

Recitation 7

Hashing

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Set

Set: collection of *distinct* objects

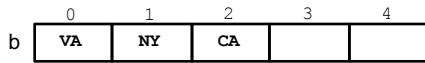
```
Set<E>
add(E ob);
remove(E ob);
contains(E ob);
isEmpty();
size();
... (a few more)
```

Sets

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Implementing a set in an array

b[0..n-1] contains the values in the set



Have to search through the list *linearly* to find values

Have to shift all values down

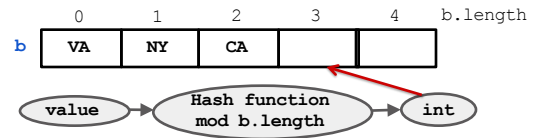
| method | expected time |
|----------|---------------|
| add | $O(n)$ |
| contains | $O(n)$ |
| remove | $O(n)$ |

Sets

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Hashing — an implementation of a Set

Idea: Use a hash function to tell where to put a value

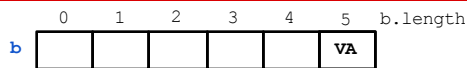


Possible hash function for an object: its address in memory (not always good, explain later)

Hashing

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Hashing



add("VA") can be done using

```
k = Math.abs(hashCode("VA")) % b.length;
if (b[k] == null) b[k] = "VA";
```

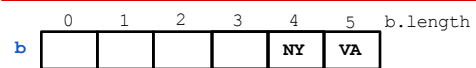
Suppose k is 5. This puts "VA" in b[5]

If b[k] != null? Handle that later

Hashing

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Hashing



add("NY")

```
k = Math.abs(hashCode("NY")) % b.length;
if (b[k] == null) b[k] = "NY";
```

Suppose k is 4. This puts "NY" in b[4]

Hashing

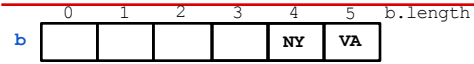
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Collision Resolution

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Collision resolution

Hashing



```
add("VT")
k= Math.abs(hashCode("VT")) % b.length;
if (b[k] == null) b[k]= "VT";
```

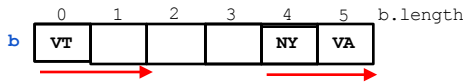
Suppose k is 4. Can't place "VT" in b[4] because "NY" is already there

Two ways to solve collisions: **Open addressing** and **chaining**.
Do open addressing first

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Open addressing: linear probing

Hashing



add("VT"). Suppose "VT" hashes to 4

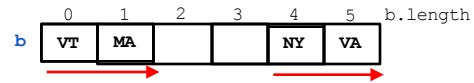
Search in successive locations (with wraparound) for the first null element, and place "VT" there.

Here, look in b[4], b[5], b[0], and place "VT" in b[0].

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Open addressing: linear probing

Hashing



add("MA"). Suppose "MA" hashes to 4

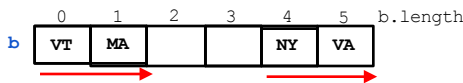
Here, look in b[4], b[5], b[0], b[1] and place "MA" in b[1].

This took 4 probes to find a null element.
"probe": a test of one array element

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Open addressing: linear probing

Hashing



basic code for add(String s):

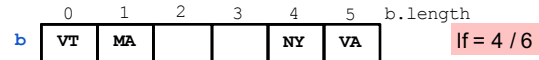
```
int k= what s hashed to;
while (b[k] != null && !b[k].equals(s))
    { k= (k+1) % b.length(); }
if (b[k] == null) { b[k]= s; } // if not null, s already in set
```

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Making linear probing take expected constant time

Hashing

Load factor lf : (# non-null elements) / b.length



Somebody proved:

Under certain assumptions about the hash function, the average number of probes used to add an element is $1 / (1 - lf)$

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Hashing

Making linear probing take expected constant time

Somebody proved:

Under certain assumptions about the hash function, the average number of probes to add an element is $1 / (1 - lf)$

So if $lf \leq 1/2$, meaning at least half the elements are null, then the average number of probes is $\leq 1/(1/2) = 2$.

WOW! Make sure at least half the elements are null and expect no more than two probes!!! How can that be?

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Hashing

Making linear probing take expected constant time

Load factor lf: (# non-null elements) / b.length

b 0 1 2 3 4 5 b.length

| | | | | | |
|----|----|--|--|--|----|
| VT | MA | | | | VA |
|----|----|--|--|--|----|

If at least half the elements are null, expect no more than two probes!!! Proof outside scope of 2110

Here's insight into it. Suppose half the elements are null. Then, half the time, you can expect to need only 1 probe.

