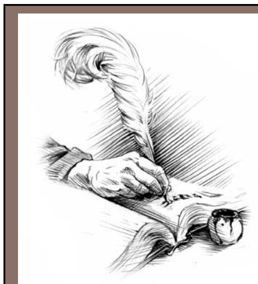
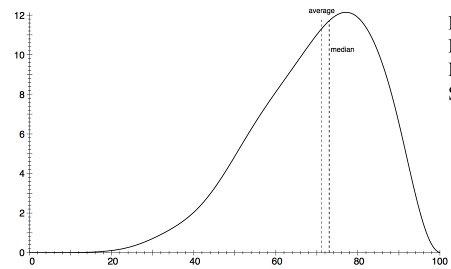


ADTS, GRAMMARS, PARSING, TREE TRAVERSALS

Lecture 12
CS2110 – Spring 2015

Prelim 1

2



Max: 99
Mean: 71.2
Median: 73
Std Dev: 14.6

Prelim 1

3

Score	Grade	%
90-99	A	26%
82-89	A-/A	
70-82	B/B+	50%
62-69	B-/B	
50-59	C-/C+	18%
40-49	D/D+	5%
< 40	F	3%

Regrades

4

- We work hard to grade exams quickly…
- … but we are not perfect!
- If you find a mistake:
  - Do not modify your exam!
  - Write up a clear explanation of the error on the regrade request form
  - Return to the handback room
  - Deadline: 4pm Friday, October 9th

Pointers to material

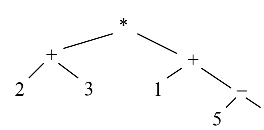
5

- Parse trees: text, section 23.36
- Definition of Java Language, sometimes useful: docs.oracle.com/javase/specs/jls/se7/html/index.html
- Grammar for most of Java, for those who are curious: docs.oracle.com/javase/specs/jls/se7/html/jls-18.html
- Tree traversals –preorder, inorder, postorder: text, sections 23.13 .. 23.15.

Expression trees

6

Can draw a tree for (2 + 3) * (1 + (5 – 4)

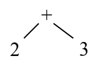



```
public abstract class Exp {
    /* return the value of this Exp */
    public abstract int eval();
}
```

Expression trees

7

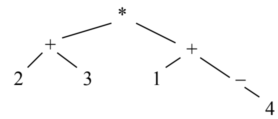
```
public abstract class Exp {
    /* return the value of this Exp */
    public abstract int eval();
}
```




```
public class Int extends Exp {
   int v;
   public int eval() {
      return v;
   }
}
```

```
public class Add extends Exp {
    Exp left;
    Exp right;
    public int eval() {
        return left.eval() + right.eval();
    }
}
```

tree for (2 + 3) * (1 + – 4)



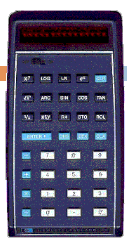
Preorder traversal:
1. Visit the root
2. Visit left subtree, in preorder
3. Visit right subtree, in preorder

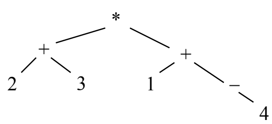
* + 2 3   + 1 - 4

prefix and postfix notation proposed by Jan Lukasiewicz in 1951

Postfix (we see it later) is often called RPN for Reverse Polish Notation

tree for (2 + 3) * (1 + – 4)



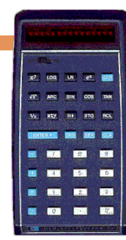


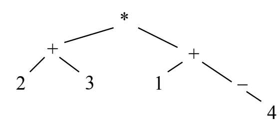
In about 1974, Gries paid $300 for an HP calculator, which had some memory and used postfix notation! Still works. Come up to see it.

Postorder traversal:
1. Visit left subtree, in postorder
2. Visit right subtree, in postorder
3. Visit the root

2 3 + 1 4 - + *
Postfix notation

tree for (2 + 3) * (1 + – 4)





	Cornell tuition	Calculator cost	Percent
1973	$5030	$300	.0596
2014	$47,050	$60	.00127

Then: (HP 45) RPN. 9 memory locations, 4-level stack, 1-line display
Now: (HP 35S) RPN and infix. 30K user memory, 2-line display

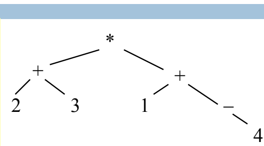
tree for (2 + 3) * (1 + – 4)

Postfix is easy to compute. Process elements left to right.

