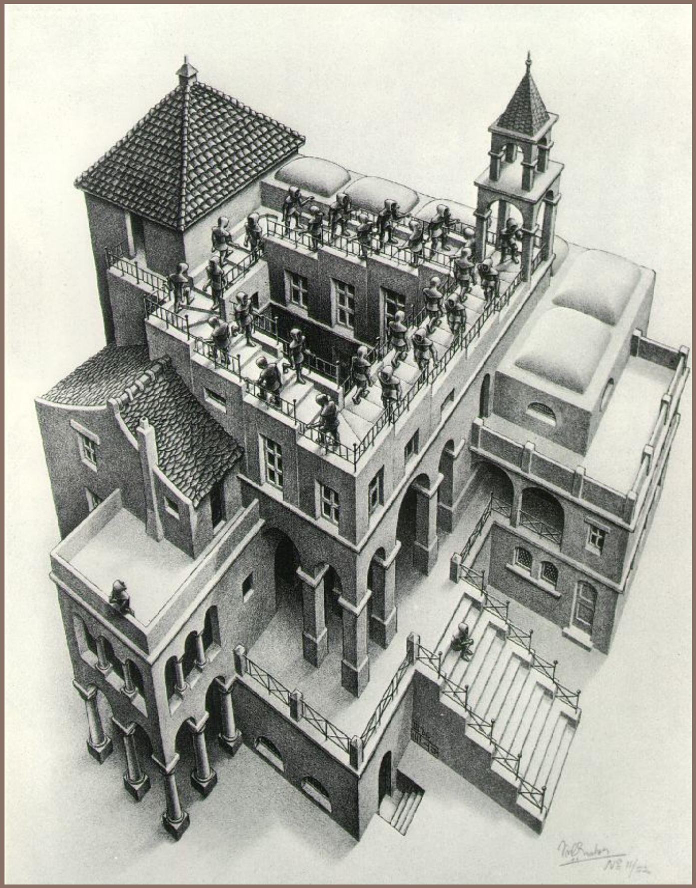


PRIORITY QUEUES AND HEAPS

Lecture 19

CS2110 Spring 2014

Readings and Homework

Read Chapter 26 to learn about heaps

Salespeople often make matrices that show all the great features of their product that the competitor's product lacks. Try this for a heap versus a BST. First, try and sell someone on a BST: List some desirable properties of a BST that a heap lacks. Now be the heap salesperson: List some good things about heaps that a BST lacks. Can you think of situations where you would favor one over the other?

With ZipUltra heaps, you've got it made in the shade my friend!

The Bag Interface

A Bag:

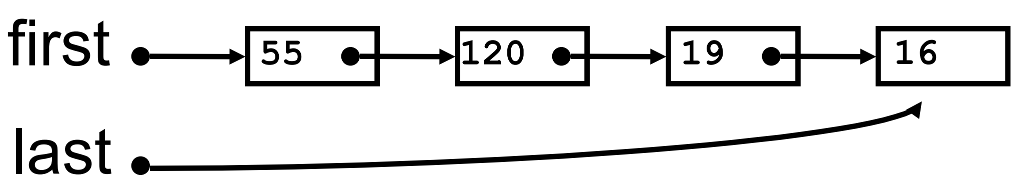
```
interface Bag<E> {
   void insert(E obj);
   E extract(); //extract some element
   boolean isEmpty();
}
```

Like a Set except that a value can be in it more than once. Example: a bag of coins

Refinements of Bag: Stack, Queue, PriorityQueue

Stacks and Queues as Lists

- Stack (LIFO) implemented as list
  - **`insert()`**, **`extract()`** from front of list
- Queue (FIFO) implemented as list
  - **`insert()`** on back of list, **`extract()`** from front of list
- All **`Bag`** operations are O(1)



Priority Queue

- A **`Bag`** in which data items are **`Comparable`**

- *lesser* elements (as determined by **`compareTo()`**) have *higher* priority

- **`extract()`** returns the element with the highest priority = least in the **`compareTo()`** ordering

- break ties arbitrarily

Priority Queue Examples

Scheduling jobs to run on a computer
default priority = arrival time
priority can be changed by operator

Scheduling events to be processed by an event handler
priority = time of occurrence

Airline check-in
first class, business class, coach
FIFO within each class

java.util.PriorityQueue<E>


```
boolean add(E e) {...} //insert an element (insert)
void clear() {...} //remove all elements
E peek() {...} //return min element without removing
               //(null if empty)
E poll() {...} //remove min element (extract)
               //(null if empty)
int size() {...}
```

Priority Queues as Lists

- Maintain as unordered list
  - **`insert()`** put new element at front – O(1)
  - **`extract()`** must search the list – O(n)

- Maintain as ordered list
  - **`insert()`** must search the list – O(n)
  - **`extract()`** get element at front – O(1)

- In either case, $O(n^2)$ to process n elements

Can we do better?

Important Special Case

- Fixed number of priority levels 0,...,p – 1
- FIFO within each level
- Example: airline check-in

- **`insert()`** – insert in appropriate queue – O(1)
- **`extract()`** – must find a nonempty queue – O(p)

Heaps

- A *heap* is a concrete data structure that can be used to implement priority queues
- Gives better complexity than either ordered or unordered list implementation:
  - **`insert()`**: O(log n)
  - **`extract()`**: O(log n)
- O(n log n) to process n elements
- Do not confuse with *heap memory*, where the Java virtual machine allocates space for objects – different usage of the word *heap*

Heaps

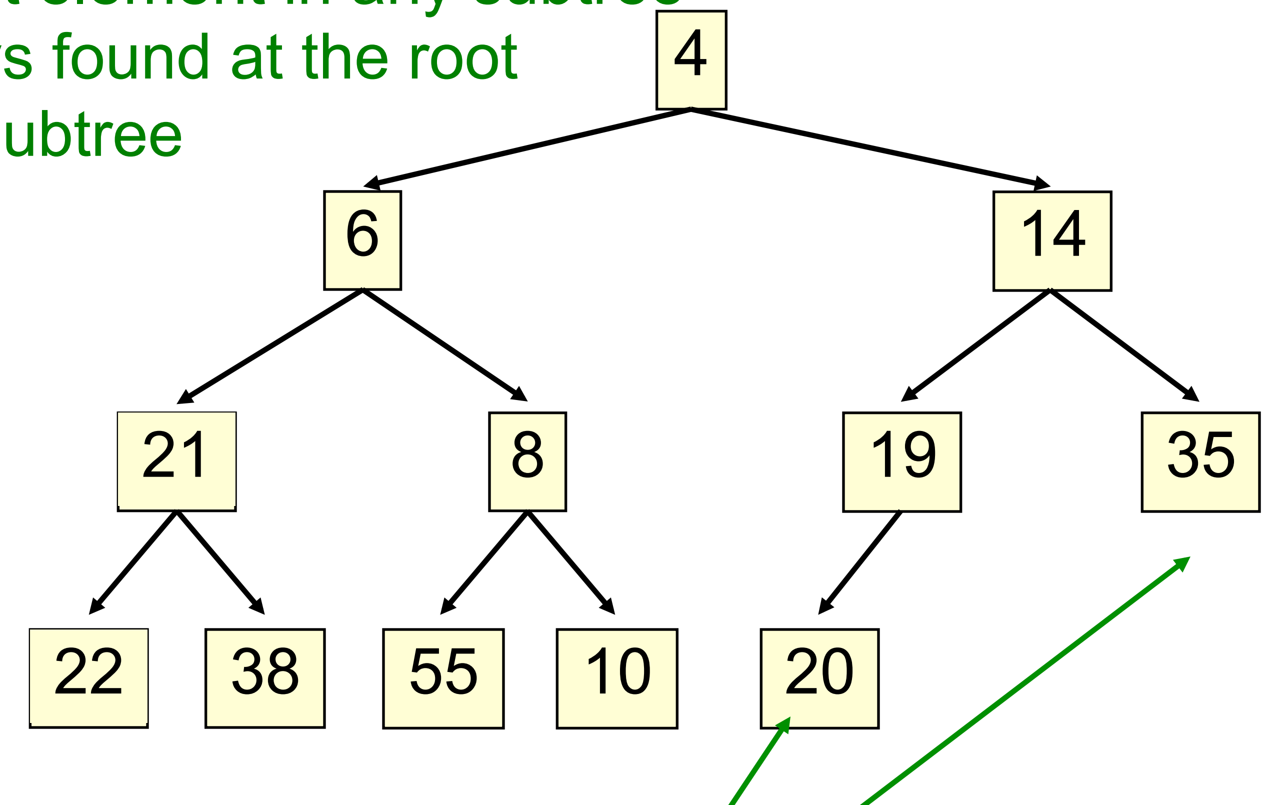
- Binary tree with data at each node
- Satisfies the *Heap Order Invariant*:

> The least (highest priority) element of any subtree is found at the root of that subtree

- Size of the heap is “fixed” at *n*. (But can usually double n if heap fills up)

Heaps

Smallest element in any subtree
is always found at the root
of that subtree



Note: 19, 20 < 35: Smaller elements
can be deeper in the tree!

