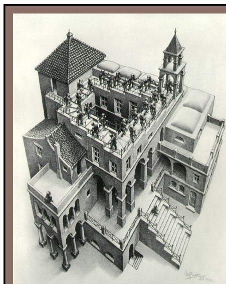

# PRIORITY QUEUES AND HEAPS

Lecture 16
CS2110 Spring 2015

## Readings and Homework

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**Read Chapter 26** "A Heap Implementation" to learn about heaps

**Exercise:** Salespeople often make matrices that show all the great features of their product that the competitor's product lacks. Try this for a heap versus a BST. First, try and sell someone on a BST: List some desirable properties of a BST that a heap lacks. Now be the heap salesperson: List some good things about heaps that a BST lacks. Can you think of situations where you would favor one over the other?

*With ZipUltra heaps, you've got it made in the shade my friend!*

## Cool data structures you now know about

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- Linked lists –singly linked, doubly linked, circular
- Binary search trees
- BST-like tree for A4 (BlockTree)
- Example of how one changes a data structure to make for efficiency purposes:

  In A4 a Shape (consisting of 1,000 Blocks?) gets moved around, rather than change the position field in each Block, have a field of Shape that gives the displacement for all Blocks.

## Interface Bag (not In Java Collections)

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```
interface Bag<E>
      implements Iterable {
   void add(E obj);
   boolean contains(E obj);
   boolean remove(E obj);
   int size();
   boolean isEmpty();
   Iterator<E> iterator()
}
```

Also called multiset

Like a set except that a value can be in it more than once. Example: a bag of coins

Refinements of Bag: Stack, Queue, PriorityQueue

## Stacks and queues are restricted lists

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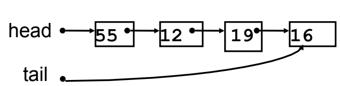
- Stack (LIFO) implemented as list
  - add(), remove() from front of list
- Queue (FIFO) implemented as list
  - add() on back of list, remove() from front of list
- These operations are O(1)

Both efficiently implementable using a singly linked list with head and tail




## Priority queue

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- **Bag** in which data items are **Comparable**
- Smaller elements (determined by **compareTo()**) have higher priority
- **remove()** return the element with the highest priority = least in the **compareTo()** ordering
- break ties arbitrarily

## Examples of Priority Queues

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Scheduling jobs to run on a computer
default priority = arrival time
priority can be changed by operator

Scheduling events to be processed by an event handler
priority = time of occurrence

Airline check-in
first class, business class, coach
FIFO within each class

Tasks that you have to carry out. You determine priority

## `java.util.PriorityQueue<E>`

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```
boolean add(E e) {...} //insert an element
void clear() {...} //remove all elements
E peek() {...} //return min element without removing
E poll() {...} //remove and return min element
boolean contains(E e)
boolean remove(E e)
int size() {...}
Iterator<E> iterator()   //an iterator over the priority queue
```

## Priority queues as lists

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- Maintain as unordered list
  - `add()` put new element at front – O(1)
  - **poll()** must search the list – O(n)
  - **peek()** must search the list – O(n)
- Maintain as ordered list
  - `add()` must search the list – O(n)
  - **poll()** must search the list – O(n)
  - **peek()** O(1)

Can we do better?

## Important Special Case

10

- Fixed number of priority levels 0,...,p – 1
- FIFO within each level
- Example: airline check-in

- `add()` – insert in appropriate queue – O(1)
- `poll()` – must find a nonempty queue – O(p)

first class   many miles   paying   frequent flier

## Heap

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- A *heap* is a concrete data structure that can be used to implement priority queues
- Gives better complexity than either ordered or unordered list implementation:
  - `add()`: O(log n)
  - `poll()`: O(log n)
- O(n log n) to process n elements
- Do not confuse with *heap memory*, where the Java virtual machine allocates space for objects – different usage of the word *heap*

## Heap

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- Binary tree with data at each node
- Satisfies the *Heap Order Invariant*:

  1. The least (highest priority) element of any subtree is at the root of that subtree.

- Binary tree is complete (no holes)

  2. Every level (except last) completely filled. Nodes on bottom level are as far left as possible.

## Heap

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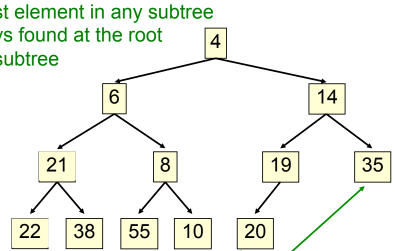
Smallest element in any subtree is always found at the root of that subtree



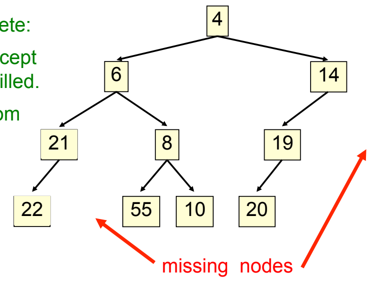

Note: 19, 20 < 35: Smaller elements can be deeper in the tree!