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Hashing

Rehash: If the load factor becomes $\geq 1/2$

If the load factor becomes $\geq 1/2$, do the following:

1. Create a new empty array b1 of size $4*b.length$
2. For each set element that is in b, hash it into array b1.
3. $b = b1$; // so from now on the new array is used

Suppose size of array goes from n to 4n. Then, can add more than n values before this has to be done again.

We can show that this does not increase the expected run time. We "amortize" this operation over the add operations that created the set.

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Hashing

What does "amortize" mean?

We bought a machine that makes fizzy water –adds fizz to plain water. Now, we don't have to buy fizzy water by the bottle. The machine cost \$100.

Use the machine to make one glass of fizzy water, that glass cost us \$100.00.

Make 100 glasses of fizzy water? Each glass cost us \$1.00.

Make 1,000 glasses? Each glass cost us 10 cents.

I are **amortizing** the cost of the machine over the use of the machine, over the number of operations "make a glass ...".

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Hashing

Deleting an element from the set

b 0 1 2 3 4 5 b.length

| | | | | | |
|----|----|--|--|----|----|
| VT | MA | | | NY | VA |
|----|----|--|--|----|----|

Does set contain "MA"?

"MA" hashes to 4. After probes of b[4], b[5], b[0], b[1], we say, yes, "MA" is in the set.

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Hashing

Deleting an element from the set

b 0 1 2 3 4 5 b.length

| | | | | | |
|----|----|--|--|----|--|
| VT | MA | | | NY | |
|----|----|--|--|----|--|

Does set contain "MA"?

"MA" hashes to 4. After probes of b[4], b[5], b[0], b[1], we say, yes, "MA" is in the set.

Now suppose we delete "VA" from the set, by setting b[5] to null.

Now ask whether the set contains "MA". Two probes say no, because the second probe finds null!!!

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Hashing

Deleting an element from the set

| | | | | | | | |
|---|----|----|---|---|----|----|----------|
| | 0 | 1 | 2 | 3 | 4 | 5 | b.length |
| b | VT | MA | | | NY | VA | |

Therefore, we can't delete a value from the set by setting its array element to null. That messes up linear probing.

Instead, in Java, use an inner class for the array elements, with two fields:

1. String value; // the value, like "VT"
2. boolean isInSet; // true iff value is in the set

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Hashing

Deleting an element from the set

| | | | | | | | |
|---|----|----|---|---|----|----|----------|
| | 0 | 1 | 2 | 3 | 4 | 5 | b.length |
| b | VT | MA | | | NY | VA | |

Instead, in Java, use an inner class for the array elements, with two fields:

1. String value; // the value, like "VT"
2. boolean isInSet; // true iff value is in the set

Above: red string means its isInSet field is true.
To delete "VA", set its isInSet field to false

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Collisions:
Chaining

Inner class HashEntry

```

class HashSet<E> {
    LinkedList<HashEntry<E>>[] b;

    private class HashEntry<E> {
        private E value;
        private boolean isInSet;
    }
}
    
```

inner class to contain value and whether it is in the set
Class is private —the user knows nothing about it

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Summary for open addressing –linear probing

1. Each non-null b[i] contains an object with two fields: a value and boolean variable isInSet.
2. add(e). Hash e to an index and linear probe. If null was found, add e at that spot. If e was found, set its isInSet field to true.
If load factor >= 1/2, move set elements to an array double the size.
3. Remove(e). Hash e to an index and linear probe. If null was found, do nothing. If e was found, set its isInSet field to false.
4. Contains(e). Hash e to an index and linear probe. If e was found and its isInSet field is true, return true; otherwise, return false.

DEMO. We have a complete implementation of this.

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Hash Functions

Class Object contains a function hashCode().
The value of C.hashCode() is the memory address where the object resides.

You can override this function in any class you write. Later slides discuss why one would do this.

For primitive types, you have to write your own hashCode function.

On the next slides, we discuss hash functions.

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Hash Functions

Requirements

Hash functions MUST:

- have the same hash for equal objects
 - In Java: if a.equals(b), then
a.hashCode() == b.hashCode()
 - if you override equals and plan on using object in a HashMap or HashSet, override hashCode too!
- be deterministic
 - calling hashCode on the same object should return the same integer
 - important to have immutable values if you override equals!