Examples of Heaps

- Ages of people in family tree
  - parent is always older than children, but you can have an uncle who is younger than you
- Salaries of employees of a company
  - bosses generally make more than subordinates, but a VP in one subdivision may make less than a Project Supervisor in a different subdivision

Balanced Heaps

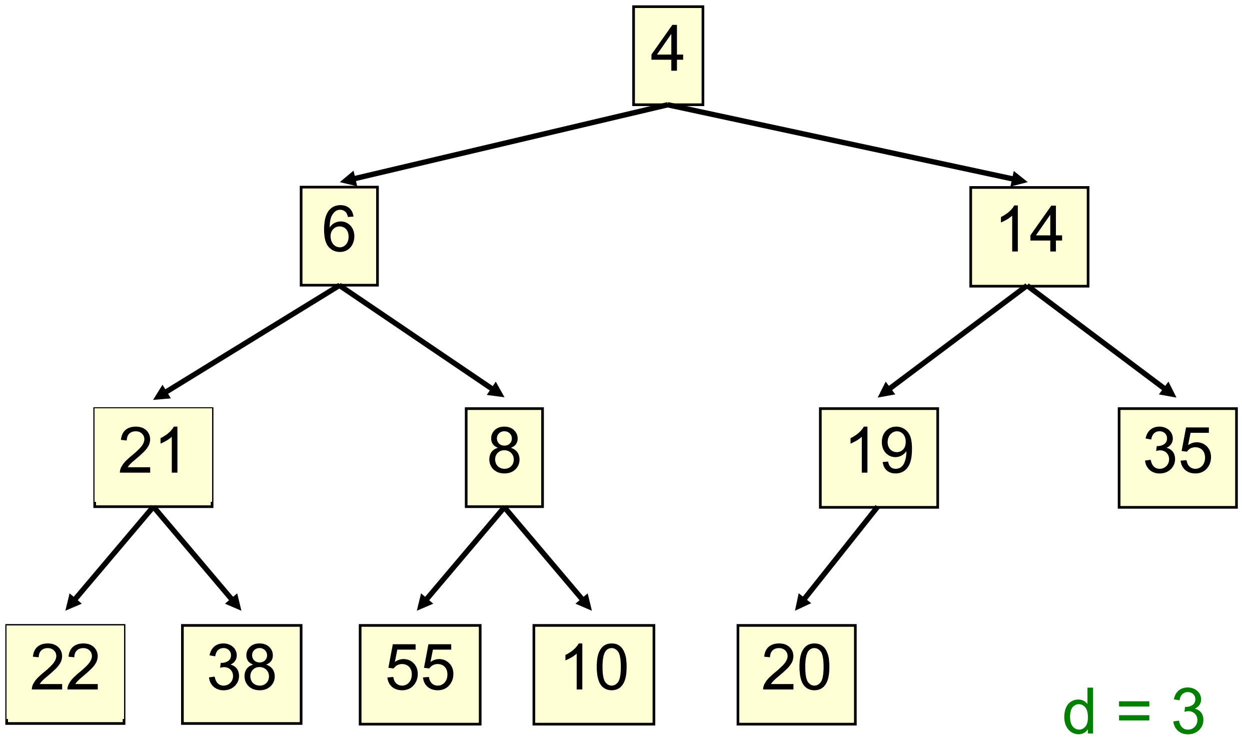
These add two restrictions:

1. Any node of depth < d – 1 has exactly 2 children, where d is the height of the tree

– implies that any two maximal paths (path from a root to a leaf) are of length d or d – 1, and the tree has at least 2^d nodes

- All maximal paths of length d are to the left of those of length d – 1

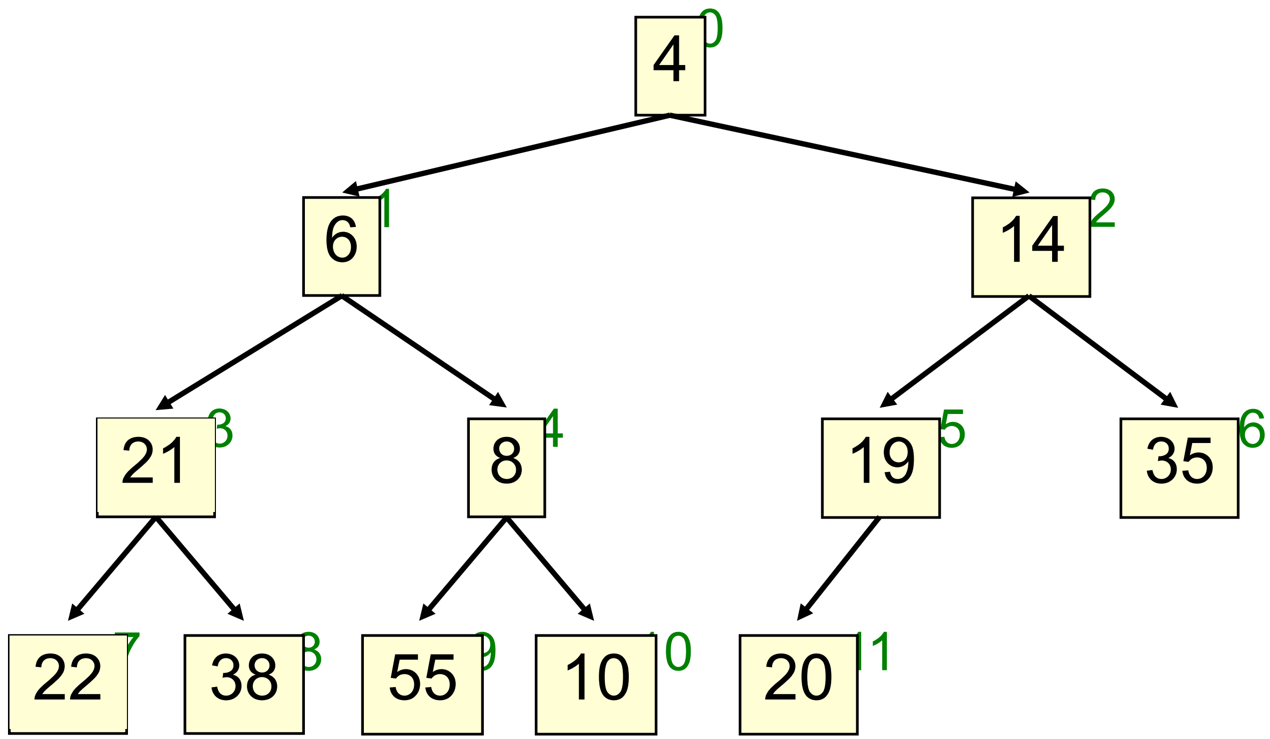
Example of a Balanced Heap



Store in an ArrayList or Vector

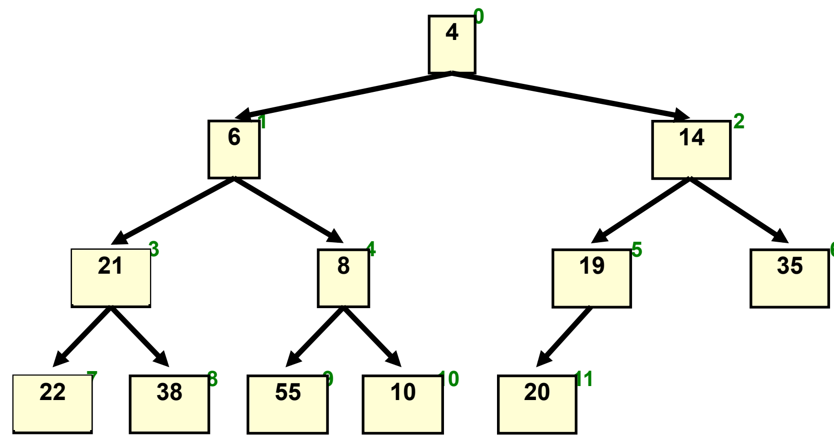
- Elements of the heap are stored in the array in order, going across each level from left to right, top to bottom
- The children of the node at array index n are at indices 2n + 1 and 2n + 2
- The parent of node n is node (n – 1)/2