## Not a heap —has two holes

Should be complete:

* Every level (except last) completely filled.

* Nodes on bottom level are as far left as possible.




## Heap: number nodes as shown

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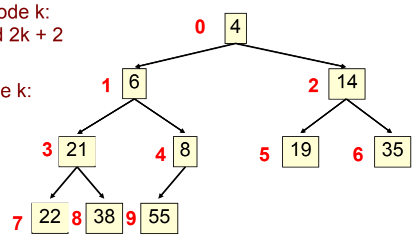
children of node k: at 2k + 1 and 2k + 2

parent of node k: at (k-1) / 2

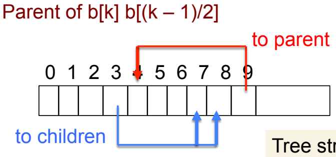



Remember, tree has no holes

## We illustrate using an array b (could also be ArrayList or Vector)

16

- Heap nodes in b in order, going across each level from left to right, top to bottom
- Children b[k] are b[2k + 1] and b[2k + 2]
- Parent of b[k] b[(k – 1)/2]




Tree structure is implicit.
No need for explicit links!

## add(e)

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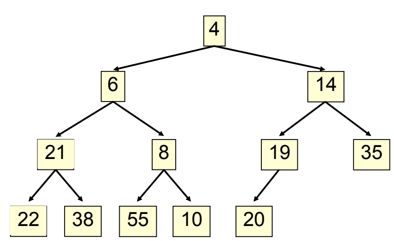
- Add e at the end of the array
- If this violates heap order because it is smaller than its parent, swap it with its parent
- Continue swapping it up until it finds its rightful place
- The heap invariant is maintained!

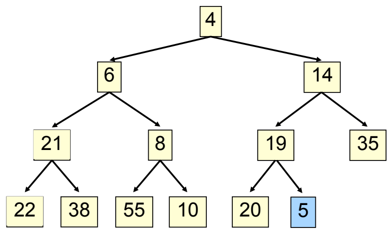
## add()

18



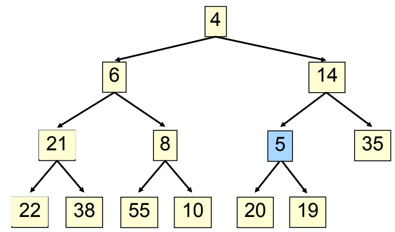
## add()

19



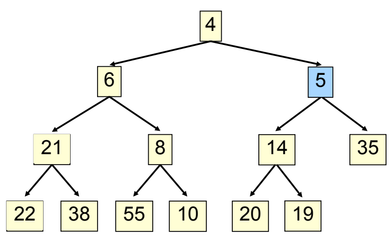

## add()

20



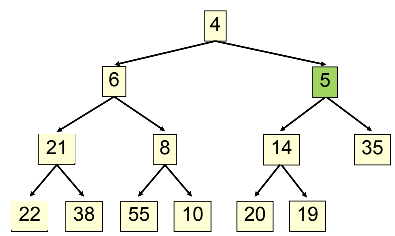

## add()

21



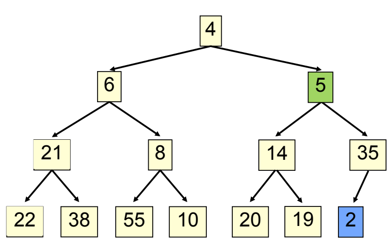

## add()

22



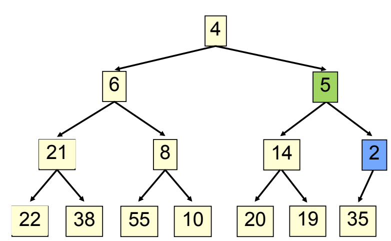

## add()

23



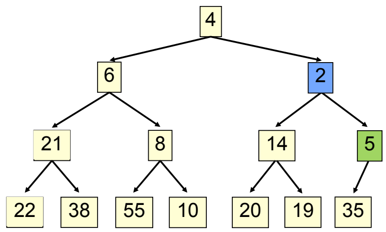

## add()

24



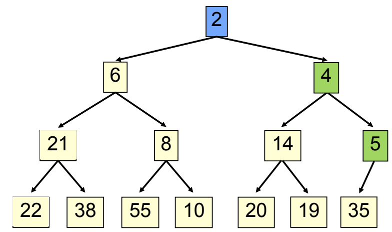
## add()

