

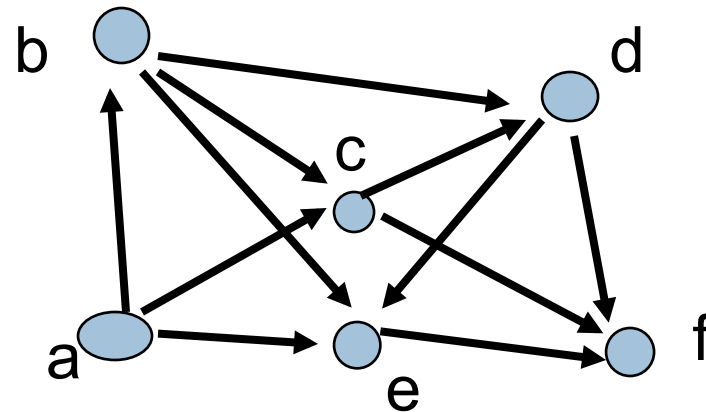
GRAPHS IV: APPLICATIONS

Lecture 22

CS2110 – Spring 2014

Example Directed Graph (Digraph)

2



$V = \{a,b,c,d,e,f\}$

$E = \{(a,b), (a,c), (a,e), (b,c), (b,d), (b,e), (c,d), (c,f), (d,e), (d,f), (e,f)\}$

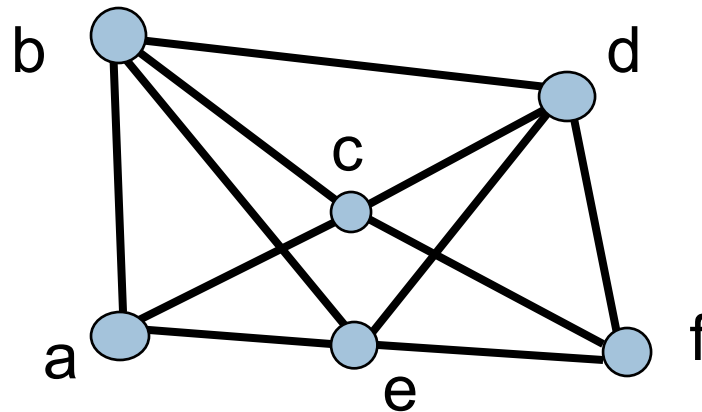
$|V| = 6, |E| = 11$

Example *Undirected Graph*

3

An *undirected graph* is just like a directed graph, except the edges are *unordered pairs (sets)* $\{u,v\}$

Example:



$$V = \{a,b,c,d,e,f\}$$

$$E = \{\{a,b\}, \{a,c\}, \{a,e\}, \{b,c\}, \{b,d\}, \{b,e\}, \{c,d\}, \{c,f\}, \{d,e\}, \{d,f\}, \{e,f\}\}$$

Graph Concepts and Algorithms

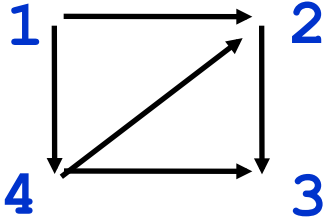
5

- Adjacency Matrix
- Topological Sort
- Search
 - depth-first search
 - breadth-first search
- Shortest paths
 - Dijkstra's algorithm
- Minimum spanning trees
 - Prim's algorithm
 - Kruskal's algorithm

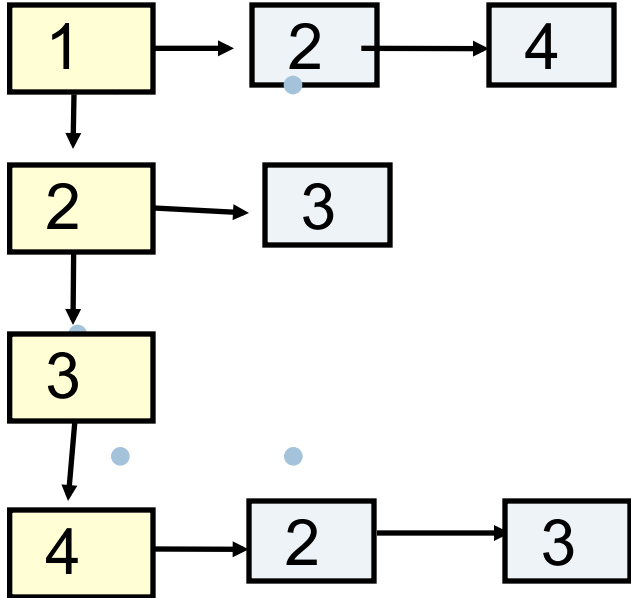
Others:

- Graph Coloring
- Planarity
- Traveling Salesman problem.