Store in an ArrayList or Vector



children of node n are found at 2n + 1 and 2n + 2

Store in an ArrayList or Vector



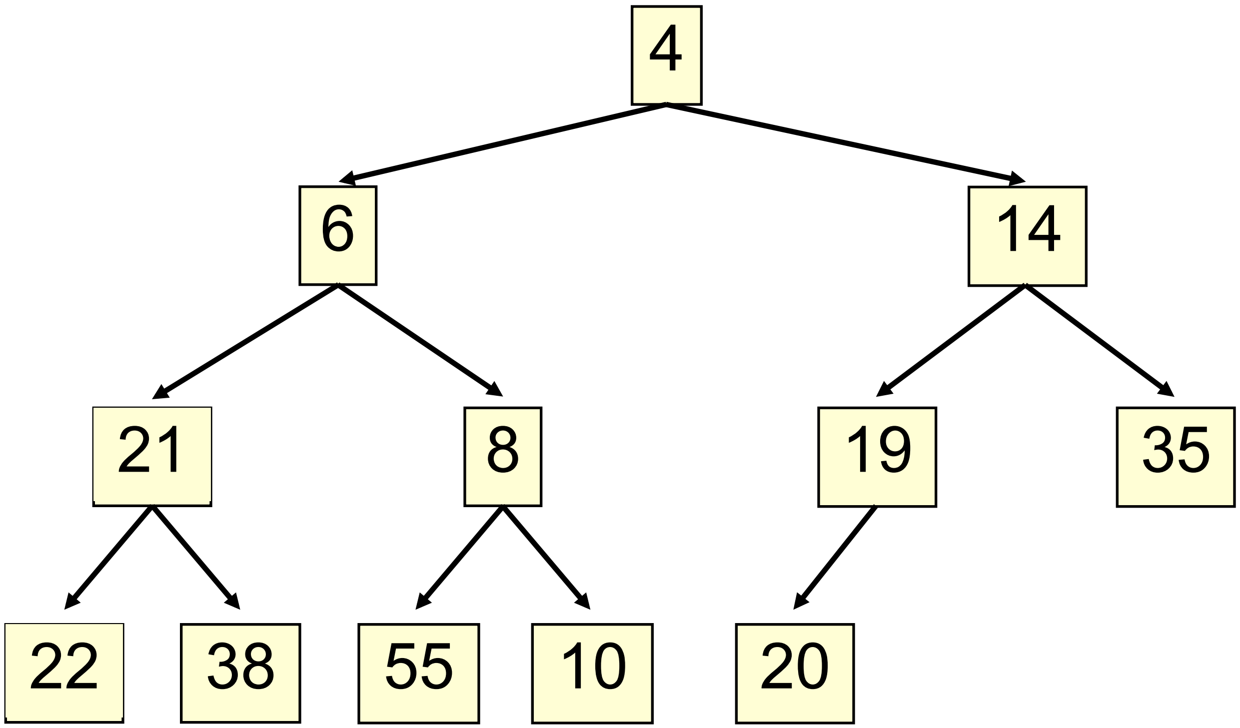
0	1	2	3	4	5	6	7	8	9	10	11
4	6	14	21	8	19	35	22	38	55	10	20

children of node n are found at 2n + 1 and 2n + 2

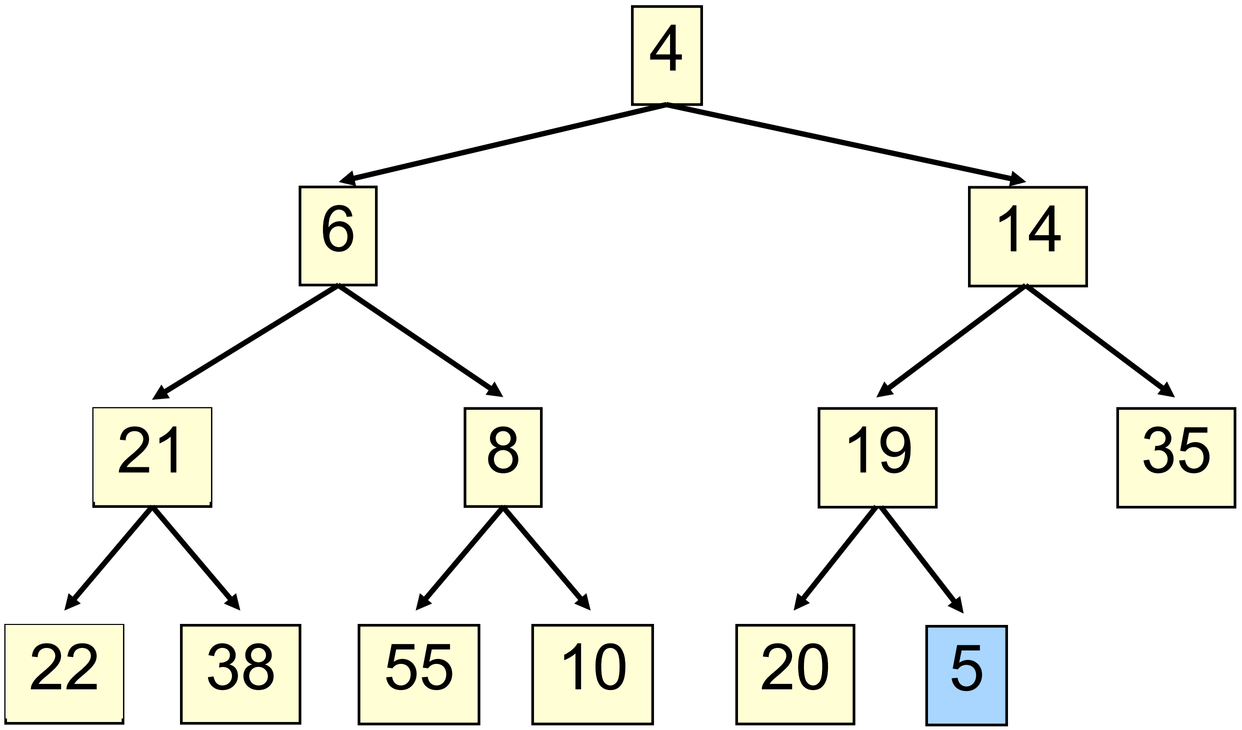
`insert()`

- Put the new element at the end of the array
- If this violates heap order because it is smaller than its parent, swap it with its parent
- Continue swapping it up until it finds its rightful place
- The heap invariant is maintained!

