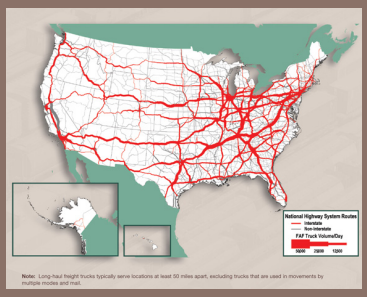
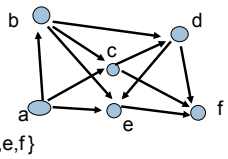


GRAPHS IV: APPLICATIONS

Lecture 22
CS2110 – Spring 2014

Example Directed Graph (Digraph)



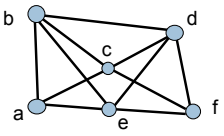
V = {a,b,c,d,e,f}
E = {(a,b), (a,c), (a,e), (b,c), (b,d), (b,e), (c,d), (c,f), (d,e), (d,f), (e,f)}

|V| = 6, |E| = 11

Example *Undirected* Graph

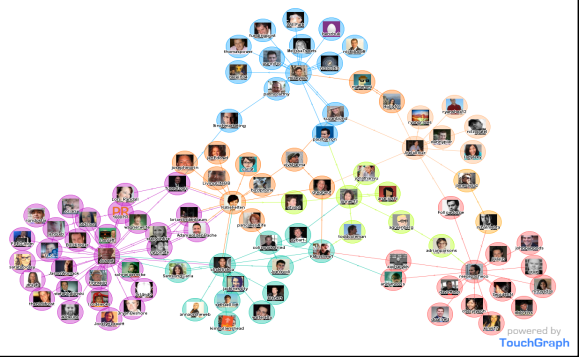
An *undirected graph* is just like a directed graph, except the edges are *unordered pairs* (*sets*) {u,v}

Example:



V = {a,b,c,d,e,f}
E = {{a,b}, {a,c}, {a,e}, {b,c}, {b,d}, {b,e}, {c,d}, {c,f}, {d,e}, {d,f}, {e,f}}

Social Network Graph



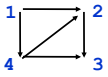
Graph Concepts and Algorithms

- Adjacency Matrix
- Topological Sort
- Search
  - depth-first search
  - breadth-first search
- Shortest paths
  - Dijkstra's algorithm
- Minimum spanning trees
  - Prim's algorithm
  - Kruskal's algorithm

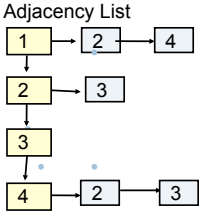
Others:

- Graph Coloring
- Planarity
- Traveling Salesman problem.

Graph Adjacency Matrix



Adjacency List

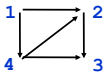


Adjacency Matrix

	1	2	3	4
1	0	1	0	1
2	0	0	1	0
3	0	0	0	0
4	0	1	1	0

Graph Adjacency Matrix

7



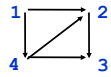
Adjacency Matrix

Squaring adjacency matrix gives number of possible paths of length 2!

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}^{2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Graph Adjacency Matrix

8



Adjacency Matrix

Cubing adjacency matrix gives number of possible paths of length 3!

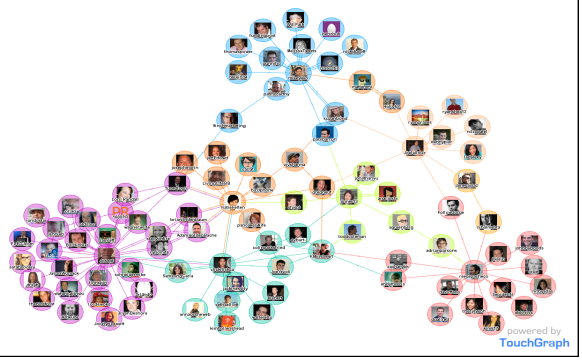
$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}^{3} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Graph Adjacency Matrix

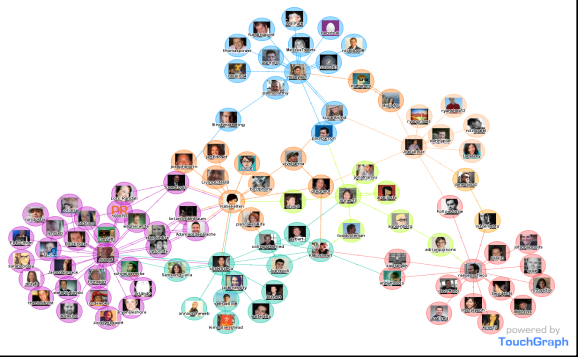
9

Same idea holds for undirected graphs.

How would you find friends of friends?



Find the number of common friends?



Graph Concepts and Algorithms

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- ✓ Adjacency Matrix
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- Search
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  - breadth-first search
- Shortest paths
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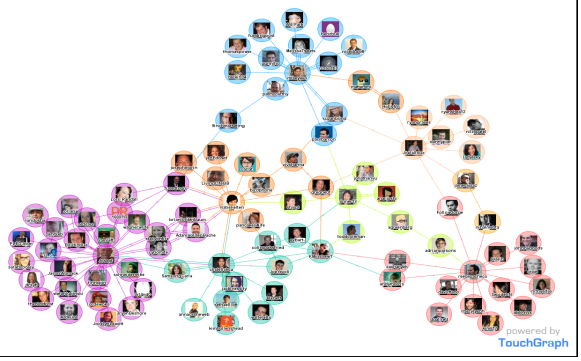
- Minimum spanning trees
  - Prim's algorithm
  - Kruskal's algorithm

Others:

- Graph Coloring
- Planarity
- Traveling Salesman problem.