Graph Adjacency Matrix



Adjacency List

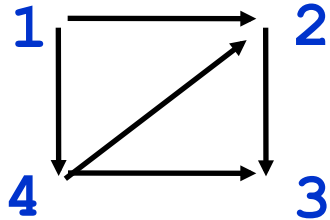


Adjacency Matrix

	1	2	3	4
1	0	1	0	1
2	0	0	1	0
3	0	0	0	0
4	0	1	1	0

Graph Adjacency Matrix

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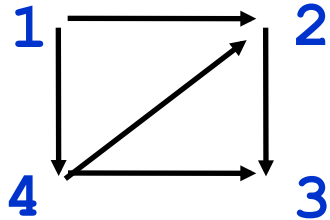
Squaring adjacency matrix gives number of possible paths of length 2!

Adjacency Matrix

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}^2 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

Graph Adjacency Matrix

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Cubing adjacency matrix gives number of possible paths of length 3!

Adjacency Matrix

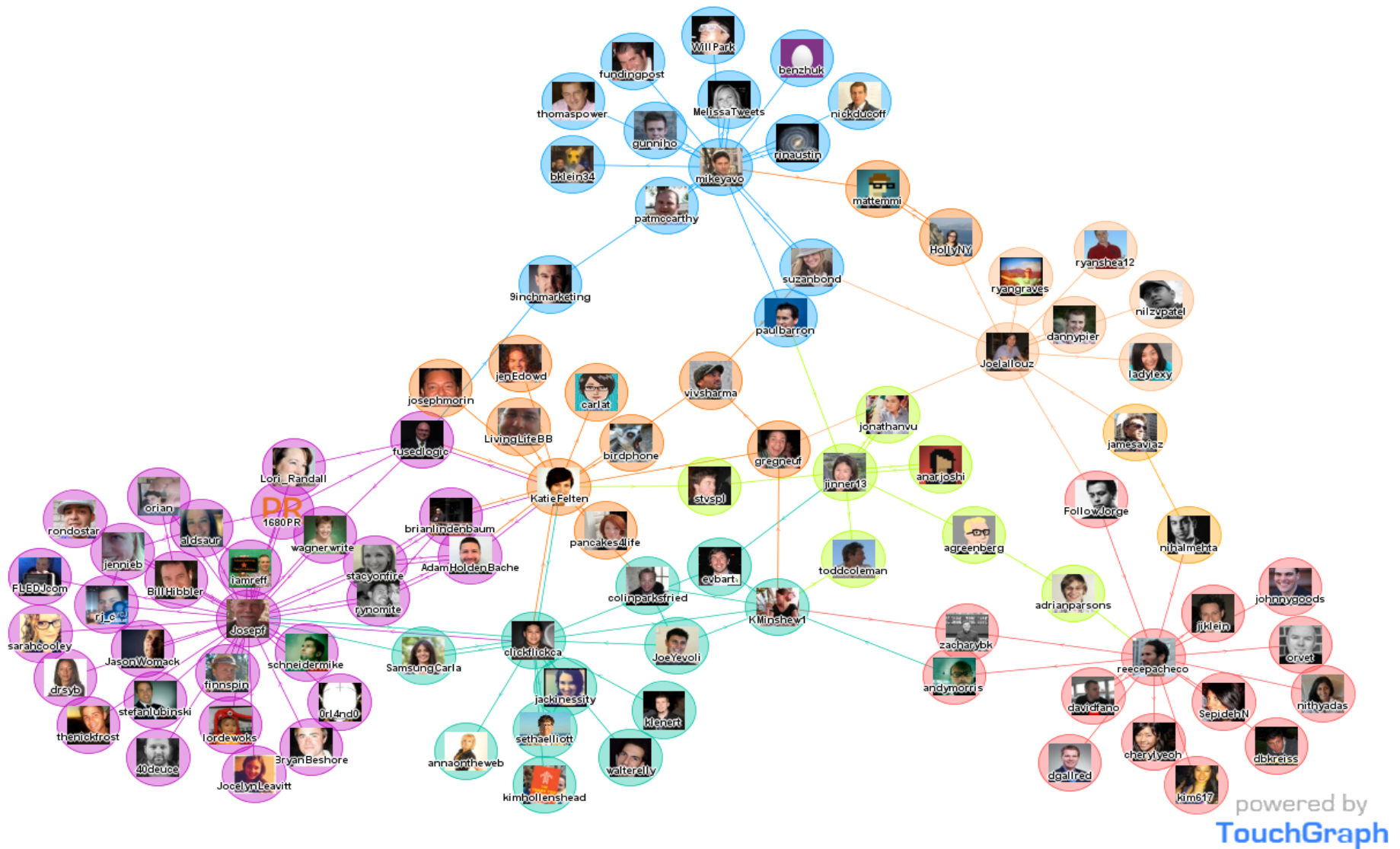
$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}^3 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

