ADTS, GRAMMARS, PARSING, TREE TRAVERSALS

Lecture 12
CS2110 – Spring 2015

ADT: Abstract Data Type

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Just like a type: Bunch of values together with operations on them.

Used often in discussing data structures

Important: The definition says ntthing about the implementation, just about the behaviour of the operations –what they do. (Could describe necessary execution times of operations)

Example: A stack is a list of values with these operations:
1. isEmpty: return true iff the stack is empty
2. Push: Put a value onto the front (top) of the stack
3. Pop: Remove and return the front (top) value of the stack

Is a stack implemented in an array? singly linked list? doubly linked list? ArrayList? We don't know.

ADT: Abstract Data Type

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Just like a type: Bunch of values together with operations on them.

```
/** an implementation of stack implements a LIFO
    list of values of type E. */
public interface<E> stack {
    /** = "the list is empty." */
    boolean isEmpty();
    /** Push v onto the top of the list. */
    void push(E v);
    /** Remove top value on the list and return it.
        Precondition: the list is not empty. */
    E pop();
```

Java interface is a great way to define an ADT!

Pointers to material

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- Parse trees: text, section 23.36
- Definition of Java Language, sometimes useful: docs.oracle.com/javase/specs/jls/se7/html/index.html
- Grammar for most of Java, for those who are curious: docs.oracle.com/javase/specs/jls/se7/html/jls-18.html
- Tree traversals –preorder, inorder, postorder: text, sections 23.13 .. 23.15.

Expression trees

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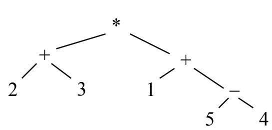
Can draw a tree for (2 + 3) * (1 + (5 – 4)




```
public abstract class Exp {
    /* return the value of this Exp */
    public abstract int eval();
}
```

Expression trees

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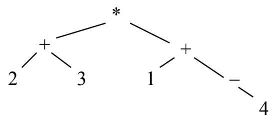
```
public abstract class Exp {
    /* return the value of this Exp */
    public abstract int eval();
}
```

```
public class Int extends Exp {
  int v;
  public int eval() {
    return v;
  }
}
```

```
public class Add extends Exp {
    Exp left;
    Exp right;
    public int eval() {
        return left.eval() + right.eval();
    }
}
```

tree for (2 + 3) * (1 + – 4)



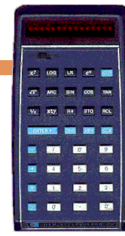
Preorder traversal:
1. Visit the root
2. Visit left subtree, in preorder
3. Visit right subtree, in preorder

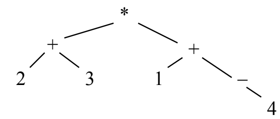
* + 2 3 + 1 - 4

prefix and postfix notation proposed by Jan Lukasiewicz in 1951

Postfix (we see it later) is often called RPN for Reverse Polish Notation

tree for (2 + 3) * (1 + – 4)





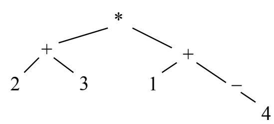
In about 1974, Gries paid $300 for an HP calculator, which had some memory and used postfix notation! Still works. Come up to see it.

Postorder traversal:
1. Visit left subtree, in postorder
2. Visit right subtree, in postorder
3. Visit the root

2 3 + 1 4 - + *

Postfix notation

tree for (2 + 3) * (1 + – 4)



	Cornell tuition	Calculator cost	Percent
1973	$5030	$300	.0596
2014	$47,050	$60	.00127

Then: (HP 45) RPN. 9 memory locations, 4-level stack, 1-line display
Now: (HP 35S) RPN and infix. 30K user memory, 2-line display

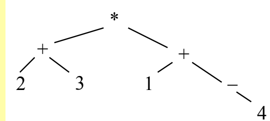
tree for (2 + 3) * (1 + – 4)

Postfix is easy to compute. Process elements left to right.