Search if a person is someone's Friend?



Graph Concepts and Algorithms

14

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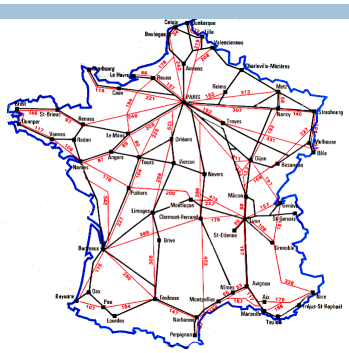
Shortest Path Algorithm: Used Everyday!

15

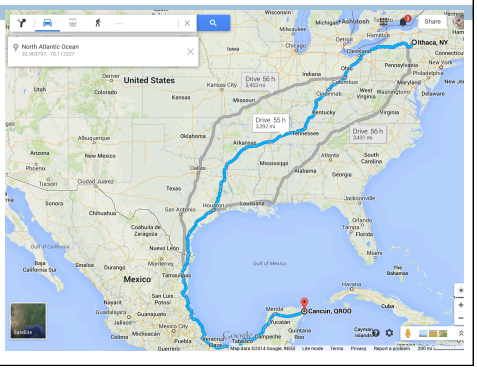
Train Network in France.



Shortest Path Algorithm: Used Everyday!

16

Maps! GPS directions!



Graph Concepts and Algorithms

17

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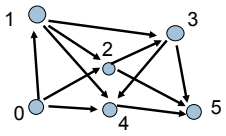
Topological Sort

18

- Topological sort of the dag

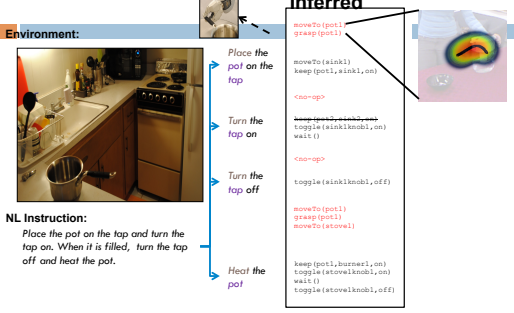
  This is a numbering of the vertices such that all edges go from lower- to higher-numbered vertices



- Useful in job scheduling with precedence constraints

Tell Me Dave: Making 'Avoffato'

Input: Output: Inferred Environment:

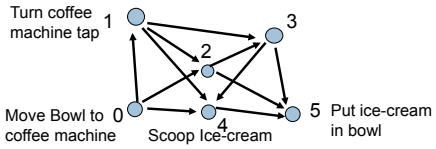


Topological Sort

21

- Topological sort of the dag

  This is a numbering of the vertices such that all edges go from lower- to higher-numbered vertices



- Useful in job scheduling with precedence constraints

Graph Concepts and Algorithms

22

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Others:

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AI: Vision and Robotics

23

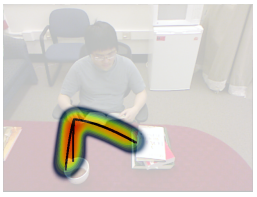
- Number of cell-phones in 2014: 7 billion!
- So many images everywhere.
- But do computers understand the images?

Without Graph Algorithms

Anticipating Future Actions

Anticipating Future Actions

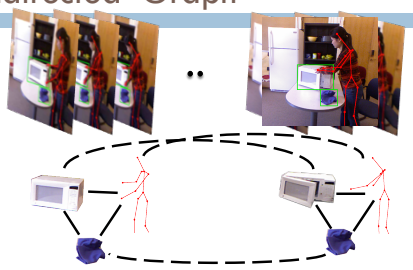

Robot's view


Future anticipations

- Humans use anticipation all the time
  - e.g., interacting with other people, playing sports, driving, etc.

Modeling the Activities with an Undirected Graph

Modeling the Activities with an Undirected Graph



$P(\mathcal{O}, \mathcal{A}|\mathcal{H}, \mathcal{L})$

Undirected Graphs: Probabilities on the Graphs

Robot Sees the Future!

Studio B, with Shepherd Smith

With Lewis Black, Comedy Central

Graph Concepts and Algorithms

30

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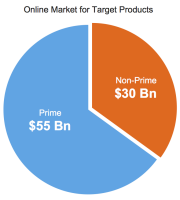
Graphs in Practice

31

- In 2012, we started a company called Cognical.
- Problem:

65 million US consumers cannot finance electronics, furniture, appliances and other durable goods online



1 in 3 US consumers cannot finance online purchases

COGNICAL

65 million US consumers

Have low FICO scores

but are broadly creditworthy

Buy at local stores

with cash or finance via Lease to Own (LTO)

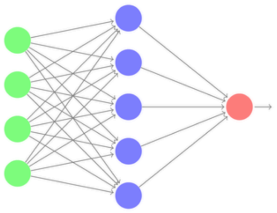
Browse Online

but lack a payment option to buy

COGNICAL

33

- Cognical collects data about customers from various sources.
- Represents over a graph, and predicts how to finance.



COGNICAL

34

1. **Q1 2014: Launch**

2. **2015: Market Penetration, Category Expansion**

$30 Bn

Target market

3. **2016: Brick & Mortar Expansion**

$96 Bn

Graphs!

35

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