Graph Adjacency Matrix

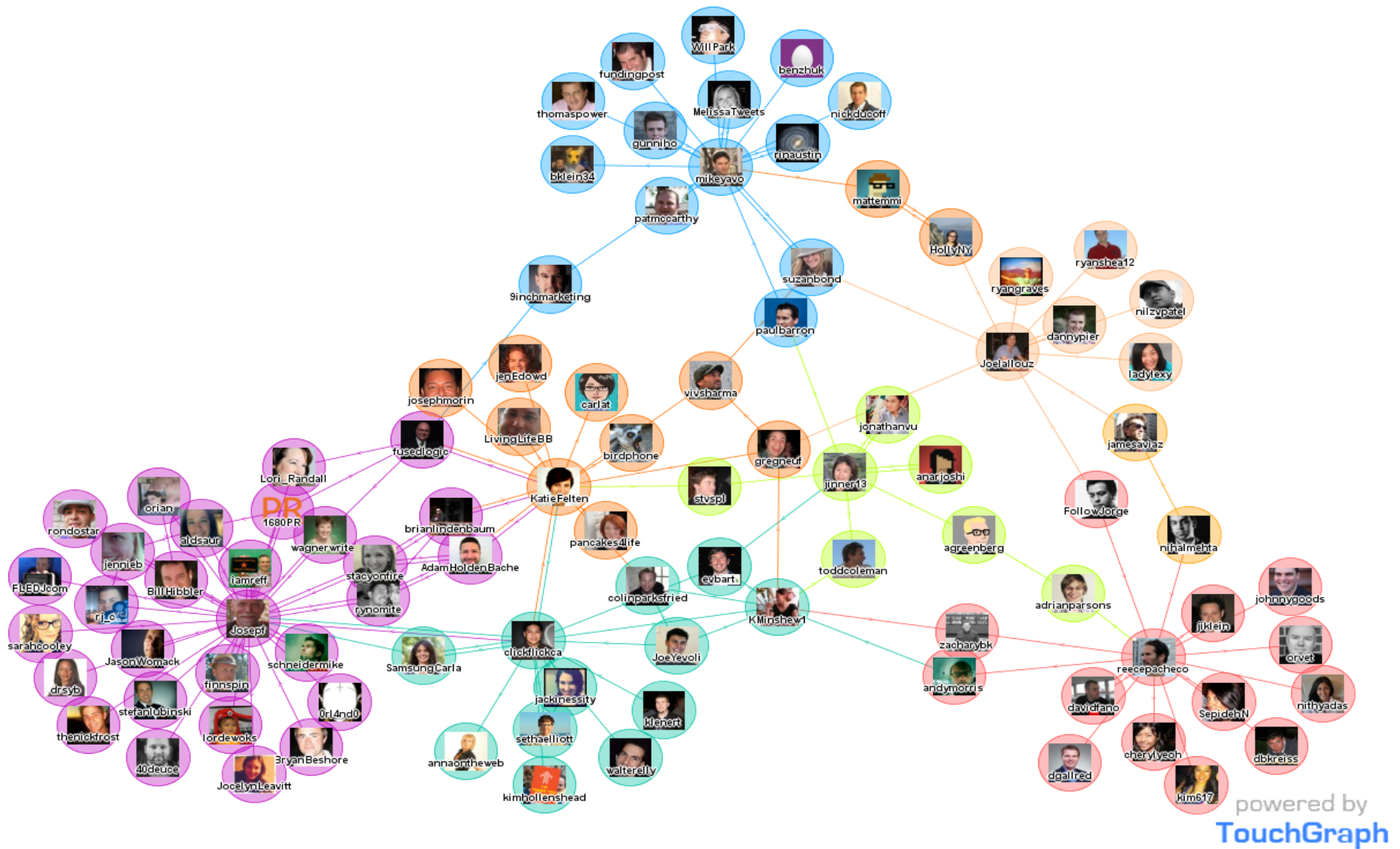
9

Same idea holds for undirected graphs.

How would you find friends of friends?



Find the number of common friends?