25



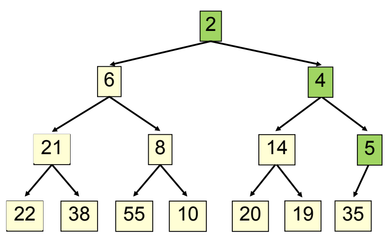

## add()

26




## add()

27




## add() to a tree of size n

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- Time is O(log n), since the tree is balanced
  - size of tree is exponential as a function of depth
  - depth of tree is logarithmic as a function of size

## add() --assuming there is space

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```
/** An instance of a heap */
Class Heap<E>{
  E[] b= new E[50];  //heap is b[0..n-1]
  int n= 0;          // heap invariant is true

  /** Add e to the heap */
  public void add(E e) {
    b[n]= e;
    n= b + 1;
    bubbleUp(n - 1); // given on next slide
  }
}
```

## add(). Remember, heap is in b[0..n-1]

30

```
class Heap<E> {
  /** Bubble element #k up to its position.
    *  Precondition: heap inv true except maybe for element k */
  private void bubbleUp(int k) {
     int p= (k-1)/2;   // p is the parent of k
     // inv: p is k's parent  and
     // Every element satisfies the heap property
     // except perhaps k (might be smaller than its parent)
    while (k>0  &&  b[k].compareTo(b[p]) < 0) {
         Swap b[k] and b[p];
         k= p;
         p= (k-1)/2;
    }
}
```

## poll()

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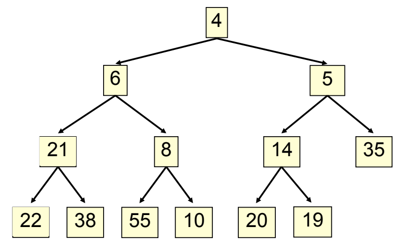
- Remove the least element and return it – (at the root)
- This leaves a hole at the root – fill it in with the last element of the array
- If this violates heap order because the root element is too big, swap it down with the smaller of its children
- Continue swapping it down until it finds its rightful place
- The heap invariant is maintained!

## poll()

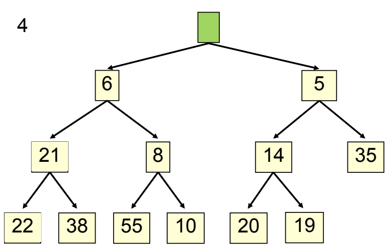
32




## poll()

33

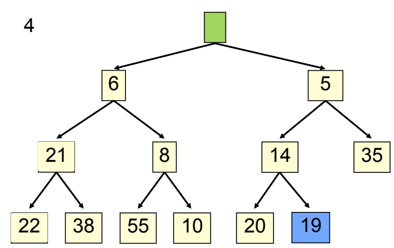
4




## poll()

34

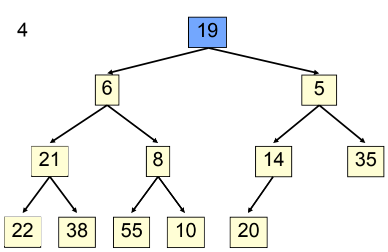
4




## poll()

35

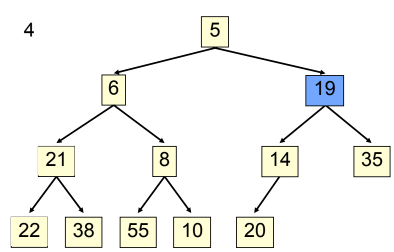
4




## poll()

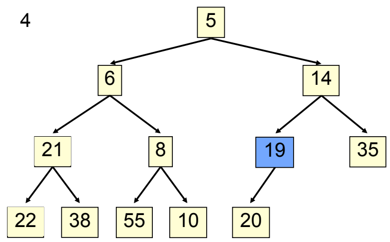
36

4



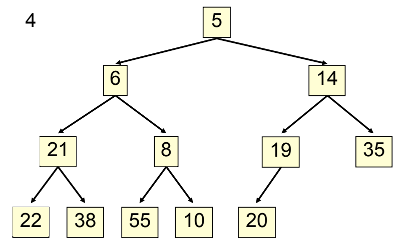
## poll()

37



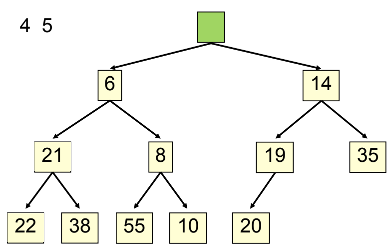

## poll()

38



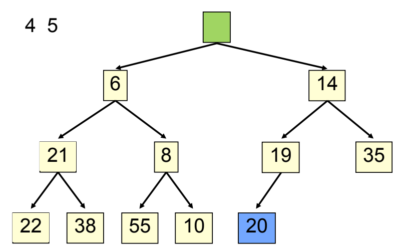

## poll()

39



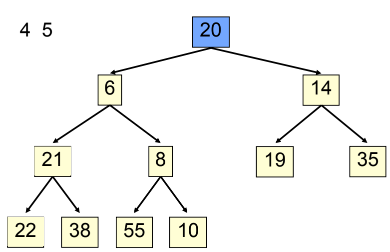

## poll()

40



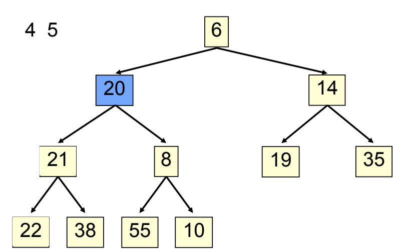

## poll()

41




## poll()

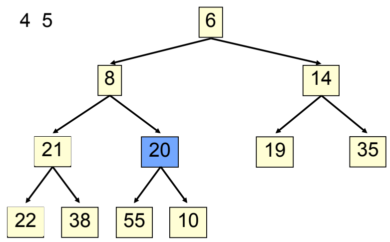
42



## poll()

43

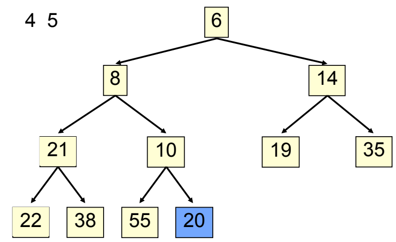
4 5




## poll()

44

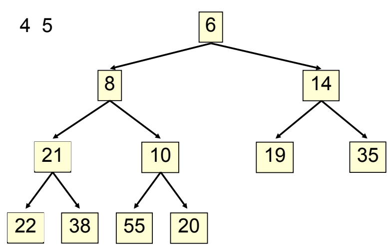
4 5




## poll()

45

4 5




## poll()

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Time is O(log n), since the tree is balanced

## poll() . Remember, heap is in b[0..n-1]

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```
/** Remove and return the smallest element
      (return null if list is empty) */
public E poll() {
    if (n == 0) return null;
    E val=  b[0];     // smallest value is at root
    b[0]= b[n-1];     // move last element to root
    n= n - 1;
    bubbleDown(0);
    return val;
}
```