Number? Push it on a stack

Binary operator? Remove two top stack elements, apply operator to it, push result on stack

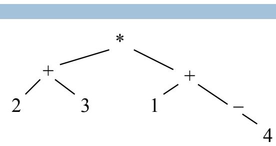
Unary operator? Remove top stack element, apply operator to it, push result on stack



Postfix notation

2 3 + 1 4 - + *

tree for (2 + 3) * (1 + – 4)



Inorder traversal:
1. Visit left subtree, in inorder
2. Visit the root
3. Visit right subtree, in inorder

To help out, put parens around expressions with operators

(2 + 3) * (1 + (- 4))

Expression trees

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```
public abstract class Exp {
    public abstract int eval();
    public abstract String pre();
    public abstract String post();
}
```

```
public class Add extends Exp {
    Exp left;
    Exp right;
    /** Return the value of this exp. */
    public int eval() {return left.eval() + right.eval();}

    /** Return the preorder.*/
    public String pre() {return "+  " + left.pre() + right.pre(); }

    /** Return the postorder.*/
    public String post() {return left.post() + right.post() + "+  "; }

}
```

Motivation for grammars

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- The cat ate the rat.
- The cat ate the rat slowly.
- The small cat ate the big rat slowly.
- The small cat ate the big rat on the mat slowly.
- The small cat that sat in the hat ate the big rat on the mat slowly, then got sick.
- …

- Not all sequences of words are legal sentences
  The ate cat rat the
- How many legal sentences are there?
- How many legal Java programs?
- How do we know what programs are legal?

http://docs.oracle.com/javase/specs/jls/se7/html/index.html

A Grammar

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Sentence → Noun Verb Noun
Noun → boys
Noun → girls
Noun → bunnies
Verb → like
| see

- White space between words does not matter
- A very boring grammar because the set of Sentences is finite (exactly 18 sentences)

Our sample grammar has these rules:
A Sentence can be a Noun followed by a Verb followed by a Noun
A Noun can be boys or girls or bunnies
A Verb can be like or see

A Grammar

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Sentence → Noun Verb Noun
Noun → boys
Noun → girls
Noun → bunnies
Verb → like
Verb → see

Grammar: set of rules for generating sentences of a language.

Examples of Sentence:
- boys see bunnies
- bunnies like girls

- The words boys, girls, bunnies, like, see are called *tokens* or *terminals*
- The words Sentence, Noun, Verb are called *nonterminals*

A recursive grammar

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Sentence → Sentence and Sentence
Sentence → Sentence or Sentence
Sentence → Noun Verb Noun
Noun → boys
Noun → girls
Noun → bunnies
Verb → like
| see

Grammar more interesting than previous one because the set of Sentences is infinite

What makes this set infinite? Answer:
Recursive definition of Sentence

Detour

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What if we want to add a period at the end of every sentence?
Sentence → Sentence and Sentence .
Sentence → Sentence or Sentence .
Sentence → Noun Verb Noun .
Noun → …
Does this work?
No! This produces sentences like:

$\underbrace{\underbrace{\text{girls like boys .}}_{\text{Sentence}} \text{ and } \underbrace{\text{boys like bunnies .}}_{\text{Sentence}} \text{ .}}_{\text{Sentence}}$

Sentences with periods

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PunctuatedSentence → Sentence .
Sentence → Sentence and Sentence
Sentence → Sentence or Sentence
Sentence → Noun VerbNoun
Noun → boys
Noun → girls
Noun → bunnies
Verb → like
Verb → see

- New rule adds a period only at end of sentence.
- Tokens are the 7 words plus the period (.)
- Grammar is ambiguous:
  boys like girls
  and girls like boys
  or girls like bunnies

Grammars for programming languages

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Grammar describes every possible legal expression

You could use the grammar for Java to list every possible Java program. (It would take forever.)

Grammar tells the Java compiler how to "parse" a Java program

docs.oracle.com/javase/specs/jls/se7/html/jls-2.html#jls-2.3

Grammar for simple expressions (not the best)

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E → integer
E → (E + E)

Simple expressions:

- An E can be an integer.
- An E can be '(' followed by an E followed by '+' followed by an E followed by ')'

Set of expressions defined by this grammar is a recursively-defined set

- Is language finite or infinite?
- Do recursive grammars always yield infinite languages?

Some legal expressions:

- 2
- (3 + 34)
- ((4+23) + 89)

Some illegal expressions:

- (3
- 3 + 4

Tokens of this grammar:
(+) and any integer

Parsing

E → integer
E → (E + E)

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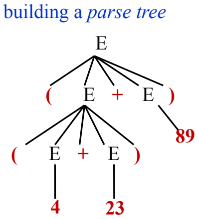
Use a grammar in two ways:

- A grammar defines a *language* (i.e. the set of properly structured *sentences*)
- A grammar can be used to *parse a sentence* (thus, checking if a string is asentence is in the *language*)

To *parse* a sentence is to build a *parse tree*: much like diagramming a sentence

- Example: Show that ((4+23) + 89) is a valid expression E by building a *parse tree*



Ambiguity

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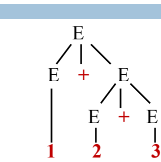
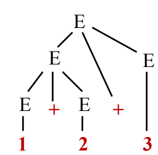
Grammar is ambiguous if it allows two parse trees for a sentence. The grammar below, using no parentheses, is ambiguous. The two parse trees to right show this. We don't know which + to evaluate first in the expression 1 + 2 + 3

E → integer
E → E + E





Recursive descent parsing

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Write a set of mutually *recursive methods* to check if a sentence is in the language (show how to generate parse tree later).

One method for each nonterminal of the grammar. The method is completely determined by the rules for that nonterminal. On the next pages, we give a high-level version of the method for nonterminal E:

E → integer
E → (E + E)

Parsing an E

E → integer
E → (E + E)

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```
/** Unprocessed input starts an E. Recognize that E, throwing
    away each piece from the input as it is recognized.
    Return false if error is detected and true if no errors.
    Upon return, processed tokens have been removed from input. */
public boolean parseE()
```