Graph Concepts and Algorithms

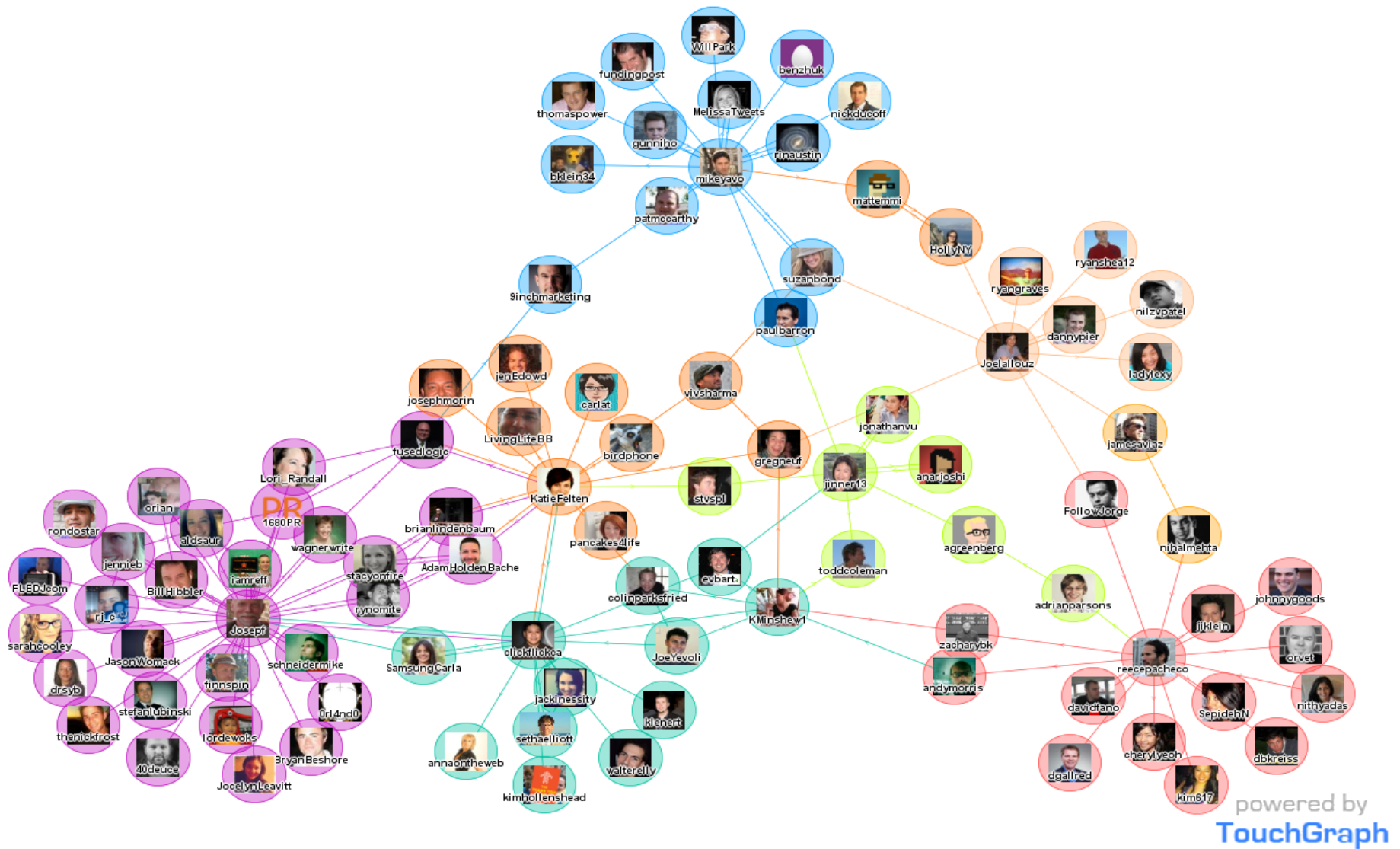
12

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Others:

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Search if a person is someone's Friend?