Number? Push it on a stack

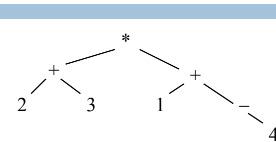
Binary operator? Remove two top stack elements, apply operator to it, push result on stack

Unary operator? Remove top stack element, apply operator to it, push result on stack



Postfix notation
2 3 + 1 4 - + *

tree for (2 + 3) * (1 + – 4)



Inorder traversal:
1. Visit left subtree, in inorder
2. Visit the root
3. Visit right subtree, in inorder

To help out, put parens around expressions with operators

(2 + 3)   *   (1 + (- 4))

Expression trees

13

```
public abstract class Exp {
    public abstract int eval();
    public abstract String pre();
    public abstract String post();
}
```

```
public class Add extends Exp {
    Exp left;
    Exp right;
    /** Return the value of this exp. */
    public int eval() {return left.eval() + right.eval();}

    /** Return the preorder.*/
    public String pre() {return "+  " + left.pre() + right.pre(); }

    /** Return the postorder.*/
    public String post() {return left.post() + right.post() + "+  "; }
}
```

Motivation for grammars

14

- The cat ate the rat.
- The cat ate the rat slowly.
- The small cat ate the big rat slowly.
- The small cat ate the big rat on the mat slowly.
- The small cat that sat in the hat ate the big rat on the mat slowly, then got sick.
- …

- Not all sequences of words are legal sentences
  The ate cat rat the
- How many legal sentences are there?
- How many legal Java programs?
- How do we know what programs are legal?

http://docs.oracle.com/javase/specs/jls/se7/html/index.html

A Grammar

15

Sentence → Noun Verb Noun
Noun → boys
Noun → girls
Noun → bunnies
Verb → like
| see

- White space between words does not matter
- A very boring grammar because the set of Sentences is finite (exactly 18 sentences)

Our sample grammar has these rules:
A Sentence can be a Noun followed by a Verb followed by a Noun
A Noun can be boys or girls or bunnies
A Verb can be like or see

A Grammar

16

Sentence → Noun Verb Noun
Noun → boys
Noun → girls
Noun → bunnies
Verb → like
Verb → see

Grammar: set of rules for generating sentences of a language.

Examples of Sentence:
- girls see bunnies
- bunnies like boys

- The words boys, girls, bunnies, like, see are called *tokens* or *terminals*
- The words Sentence, Noun, Verb are called *nonterminals*

A recursive grammar

17

Sentence → Sentence and Sentence
Sentence → Sentence or Sentence
Sentence → Noun Verb Noun
Noun → boys
Noun → girls
Noun → bunnies
Verb → like
| see

This grammar is more interesting than previous one because the set of Sentences is infinite

What makes this set infinite? Answer:
Recursive definition of Sentence

Detour

18

What if we want to add a period at the end of every sentence?
Sentence → Sentence and Sentence .
Sentence → Sentence or Sentence .
Sentence → Noun Verb Noun .
Noun → …
Does this work?
No! This produces sentences like:

girls like boys . and boys like bunnies . .

Sentence Sentence
Sentence

Sentences with periods

19

PunctuatedSentence → Sentence .
Sentence → Sentence and Sentence
Sentence → Sentence or Sentence
Sentence → Noun VerbNoun
Noun → boys
Noun → girls
Noun → bunnies
Verb → like
Verb → see

- New rule adds a period only at end of sentence.
- Tokens are the 7 words plus the period (.)
- Grammar is ambiguous:
  - **boys like girls**
  - **and girls like boys**
  - **or girls like bunnies**

Grammars for programming languages

20

Grammar describes every possible legal expression

You could use the grammar for Java to list every possible Java program. (It would take forever.)

Grammar tells the Java compiler how to "parse" a Java program

docs.oracle.com/javase/specs/jls/se7/html/jls-2.html#jls-2.3

Grammar for simple expressions (not the best)

21

E → integer
E → (E + E)

Simple expressions:

- An E can be an integer.
- An E can be '(' followed by an E followed by '+' followed by an E followed by ')'

Set of expressions defined by this grammar is a recursively-defined set

- Is language finite or infinite?
- Do recursive grammars always yield infinite languages?