before call: already processed unprocessed

(2 + (4 + 8) + 9)

after call: already processed unprocessed
(call returns true)

(2 + (4 + 8) + 9)

Specification: /** Unprocessed input starts an E. …*/

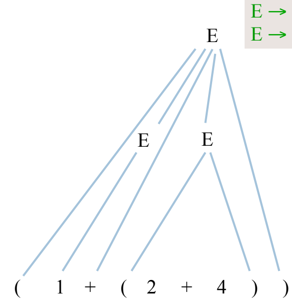
E → integer
E → (E + E)

```
public boolean parseE() {
    if (first token is an integer) remove it from input and return true;
    if (first token is not '(' ) return false else remove it from input;
    if (!parseE()) return false;
    if (first token is not '+' ) return false else remove it from input;
    if (!parseE()) return false;
    if (first token is not ')' ) return false else remove it from input;
    return true;
}
```

Same code used 3 times. Cries out for a method to do that

Illustration of parsing to check syntax

E → integer
E → (E + E)



The scanner constructs tokens

An object scanner of class Scanner is in charge of the input String. It constructs the tokens from the String as necessary.

e.g. from the string "1464+634" build the token "1464", the token "+", and the token "634".

It is ready to work with the part of the input string that has not yet been processed and has thrown away the part that is already processed, in left-to-right fashion.

already processed | unprocessed

(2 + (4 + 8) + 9)

Change parser to generate a tree

E → integer
E → (E + E)

```
/** … Return a Tree for the E if no error.
       Return null if there was an error*/
public Tree parseE() {
    if (first token is an integer) remove it from input and return true;

    if (first token is an integer) {
      Tree t= new Tree(the integer);
      Remove token from input;
      return t;
    }

    …
}
```