Graph Concepts and Algorithms

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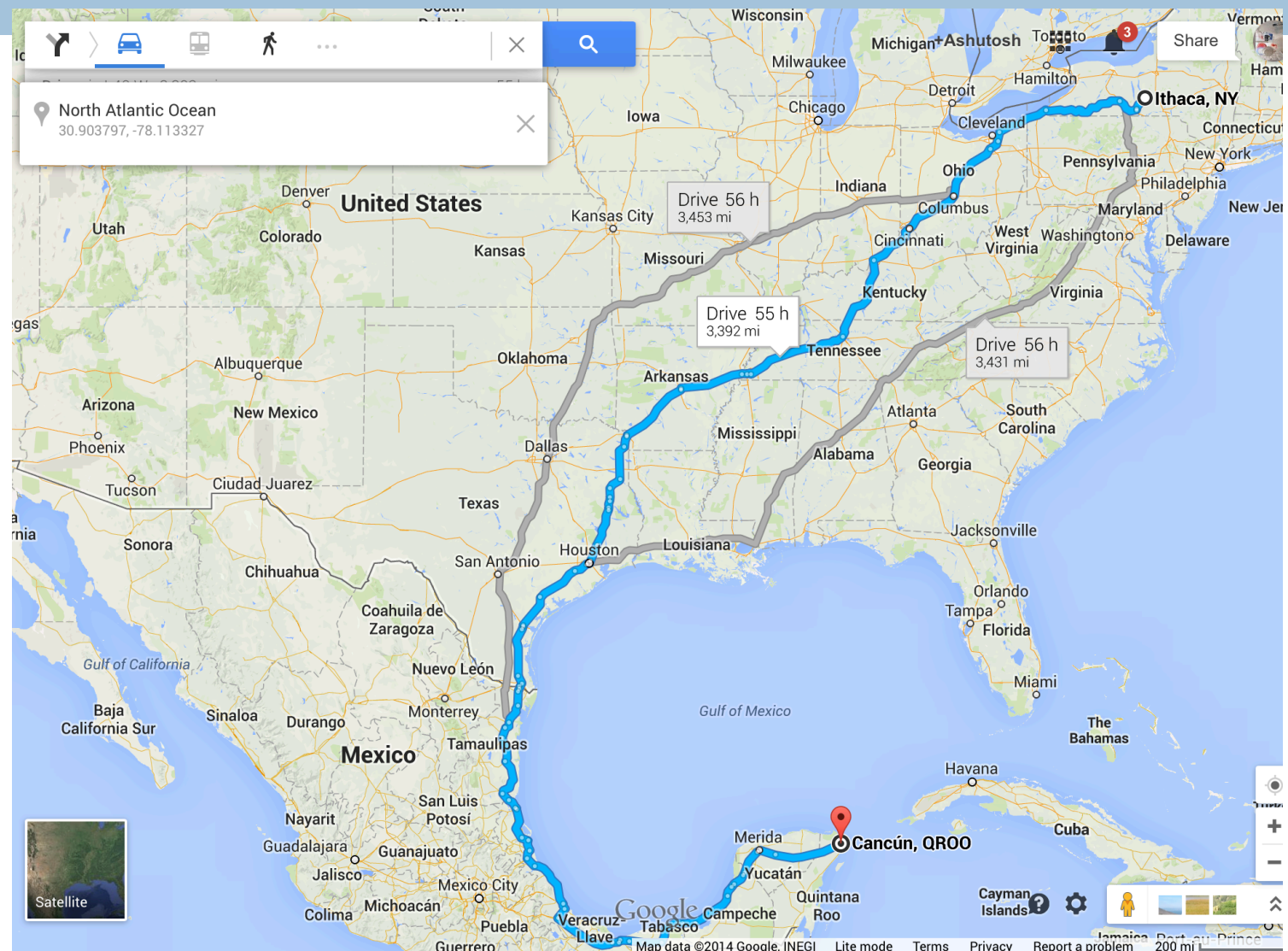
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Shortest Path Algorithm: Used Everyday!

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Maps!
GPS
directions!



Graph Concepts and Algorithms

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Others:

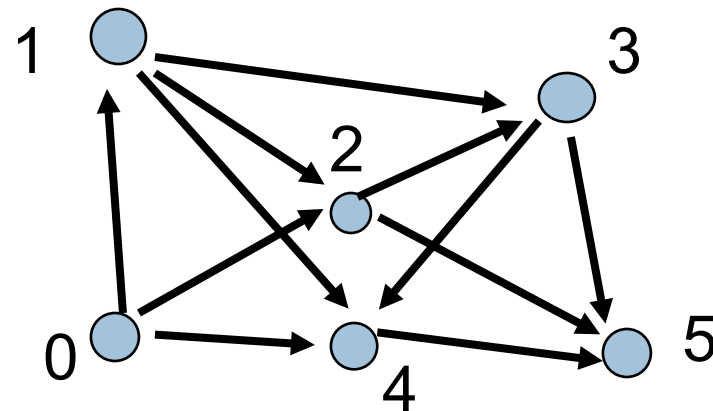
- Graph Coloring
- Planarity
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Topological Sort

18

□ Topological sort of the dag

This is a numbering of the vertices such that all edges go from lower- to higher-numbered vertices



□ Useful in job scheduling with precedence constraints

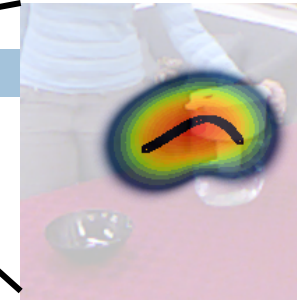
Tell Me Dave: Making 'Avoffato'



Input:



Output: Inferred



Environment:



NL Instruction:

Place the pot on the tap and turn the tap on. When it is filled, turn the tap off and heat the pot.