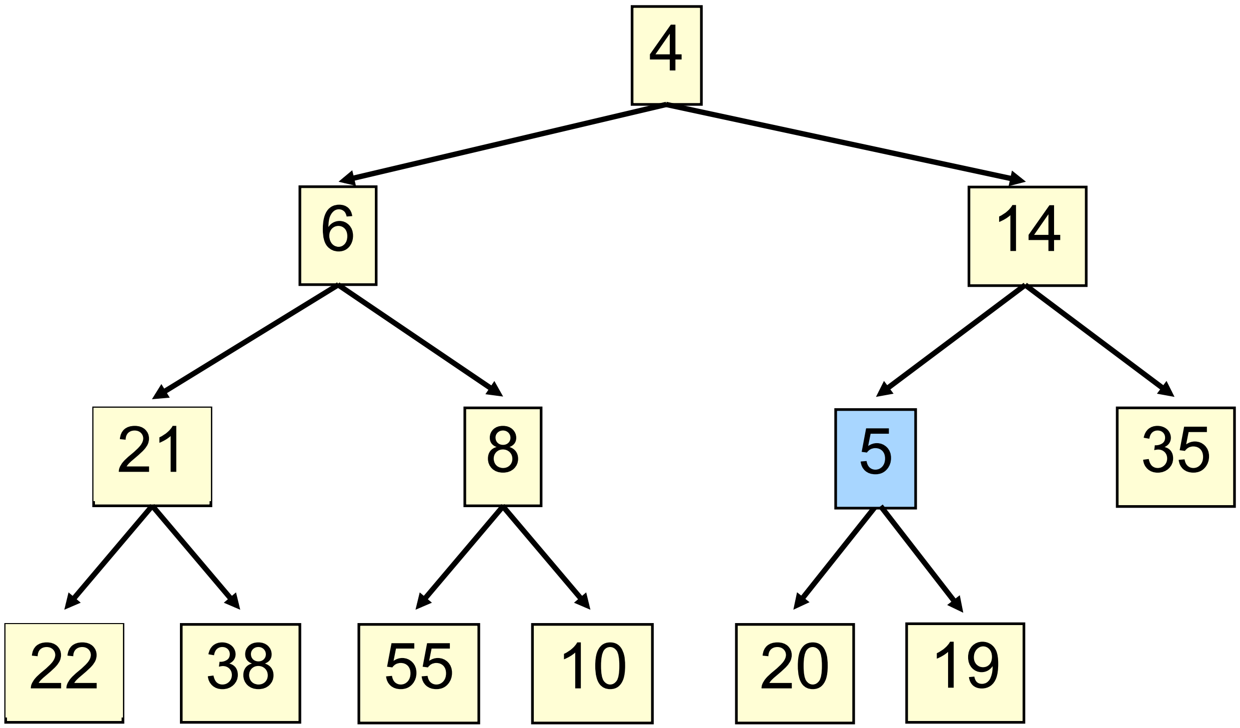
`insert()`



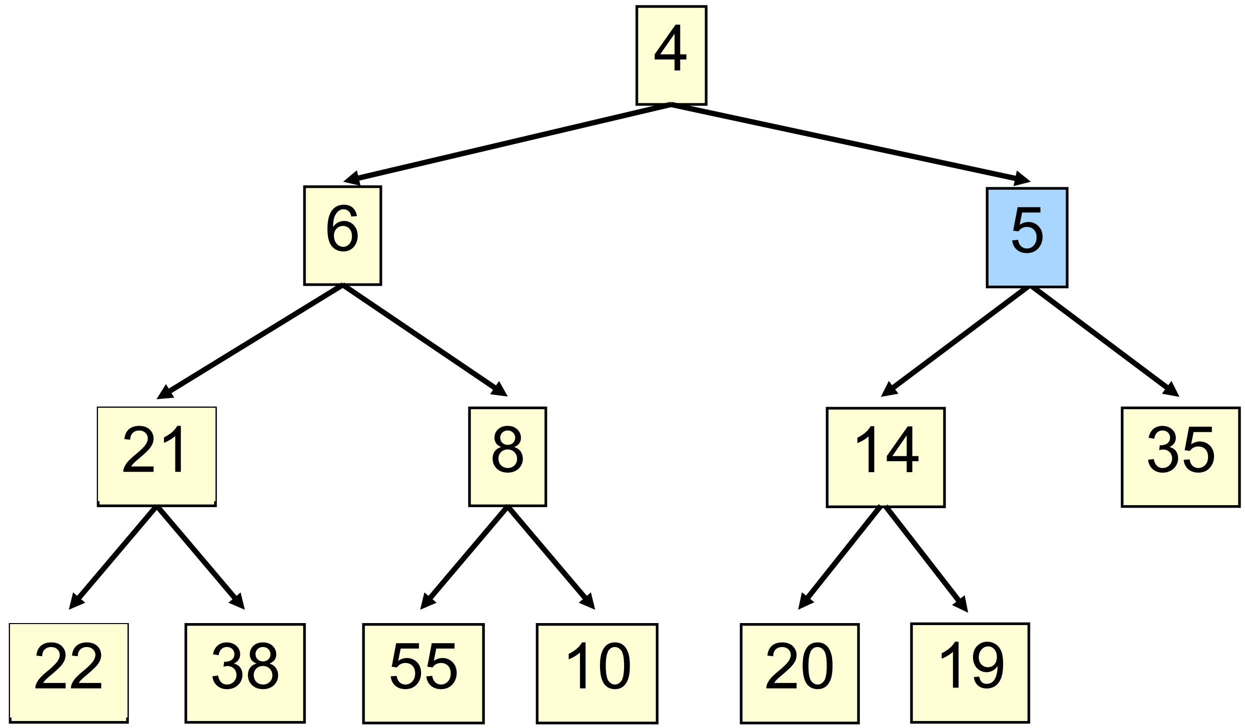
insert()