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```
/** Bubble root down to its heap position.
    Pre: b[0..n-1] is a heap except: b[0] may be > than a child */
private void bubbleDown() {
    int k= 0;
   // Set c to smaller of k’s children
   int c= 2*k + 2;     // k’s right child
   if (c >= n || b[c-1].compareTo(b[c]) < 0)  c= c-1;

   // inv: b[0..n-1] is a heap except: b[k] may be > than a child.
   //        Also, b[c] is b[k]’s smallest child
   while (c < n   &&  b[k].compareTo(b[c]) > 0) {
      Swap b[k] and b[c];
      k= c;
      c= 2*k + 2;     // k’s right child
      if (c >= n || b[c-1].compareTo(b[c]) < 0)  c= c-1;
   }
}
```

## HeapSort(b, n) —Sort b[0..n-1]



Whet your appetite –use heap to get exactly n log n in-place sorting algorithm. 2 steps, each is O(n log n)

1. Make b[0..n-1] into a max-heap (in place)
2. for (k= n-1; k > 0; k= k-1) {
   b[k]= poll –i.e. take max element out of heap.
   }

We'll post this algorithm on course website

A max-heap has max value at root

## Many uses of priority queues & heaps



Surface simplification [Garland and Heckbert 1997]

- Mesh compression: quadric error mesh simplification
- Event-driven simulation: customers in a line
- Collision detection: "next time of contact" for colliding bodies
- Data compression: Huffman coding
- Graph searching: Dijkstra's algorithm, Prim's algorithm
- AI Path Planning: A* search
- Statistics: maintain largest M values in a sequence
- Operating systems: load balancing, interrupt handling
- Discrete optimization: bin packing, scheduling
- Spam filtering: Bayesian spam filter