Place the
pot on the
tap

Turn the
tap on

Turn the
tap off

Heat the
pot

```
moveTo (pot1)  
grasp (pot1)
```

```
moveTo (sink1)  
keep (pot1, sink1, on)
```

```
<no-op>
```

```
keep (pot2, sink2, on)  
toggle (sink1knob1, on)  
wait ()
```

```
<no-op>
```

```
toggle (sink1knob1, off)
```

```
moveTo (pot1)  
grasp (pot1)  
moveTo (stove1)
```

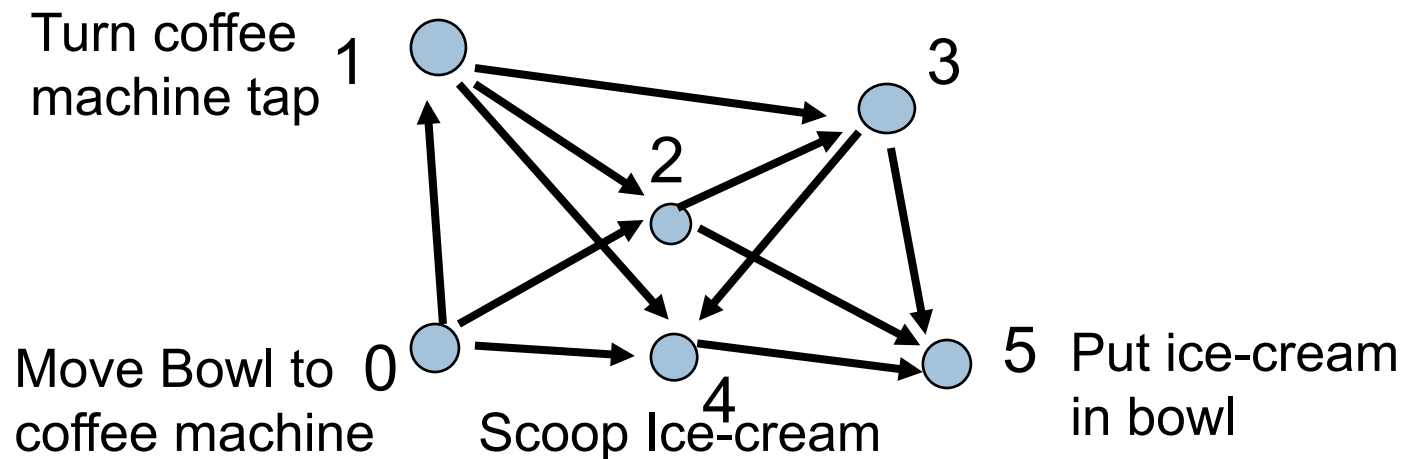
```
keep (pot1, burner1, on)  
toggle (stove1knob1, on)  
wait ()  
toggle (stove1knob1, off)
```

Topological Sort

21

□ Topological sort of the dag

This is a numbering of the vertices such that all edges go from lower- to higher-numbered vertices



□ Useful in job scheduling with precedence constraints

Graph Concepts and Algorithms

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AI: Vision and Robotics

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- Number of cell-phones in 2014: 7 billion!
- So many images everywhere.
- But do computers understand the images?