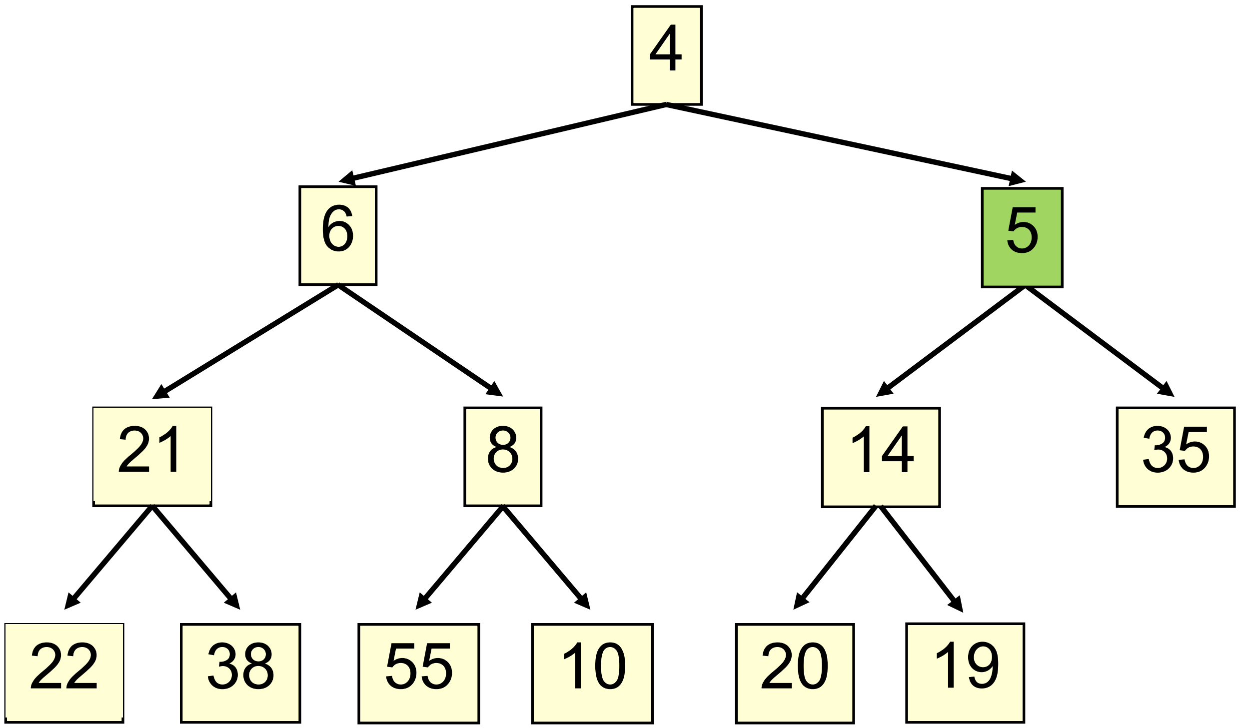
`insert()`



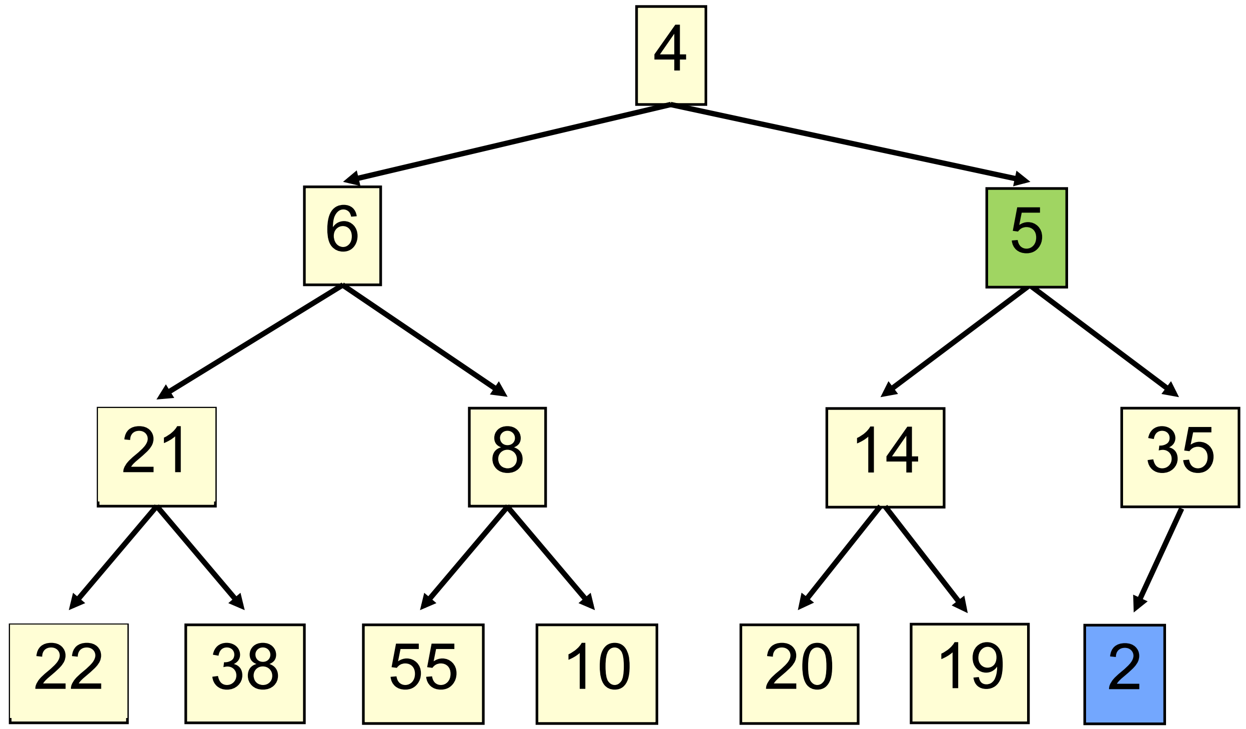
`insert()`



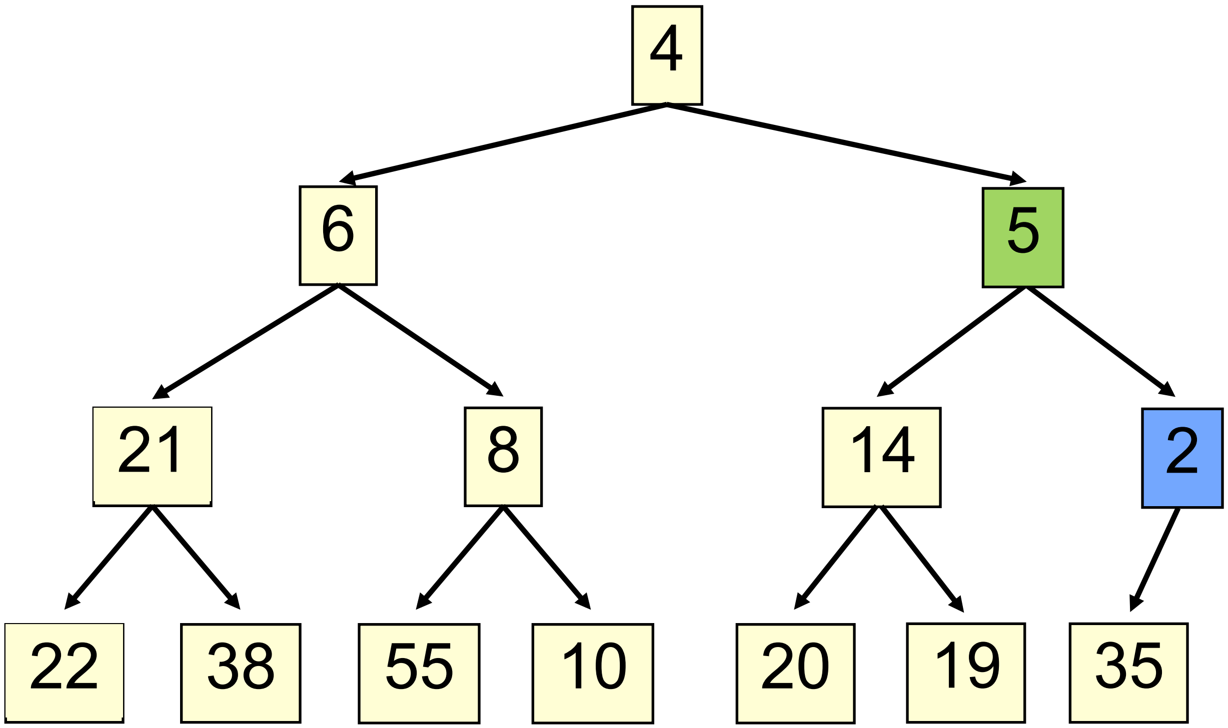
insert()



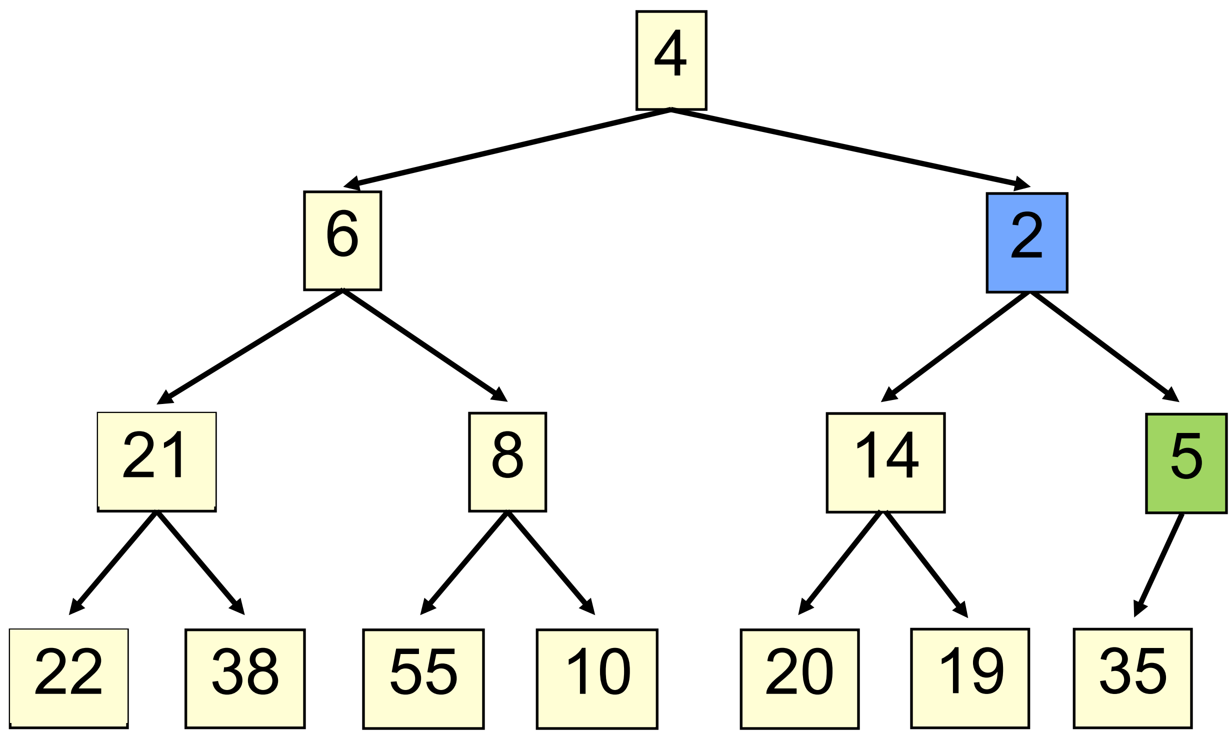
`insert()`



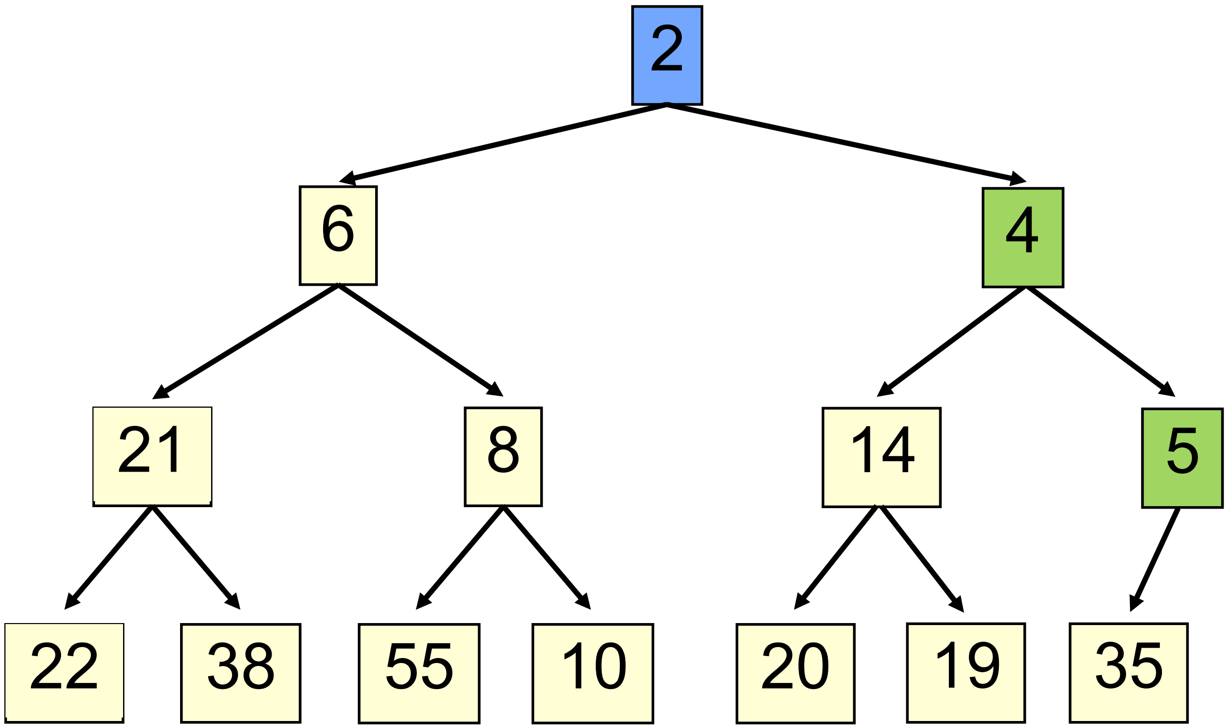
insert()



