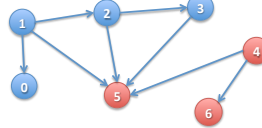Nodes REACHABLE from node 1:
{1, 0, 2, 3, 5}

Depth-First Search

boolean[ ] visited;

- Node u is visited means: visited[u] is true
- To visit u means to: set visited[u] to true
- Node v is REACHABLE from node u if there is a path (u, ..., v) in which all nodes of the path are unvisited.

Suppose all nodes are unvisited.

Nodes REACHABLE from node 1:
{1, 0, 2, 3, 5}

Nodes REACHABLE from 4: {4, 5, 6}

Depth-First Search

boolean[ ] visited;

- Node u is visited means: visited[u] is true
- To visit u means to: set visited[u] to true
- Node v is REACHABLE from node u if there is a path (u, …, v) in which all nodes of the path are unvisited.

Green: visited
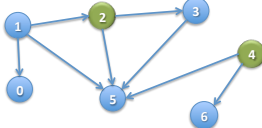**Blue**: unvisited

---

Depth-First Search

boolean[ ] visited;

- Node u is visited means: visited[u] is true
- To visit u means to: set visited[u] to true
- Node v is REACHABLE from node u if there is a path (u, …, v) in which all nodes of the path are unvisited.

Green: visited
Blue: unvisited

Nodes REACHABLE from node **1**:
{1, 0, 5}

Depth-First Search

boolean[ ] visited;

- Node u is visited means: visited[u] is true
- To visit u means to: set visited[u] to true
- Node v is REACHABLE from node u if there is a path (u, …, v) in which all nodes of the path are unvisited.

Green: visited
Blue: unvisited

Nodes REACHABLE from node **1**:
{1, 0, 5}

Nodes REACHABLE from **4**: none

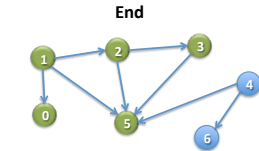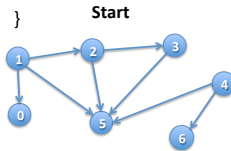Not even **4** itself, because it's already been visited!

Depth-First Search

/** Node u is unvisited. Visit all nodes that are REACHABLE from u. */
public static void dfs(**int** u) {

}

Let u be **1**

The nodes REACHABLE from 1 are 1, 0, 2, 3, 5

Start   **End**
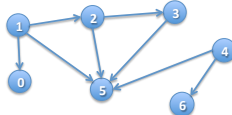

Depth-First Search

/** Node u is unvisited. Visit all nodes that are REACHABLE from u. */
public static void dfs(**int** u) {

}

Let u be **1**

The nodes REACHABLE from 1 are 1, 0, 2, 3, 5

Depth-First Search

/** Node u is unvisited. Visit all nodes that are REACHABLE from u. */
public static void dfs(**int** u) {
    visited[u] = true;

}

Let u be **1**

The nodes REACHABLE from 1 are 1, 0, 2, 3, 5

Depth-First Search

/** Node u is unvisited. Visit all nodes that are REACHABLE from u. */
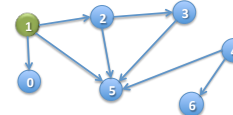
```
public static void dfs(int u) {
    visited[u] = true;


}
```
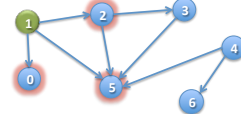
Let u be **1** (visited)

The nodes to be visited are 0, 2, 3, 5

Depth-First Search

/** Node u is unvisited. Visit all nodes that are REACHABLE from u. */

```
public static void dfs(int u) {
    visited[u] = true;
    for all edges (u, v) leaving u:
        if v is unvisited then dfs(v);
}
```

Let u be **1** (visited)

The nodes to be visited are 0, 2, 3, 5

Have to do DFS on all unvisited neighbors of u!