Without Graph Algorithms



Anticipating Future Actions



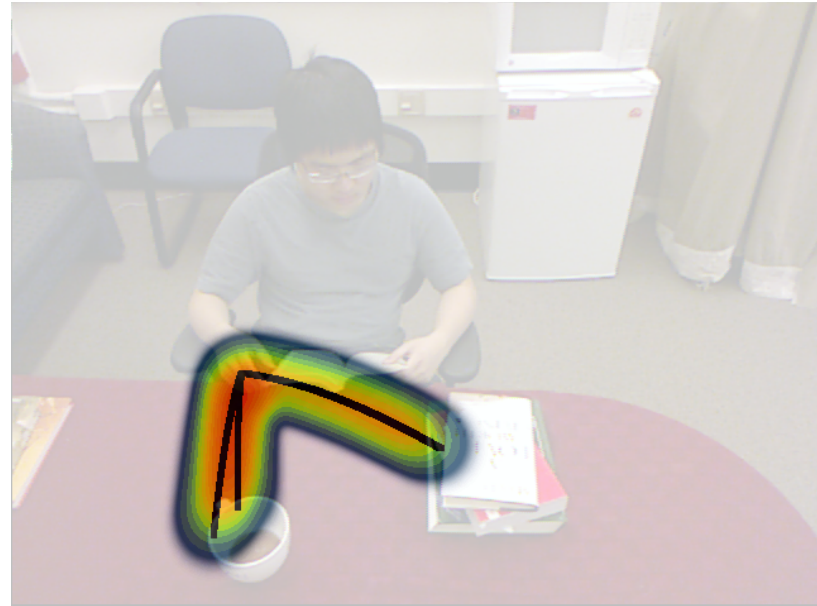
Robot's view

Future anticipations

Anticipating Future Actions



Robot's view



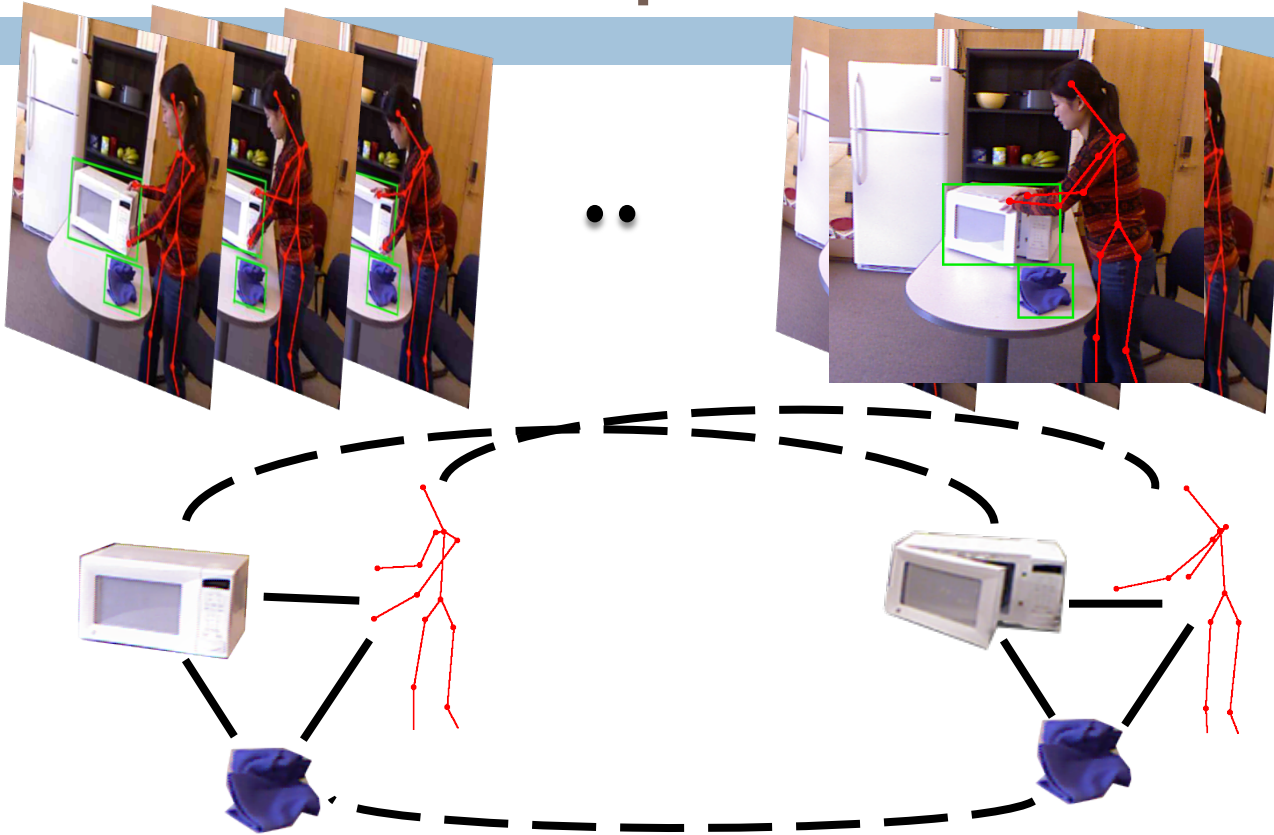
Future anticipations

- Humans use anticipation all the time
 - ▣ e.g., interacting with other people, playing sports, driving, etc.