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Hash Functions

Good hash functions

- As often as possible, if !a.equals(b), then a.hashCode() != b.hashCode()
 - this helps avoid collisions and clustering
- Good distribution of hash values across all possible keys
- FAST. add, contains, and remove take time proportional to speed of hash function

A bad hash function won't break a hash set but it could seriously slow it down

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Hash Functions

String.hashCode()

Don't hash long strings, not O(1) but O(length of string)!

```

/** Return a hash code for this string.
 * Computes it as
 *   s[0]*31^(n-1) + s[1]*31^(n-2) + ... + s[n-1]
 * using int arithmetic.
 */
public int hashCode() { ... }
    
```

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Hash Functions

Designing good hash functions

```

class Thingy {
    private String s1, s2;

    public boolean equals(Object obj) {
        return s1.equals(obj.s1) &&
            s2.equals(obj.s2);
    }

    public int hashCode() {
        return 37 * s1.hashCode() + 97 * s2.hashCode();
    }
}
    
```

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Collisions: Chaining

an alternative to open addressing (probing)

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Collisions:
Chaining

Chaining definition

Each b[k] contains a linked list of values in the set that hashed to k.

b[5] is an empty list

add(e): hash e to some k. If e is not on linked list b[k], add it to the list

remove(e): hash e to some k. If e is on linked list b[k], remove it

You can figure out other operations yourself.

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Collisions:
Chaining

Chaining

Each b[k] contains a linked list of values in the set that hashed to k.

Load factor is $3/6 = 1/2$

The load factor: $(\text{number of values in list}) / \text{size of array}$

It must be kept under $1/2$, as with open addressing

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Linear probing versus quadratic probing

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Collisions: Open Addressing

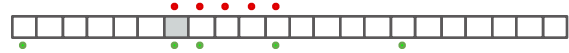
Linear vs quadratic probing

When a collision occurs, how do we search for an empty space?

linear probing: search the array in order:
 $i, i+1, i+2, i+3 \dots$

quadratic probing: search the array in nonlinear sequence:
 $i, i+1^2, i+2^2, i+3^2 \dots$

For quadratic probing, the size of the array should be a prime. Someone proved that then, every single array element will be covered.



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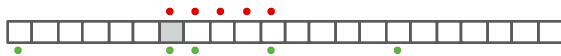
Collisions: Open Addressing

Why use quadratic probing

linear probing:
 $i, i+1, i+2, i+3 \dots$

quadratic probing:
 $i, i+1^2, i+2^2, i+3^2 \dots$

Collisions can lead to clustering: many full elements in a row. Quadratic probing spreads the values out more, leading to less clustering than with linear probing.



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Big O!

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Big O of Hashing

Runtime analysis

| | Open Addressing | Chaining |
|-----------------|--|--|
| Expected | $O(\text{hash function})$ (since load factor kept $< 1/2$) | $O(\text{hash function})$ (since load factor kept $< 1/2$) |
| Worst | $O(n)$ (no null between values) | $O(n)$ (all values in one linked list) |

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Big O of Hashing

Amortized runtime

Insert n items: $n + 2n$ (from copying) = $3n$ inserts $\rightarrow O(3n) \rightarrow O(n)$
 Amortized to constant time per insert

| | Copying Work |
|---|---------------------------------|
| Everything has just been copied | n inserts |
| Half were copied in previous doubling | $n/2$ inserts |
| Half of those were copied in doubling before previous one | $n/4$ inserts |
| ... | ... |
| Total work | $n + n/2 + n/4 + \dots \leq 2n$ |

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Limitations of hash sets

1. Due to rehashing, adding elements may take $O(n)$
 - a. not always ideal for time-critical applications
1. No ordering among elements, very slow to find nearby elements

Alternatives (out of scope of the course):

1. hash set with incremental resizing prevents $O(n)$ rehashing
1. self-balancing binary search trees are worst case $O(\log n)$ and keep the elements ordered

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Hashing Extras

Hashing has wide applications in areas such as security

- cryptographic hash functions are ones that are very hard to invert (figure out original data from hash code), changing the data almost always changes the hash, and two objects almost always have different hashes
- md5 hash: `md5 filename` in Terminal

| By | Size | MD5 |
|-------|----------|----------------------------------|
| erc73 | 1.449 MB | 13188ff26829b7cc4f4a45b84ee6deb7 |

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