Depth-First Search

/** Node u is unvisited. Visit all nodes that are REACHABLE from u. */

```
public static void dfs(int u) {
    visited[u] = true;
    for all edges (u, v) leaving u:
        if v is unvisited then dfs(v);
}
```

Suppose the **for** loop visits neighbors in numerical order. Then **dfs(1)** visits the nodes in this order: 1 …

Depth-First Search

/** Node u is unvisited. Visit all nodes that are REACHABLE from u. */

```
public static void dfs(int u) {
    visited[u] = true;
    for all edges (u, v) leaving u:
        if v is unvisited then dfs(v);
}
```
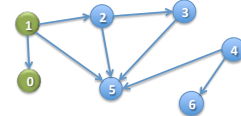
Suppose the **for** loop visits neighbors in numerical order. Then **dfs(1)** visits the nodes in this order: 1, 0 …

Depth-First Search

/** Node u is unvisited. Visit all nodes that are REACHABLE from u. */

```
public static void dfs(int u) {
    visited[u] = true;
    for all edges (u, v) leaving u:
        if v is unvisited then dfs(v);
}
```
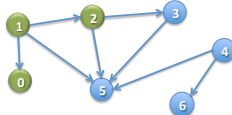
Suppose the **for** loop visits neighbors in numerical order. Then **dfs(1)** visits the nodes in this order: 1, 0, 2 …

Depth-First Search

/** Node u is unvisited. Visit all nodes that are REACHABLE from u. */

```
public static void dfs(int u) {
    visited[u] = true;
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Suppose the **for** loop visits neighbors in numerical order. Then **dfs(1)** visits the nodes in this order: 1, 0, 2, 3 …

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        if v is unvisited then dfs(v);
}

> Suppose the **for** loop visits neighbors in numerical order. Then **dfs(1)** visits the nodes in this order: 1, 0, 2, 3, 5

Depth-First Search

/** Node u is unvisited. Visit all nodes that are REACHABLE from u. */

public static void dfs(**int** u) {
    visited[u] = true;
    for all edges (u, v) leaving u:
        if v is unvisited then dfs(v);
}

> Suppose n nodes are REACHABLE along e edges (in total). What is
> • Worst-case execution?
> • Worst-case space?

Depth-First Search

/** Node u is unvisited. Visit all nodes that are REACHABLE from u. */

public static void dfs(**int** u) {
    visited[u] = true;
    for all edges (u, v) leaving u:
        if v is unvisited then dfs(v);
}

Example: Use different way (other than array visited) to know whether a node has been visited

Example: We really haven't said what data structures are used to implement the graph

> That's all there is to basic DFS. You may have to change it to fit a particular situation.

> If you don't have this spec and you do something different, it's probably wrong.

Depth-First Search in OO fashion

public class Node {
    boolean visited;
    List<Node> neighbors;

    /** This node is unvisited. Visit all nodes
       REACHABLE from this node */
    public void dfs() {
        visited= true;
        for (Node n: neighbors) {
            if (!n.visited) n.dfs();
        }
    }
}

> Each node of the graph is an object of type Node

> No need for a parameter. The object is the node.
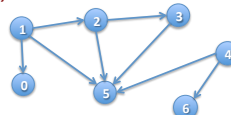

Depth-First Search written iteratively

/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);   // Not Java!
    // inv: all nodes that have to be visited are
    //   REACHABLE from some node in s
    while (s is not empty) {
        u= s.pop();   // Remove top stack node, put in u
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}

Depth-First Search written iteratively

/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
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    while (s is not empty) {
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                s.push(v);
        }
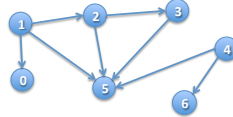   }
}

> Call dfs(1)

1

Stack s

Slide 1

Depth-First Search written iteratively

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

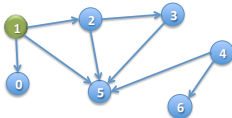Call dfs(1)   Iteration 0

```
1
```
Stack s

Slide 2

Depth-First Search written iteratively

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```
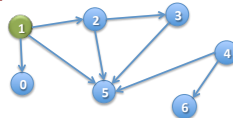
Call dfs(1)   Iteration 0

Stack s

Slide 3

Depth-First Search written iteratively

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

Call dfs(1)   Iteration 0

Stack s

Slide 4

Depth-First Search written iteratively

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

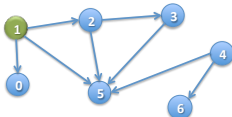Call dfs(1)   Iteration 0

```
0
2
5
```
Stack s

Slide 5

Depth-First Search written iteratively

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

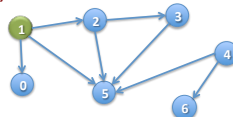Call dfs(1)   Iteration 1

```
0
2
5
```
Stack s

Slide 6

Depth-First Search written iteratively

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

Call dfs(1)   Iteration 1