Some legal expressions:

- 2
- (3 + 34)
- ((4+23) + 89)

Some illegal expressions:

- (3
- 3 + 4

Tokens of this grammar:
(+) and any integer

Parsing

22

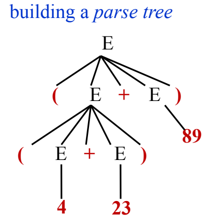
E → integer
E → (E + E)

Use a grammar in two ways:

- A grammar defines a *language* (i.e. the set of properly structured *sentences*)
- A grammar can be used to *parse* a *sentence* (thus, checking if a string is a*sentence* is in the *language*)

To *parse* a sentence is to build a *parse tree*: much like diagramming a sentence

- Example: Show that ((4+23) + 89) is a valid expression E by building a *parse tree*

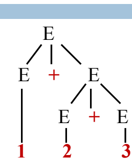


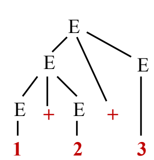
Ambiguity

23

Grammar is ambiguous if it allows two parse trees for a sentence. The grammar below, using no parentheses, is ambiguous. The two parse trees to right show this. We don't know which + to evaluate first in the expression 1 + 2 + 3

E → integer
E → E + E





Recursive descent parsing

24

Write a set of mutually *recursive methods* to check if a sentence is in the language (show how to generate parse tree later).

One method for each nonterminal of the grammar. The method is completely determined by the rules for that nonterminal. On the next pages, we give a high-level version of the method for nonterminal E:

E → integer
E → (E + E)

Parsing an E

E → integer
E → (E + E)

25

/** Unprocessed input starts an E. Recognize that E, throwing away each piece from the input as it is recognized. Return false if error is detected and true if no errors. Upon return, processed tokens have been removed from input. */
public boolean parseE()

before call: already processed unprocessed

(2 + (4 + 8) + 9)

after call: already processed unprocessed
(call returns true)

(2 + (4 + 8) + 9)

Specification: /** Unprocessed input starts an E. …*/

E → integer
E → (E + E)

26

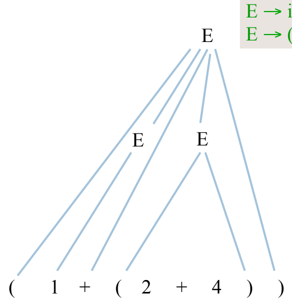
public boolean parseE() {
  if (first token is an integer) remove it from input and return true;
  if (first token is not '(') return false **else** remove it from input;
  if (!parseE()) return false;
  if (first token is not '+') return false **else** remove it from input;
  if (!parseE()) return false;
  if (first token is not ')') return false **else** remove it from input;
  return true;
}

Same code used 3 times. Cries out for a method to do that

Illustration of parsing to check syntax

27

E → integer
E → (E + E)



The scanner constructs tokens

28

An object scanner of class Scanner is in charge of the input String. It constructs the tokens from the String as necessary.

e.g. from the string "1464+634" build the token "1464", the token "+", and the token "634".

It is ready to work with the part of the input string that has not yet been processed and has thrown away the part that is already processed, in left-to-right fashion.

already processed unprocessed

(2 + (4 + 8) + 9)

Change parser to generate a tree

E → integer
E → (E + E)

29

/** … Return a Tree for the E if no error.
  Return null if there was an error*/
public Tree parseE() {
  ~~**if** (first token is an integer) remove it from input and return true;~~

  if (first token is an integer) {
    Tree t= **new** Tree(the integer);
    Remove token from input;
    return t;
  }

  …
}

Change parser to generate a tree

30

E → integer
E → (E + E)

/** … Return a Tree for the E if no error.
  Return null if there was an error*/
public Tree parseE() {
  if (first token is an integer) … ;
  if (first token is not '(') return **null else** remove it from input;
  Tree t1= parse(E); **if** (t1 == **null**) **return null**;
  if (first token is not '+') return **null else** remove it from input;
  Tree t2= parse(E); **if** (t2 == **null**) **return null**;
  if (first token is not ')') return false **else** remove it from input;
  return new Tree(t1, '+', t2);
}

Code for a stack machine

31

Code for 2 + (3 + 4)

PUSH 2
PUSH 3
PUSH 4
ADD
ADD

ADD: remove two top values from stack, add them, and place result on stack

4
3
2
Stack

It's postfix notation! **2 3 4 + +**

Code for a stack machine

32

Code for 2 + (3 + 4)

PUSH 2
PUSH 3
PUSH 4
ADD
ADD

ADD: remove two top values from stack, add them, and place result on stack

7
9
Stack

It's postfix notation! **2 3 4 + +**

Use parser to generate code for a stack machine

33

Code for 2 + (3 + 4)