Modeling the Activities with an Undirected Graph



Modeling the Activities with an Undirected Graph



$$P(\mathcal{O}, \mathcal{A} | \mathcal{H}, \mathcal{L})$$

Undirected Graphs: Probabilities on the Graphs

Robot Sees the Future!



Studio B, with
Shepherd Smith



With Lewis Black,
Comedy Central

Graph Concepts and Algorithms

30

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Graphs in Practice

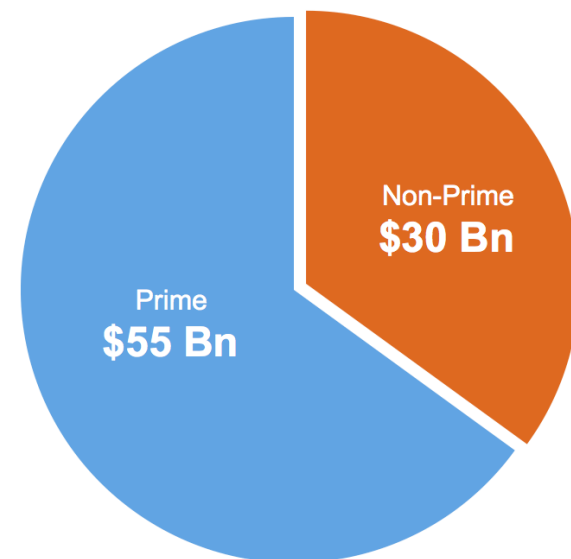
31

- In 2012, we started a company called Cognical.

- Problem:

65 million US consumers cannot finance electronics, furniture, appliances and other durable goods online

Online Market for Target Products



1 in 3 US consumers cannot finance online purchases



65 million US consumers

Have low FICO scores

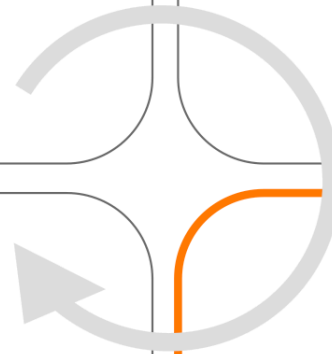
but are broadly creditworthy

Buy at local stores

with cash or finance via Lease to Own (LTO)

Browse Online

but lack a payment option to buy

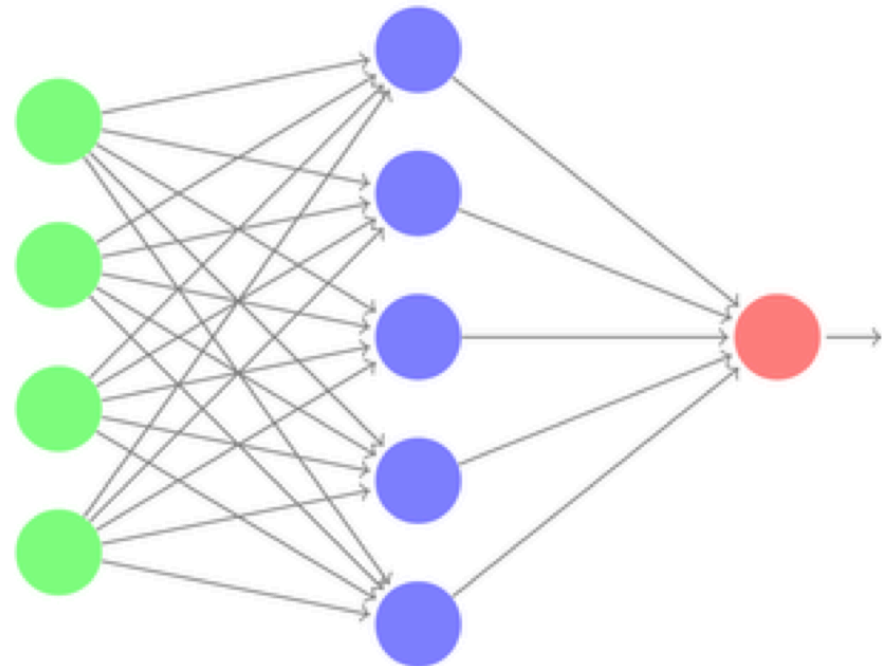




COGNICAL

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- Cognical collects data about customers from various sources.
- Represents over a graph, and predicts how to finance.





COGNICAL

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1 Q1 2014: Launch



Electronics



Furniture



Appliances



Instruments

2 2015: Market Penetration, Category Expansion

\$30 Bn

Target market

3 2016: Brick & Mortar Expansion

\$96 Bn

Graphs!

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