Change parser to generate a tree

E → integer
E → (E + E)

```
/** … Return a Tree for the E if no error.
       Return null if there was an error*/
public Tree parseE() {
    if (first token is an integer) … ;
    if (first token is not '(' ) return null else remove it from input;
    Tree t1= parse(E); if (t1 == null) return null;
    if (first token is not '+' ) return null else remove it from input;
    Tree t2= parse(E); if (t2 == null) return null;
    if (first token is not ')' ) return false else remove it from input;
    return new Tree(t1, '+', t2);
}
```

Code for a stack machine

Code for 2 + (3 + 4)

```
PUSH 2
PUSH 3
PUSH 4
ADD
ADD
```

4
3
2
Stack

ADD: remove two top values from stack, add them, and place result on stack

It's postfix notation! **2 3 4 + +**

Code for a stack machine

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Code for 2 + (3 + 4)

PUSH 2
PUSH 3
PUSH 4
ADD
ADD

ADD: remove two top values from stack, add them, and place result on stack

7
9
Stack

It's postfix notation! **2 3 4 + +**

Use parser to generate code for a stack machine

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Code for 2 + (3 + 4)

PUSH 2
PUSH 3
PUSH 4
ADD
ADD

ADD: remove two top values from stack, add them, and place result on stack

It's postfix notation! **2 3 4 + +**

parseE can generate code as follows:

- For integer i, return string "PUSH " + i + "\n"
- For (E1 + E2), return a string containing
  - Code for E1
  - Code for E2
  - "ADD\n"

Grammar that gives precedence to * over +

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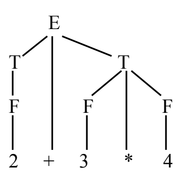
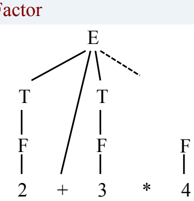
E -> T { + T }
T -> F { * F }
F -> integer
F -> (E)

Notation: { xxx } means 0 or more occurrences of xxx.
E: Expression **T:** Term
F: Factor



says do * first



Try to do + first, can't complete tree

Does recursive descent always work?

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Some grammars cannot be used for recursive descent

Trivial example (causes infinite recursion):

S → b
S → Sa

Can rewrite grammar

S → b
S → bA
A → a
A → aA

For some constructs, recursive descent is hard to use

Other parsing techniques exist – take the compiler writing course

Syntactic ambiguity

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Sometimes a sentence has more than one parse tree

S → A | aaxB
A → x | aAb
B → b | bB

aaxbb can be parsed in two ways

This kind of ambiguity sometimes shows up in programming languages. In the following, which **then** does the **else** go with?

if E1 **then if** E2 **then** S1 **else** S2

Syntactic ambiguity

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This kind of ambiguity sometimes shows up in programming languages. In the following, which **then** does the **else** go with?

if E1 **then if** E2 **then** S1 **else** S2

This ambiguity actually affects the program's meaning

Resolve it by either
(1) Modify the grammar to eliminate the ambiguity (best)
(2) Provide an extra non-grammar rule (e.g. else goes with closest if)

Can also think of modifying the language (require end delimiters)

Summary: What you should know

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- preorder, inorder, and postorder traversal. How they can be used to get prefix notation, infix notation, and postfix notation for an expression tree.
- Grammars: productions or rules, tokens or terminals, nonterminals. The parse tree for a sentence of a grammar.
- Ambiguous grammar, because a sentence is ambiguous (has two different parse trees).
- You should be able to tell whether string is a sentence of a simple grammar or not. You should be able to tell whether a grammar has an infinite number of sentences.
- You are *not* responsible for recursive descent parsing

Exercises

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Write a grammar and recursive descent parser for sentence palindromes that ignores white spaces & punctuation

Was it Eliot's toilet I saw? No trace, not one carton

Go deliver a dare, vile dog! Madam, I'm Adam

Write a grammar and recursive program for strings A^nB^n

AB AABB

AAAAAAABBBBBBB

Write a grammar and recursive program for Java identifiers

<letter> [<letter> or <digit>]$^{0\ldots N}$

j27, but not 2j7