PUSH 2
PUSH 3
PUSH 4
ADD
ADD

ADD: remove two top values from stack, add them, and place result on stack

It's postfix notation! **2 3 4 + +**

parseE can generate code as follows:

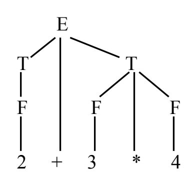
- For integer i, return string "PUSH " + i + "\n"
- For (E1 + E2), return a string containing
  - Code for E1
  - Code for E2
  - "ADD\n"

Grammar that gives precedence to * over +

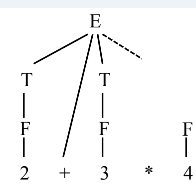
34

E -> T { + T }
T -> F { * F }
F -> integer
F -> (E)

Notation: { xxx } means 0 or more occurrences of xxx.
E: Expression **T:** Term
F: Factor



says do * first



Try to do + first, can't complete tree

Does recursive descent always work?

35

Some grammars cannot be used for recursive descent

Trivial example (causes infinite recursion):

S → b
S → Sa

Can rewrite grammar

S → b
S → bA
A → a
A → aA

For some constructs, recursive descent is hard to use

Other parsing techniques exist – take the compiler writing course

Syntactic ambiguity

36

Sometimes a sentence has more than one parse tree

S → A | aaxB
A → x | aAb
B → b | bB

aaxbb can be parsed in two ways

This kind of ambiguity sometimes shows up in programming languages. In the following, which **then** does the **else** go with?

if E1 **then if** E2 **then** S1 **else** S2

Syntactic ambiguity

37

This kind of ambiguity sometimes shows up in programming languages. In the following, which **then** does the **else** go with?

if E1 **then if** E2 **then** S1 **else** S2

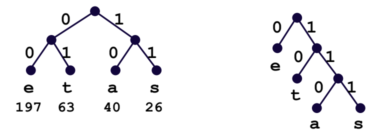
This ambiguity actually affects the program's meaning

Resolve it by either
(1) Modify the grammar to eliminate the ambiguity (best)
(2) Provide an extra non-grammar rule (e.g. else goes with closest if)

Can also think of modifying the language (require end delimiters)

Huffman trees

38



Fixed length encoding
197*2 + 63*2 + 40*2 + 26*2 = 652

Huffman encoding
197*1 + 63*2 + 40*3 + 26*3 = 521

Huffman compression of "Ulysses"

39

' '	242125	00100000	3	110
'e'	139496	01100101	3	000
't'	95660	01110100	4	1010
'a'	89651	01100001	4	1000
'o'	88884	01101111	4	0111
'n'	78465	01101110	4	0101
'i'	76505	01101001	4	0100
's'	73186	01110011	4	0011
'h'	68625	01101000	5	11111
'r'	68320	01110010	5	11110
'l'	52657	01101100	5	10111
'u'	32942	01110101	6	111011
'g'	26201	01100111	6	101101
'f'	25248	01100110	6	101100
'.'	21361	00101110	6	011010
'p'	20661	01110000	6	011001

39

Huffman compression of "Ulysses"

40

...

'7'	68	00110111	15	111010101001111
'/'	58	00101111	15	111010101001110
'X'	19	01011000	16	0110000000100011
'&'	3	00100110	18	011000000010001010
'%'	3	00100101	19	0110000000100010111
'+'	2	00101011	19	0110000000100010110

- original size 11904320
- compressed size 6822151
- 42.7% compression

40

Summary: What you should know

41

- preorder, inorder, and postorder traversal. How they can be used to get prefix notation, infix notation, and postfix notation for an expression tree.
- Grammars: productions or rules, tokens or terminals, nonterminals. The parse tree for a sentence of a grammar.
- Ambiguous grammar, because a sentence is ambiguous (has two different parse trees).
- You should be able to tell whether string is a sentence of a simple grammar or not. You should be able to tell whether a grammar has an infinite number of sentences.
- You are *not* responsible for recursive descent parsing

Exercises

42

Write a grammar and recursive descent parser for sentence palindromes that ignores white spaces & punctuation

Was it Eliot's toilet I saw? No trace, not one carton
Go deliver a dare, vile dog! Madam, I'm Adam

Write a grammar and recursive program for strings A^nB^n

AB AABB
AAAAAAABBBBBBB

Write a grammar and recursive program for Java identifiers

<letter> [<letter> or <digit>]$^{0…N}$
j27, but not 2j7