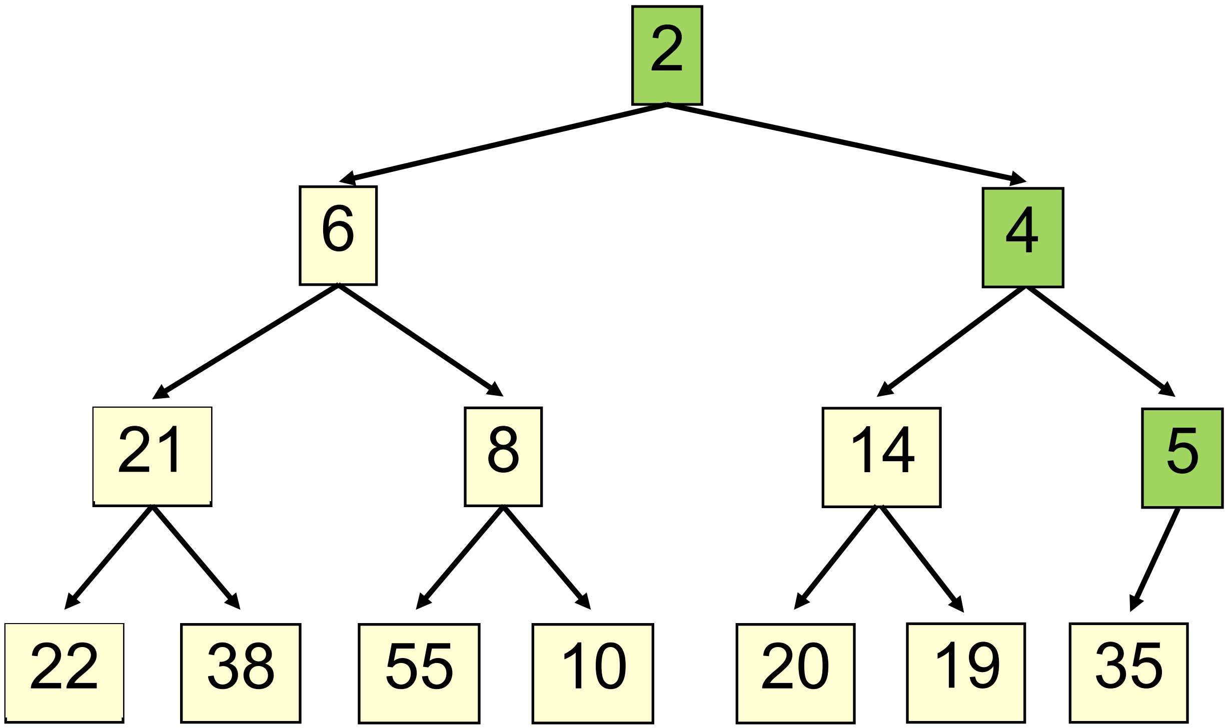
insert()



insert()



insert()



`insert()`

- Time is O(log n), since the tree is balanced
  - size of tree is exponential as a function of depth
  - depth of tree is logarithmic as a function of size

insert()


```
/** An instance of a priority queue */
class PriorityQueue<E> extends java.util.Vector<E> {

  /** Insert e into the priority queue */
  public void insert(E e) {
    super.add(e); //add to end of array
    bubbleUp(size() - 1); // given on next slide
  }
```

`insert()`

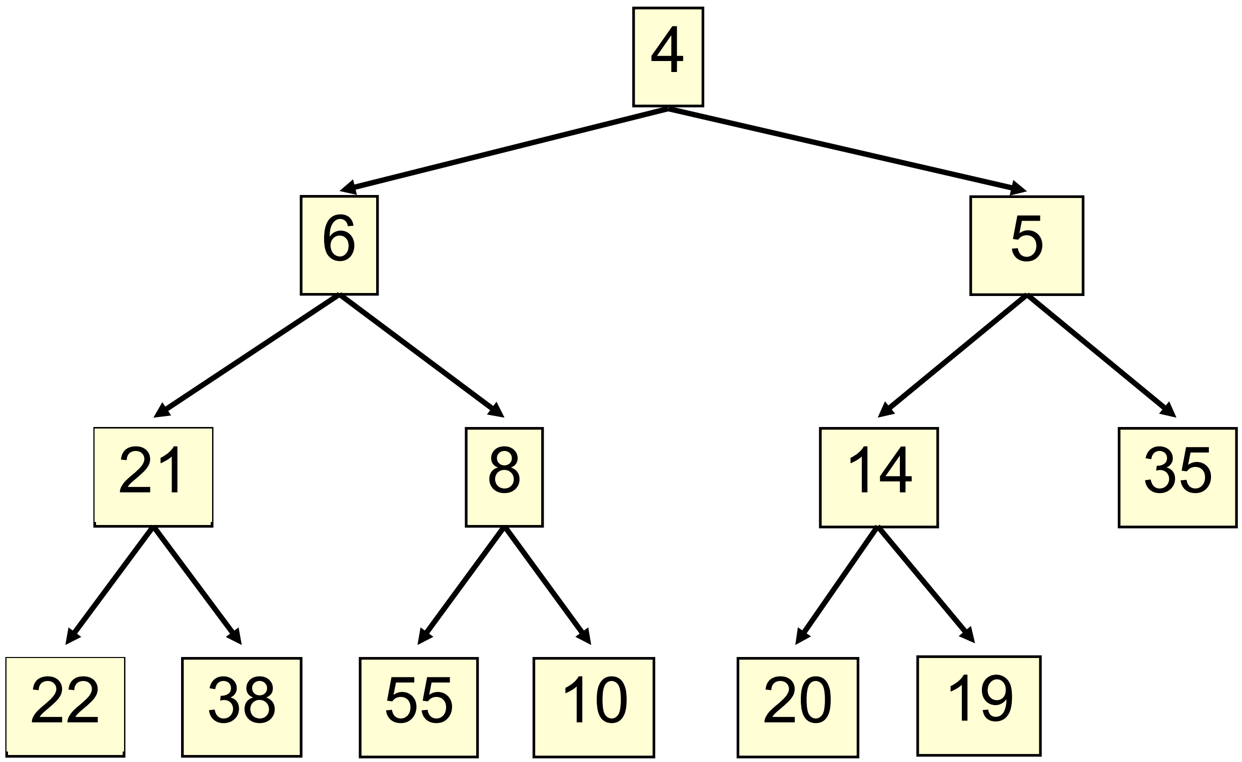

```
class PriorityQueue<E> extends java.util.Vector<E> {

 /** Bubble element k up the tree */
 private void bubbleUp (int k) {
    int p= (k-1)/2;   // p is the parent of k
    // inv: Every element satisfies the heap property except
    //         element k might be smaller than its parent
    while (k > 0   &&   get(k).compareTo(get(p)) < 0) {
        swap elements k and p;
        k= p;
        p= (k-1)/2;
    }
}
```