```
2
5
```
Stack s

Slide 1 — Depth-First Search written iteratively

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);                    Call dfs(1)    Iteration 1
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```
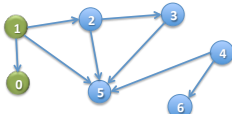
2
5

Stack s

Slide 2 — Depth-First Search written iteratively

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);                    Call dfs(1)    Iteration 2
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```
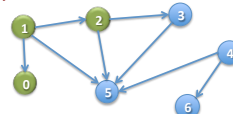
2
5

Stack s

Slide 3 — Depth-First Search written iteratively

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);                    Call dfs(1)    Iteration 2
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

5

Stack s

Slide 4 — Depth-First Search written iteratively

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);                    Call dfs(1)    Iteration 2
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

5

Stack s

Slide 5 — Depth-First Search written iteratively

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);                    Call dfs(1)    Iteration 2
    while (s is not empty) {
        u= s.pop();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                s.push(v);
        }
    }
}
```

Yes, 5 is put on the stack twice, once for each edge to it. It will be visited only once.

3
5
5

Stack s

Slide 6 — Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
    Queue q= (u);   // Not Java!
    // inv: all nodes that have to be visited are
    //      REACHABLE from some node in s
    while ( q is not empty ) {
        u= q.popFirst();   // Remove first node in queue, put in u
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);   // Add to end of queue
        }
    }
}
```
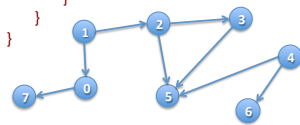
7

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
    Queue q= (u);                      Call bfs(1)
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```
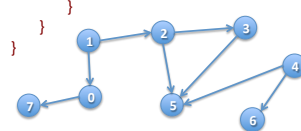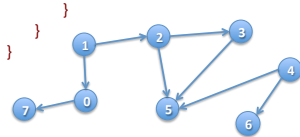
1
Queue q

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
    Queue q= (u);                      Call bfs(1)    Iteration 0
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```
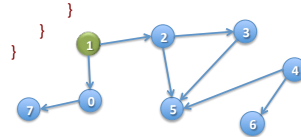
1
Queue q

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
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}
```

Queue q

Breadth-First Search

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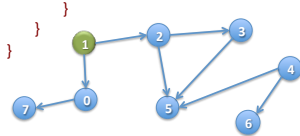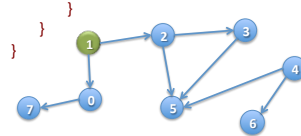
Queue q

Breadth-First Search

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/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
    Queue q= (u);                      Call bfs(1)    Iteration 0
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```

0 2
Queue q

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
    Queue q= (u);                      Call bfs(1)    Iteration 1
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
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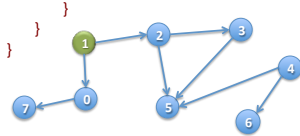
0 2
Queue q

Breadth-First Search

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/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```

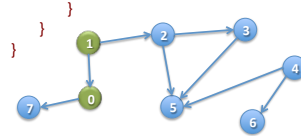Call bfs(1) Iteration 1

2
Queue q

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
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        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```

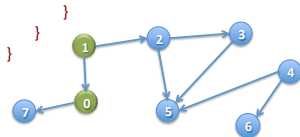Call bfs(1) Iteration 1

2
Queue q

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
    Queue q= (u);
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            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```

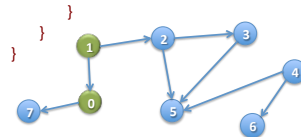Call bfs(1) Iteration 1

2 7
Queue q

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```

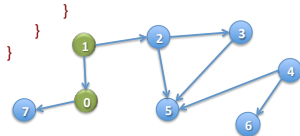Call bfs(1) Iteration 2

2 7
Queue q

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```
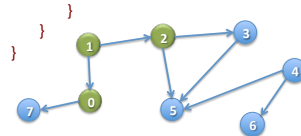
Call bfs(1) Iteration 2

7
Queue q

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```

Call bfs(1) Iteration 2

7
Queue q

Breadth-First Search

```
/** Node u is unvisited. Visit all nodes REACHABLE from u. */
public static void bfs(int u) {
    Queue q= (u);
    while q is not empty) {
        u= q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v) leaving u:
                q.append(v);
        }
    }
}
```

Call bfs(1)   Iteration 2

Breadth first:
(1) Node u
(2) All nodes 1 edge from u
(3) All nodes 2 edges from u
(4) All nodes 3 edges from u
...

7 3 5
Queue q