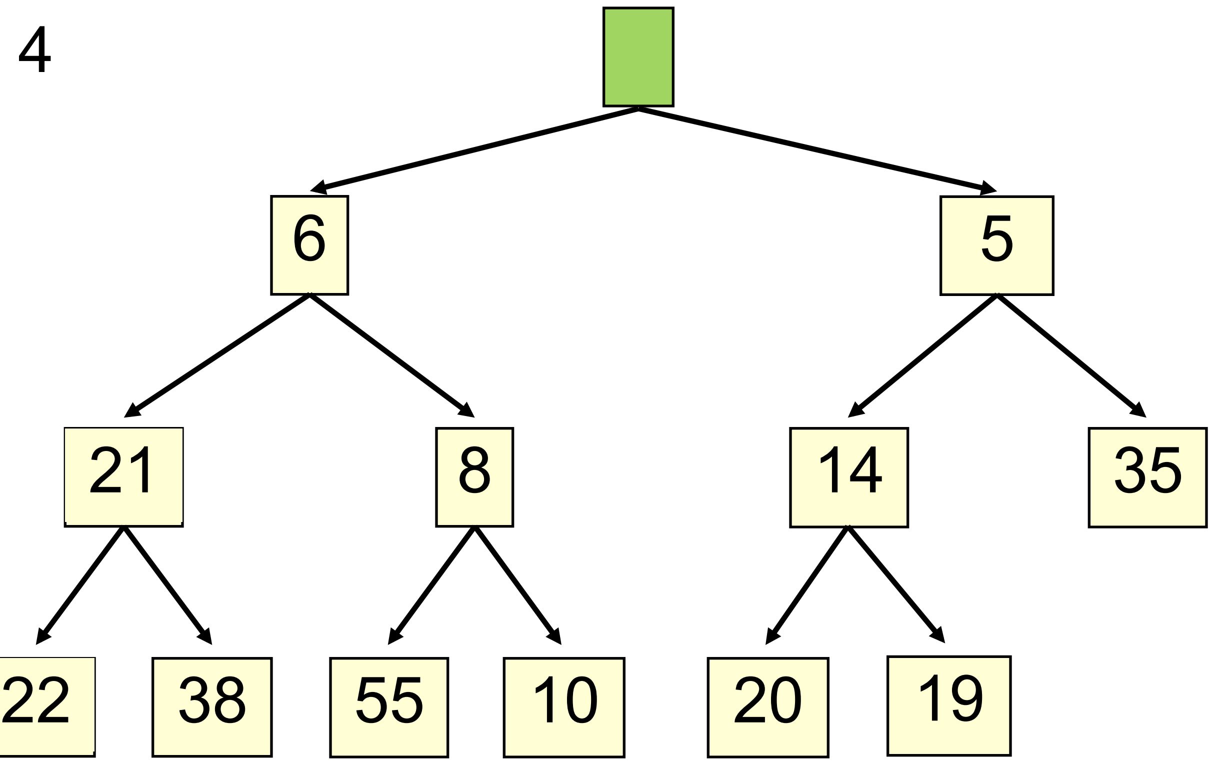
`extract()`

- Remove the least element – it is at the root
- This leaves a hole at the root – fill it in with the last element of the array
- If this violates heap order because the root element is too big, swap it down with the smaller of its children
- Continue swapping it down until it finds its rightful place
- The heap invariant is maintained!

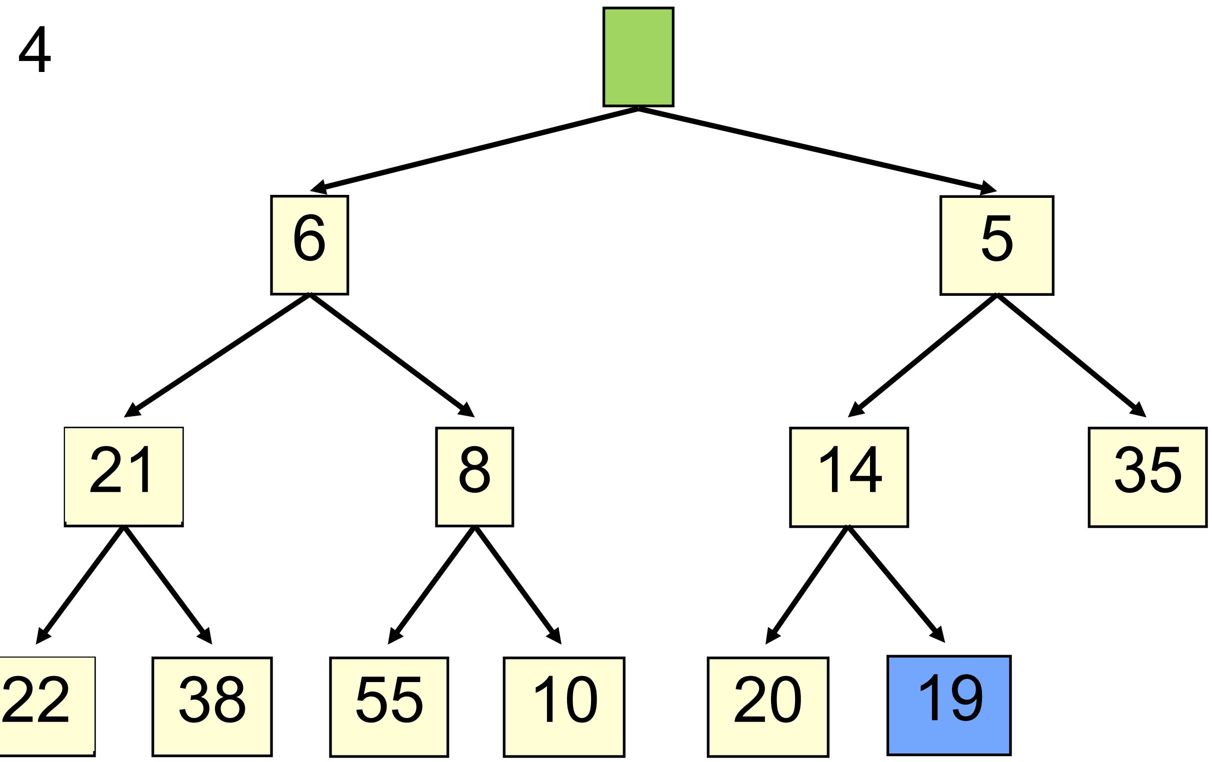
extract()



`extract()`

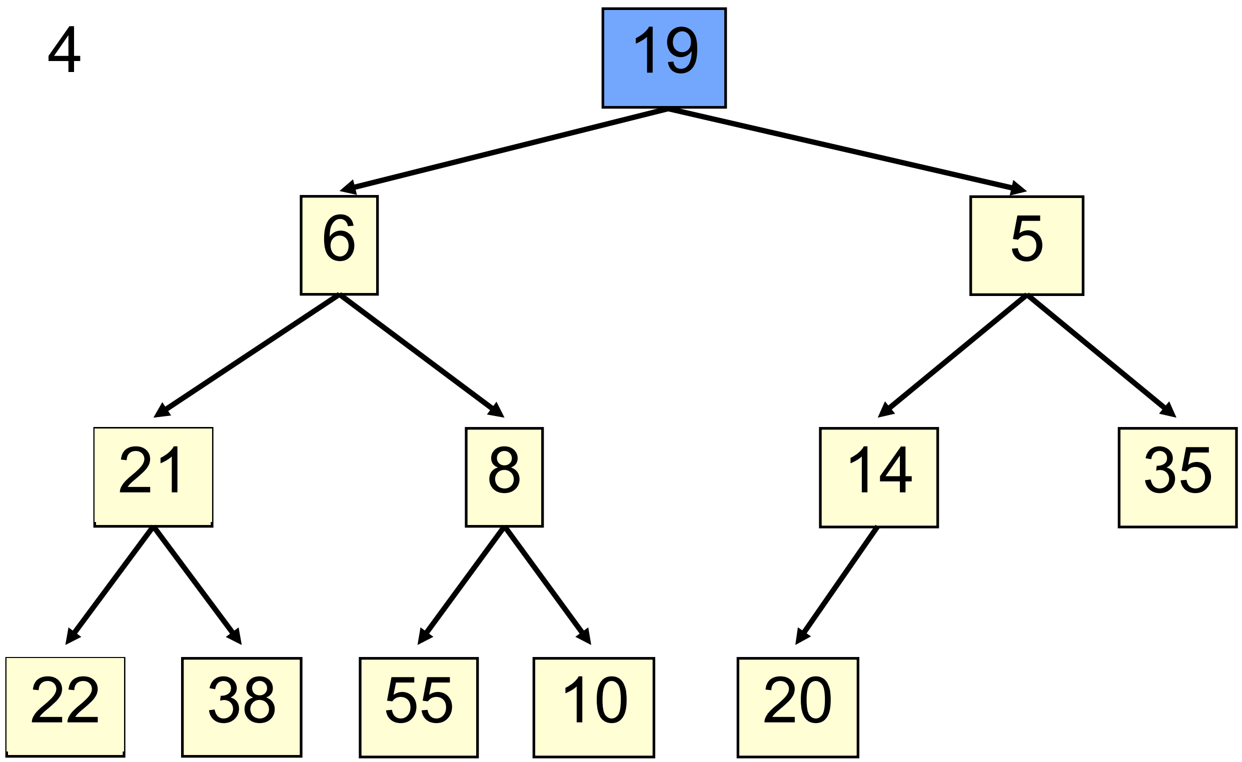


`extract()`



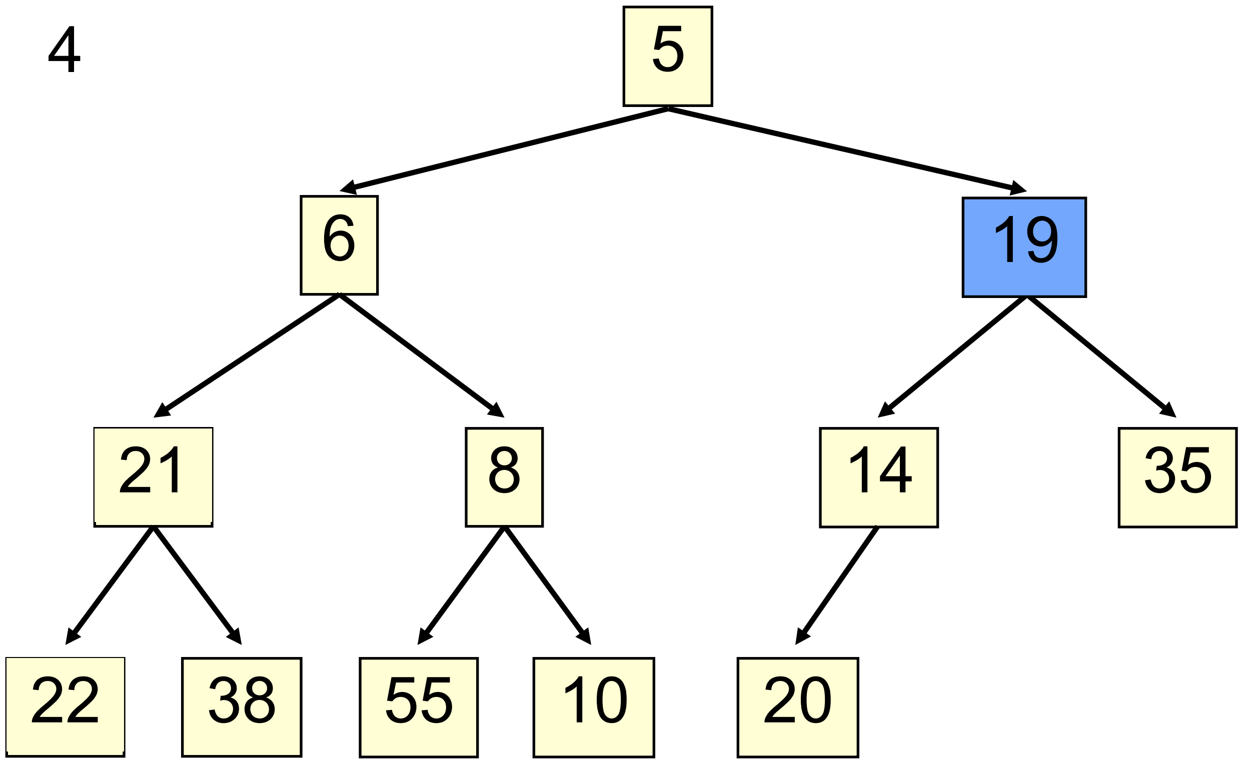
extract()

4



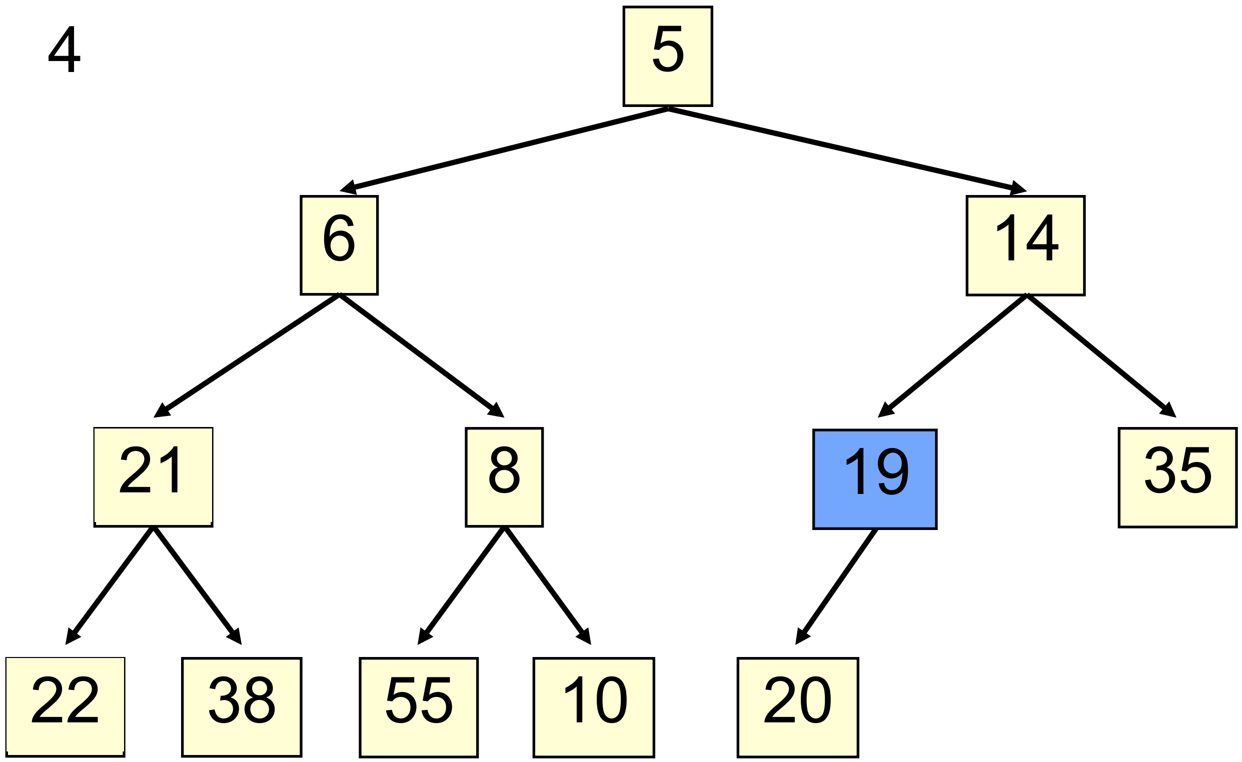
`extract()`

4

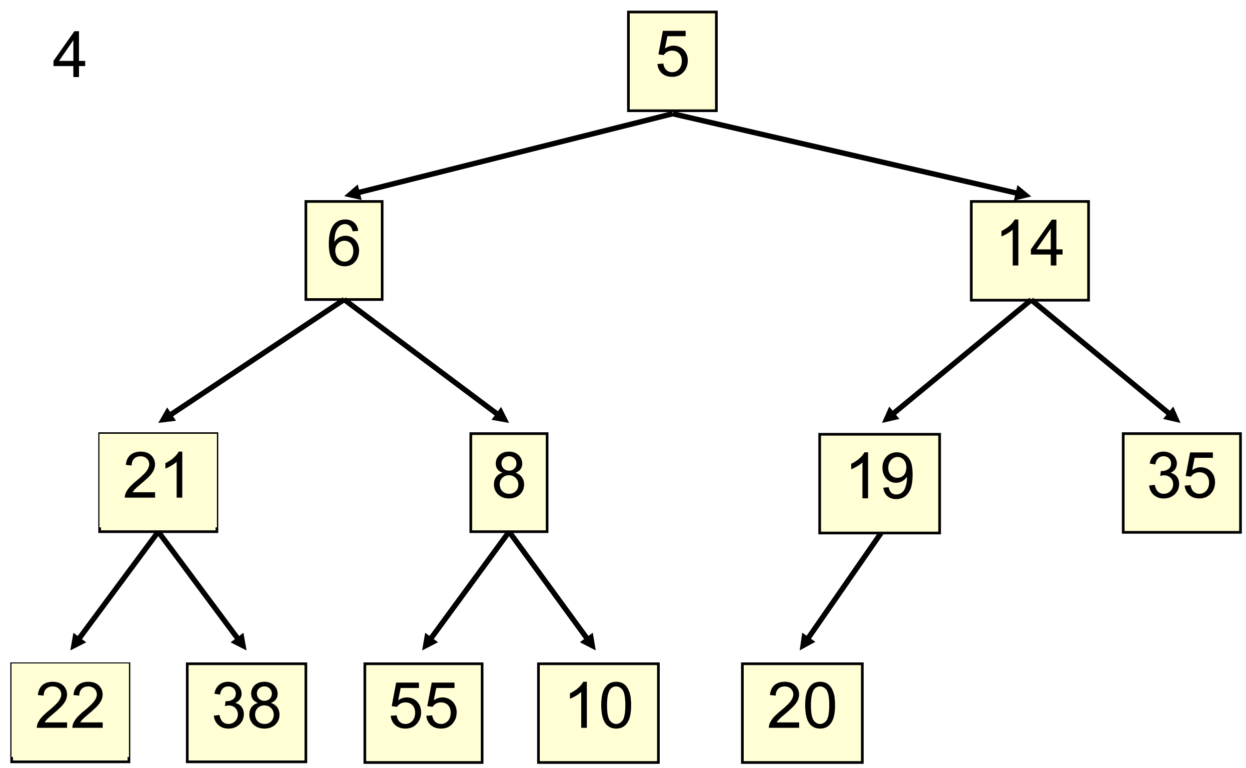


extract()

4

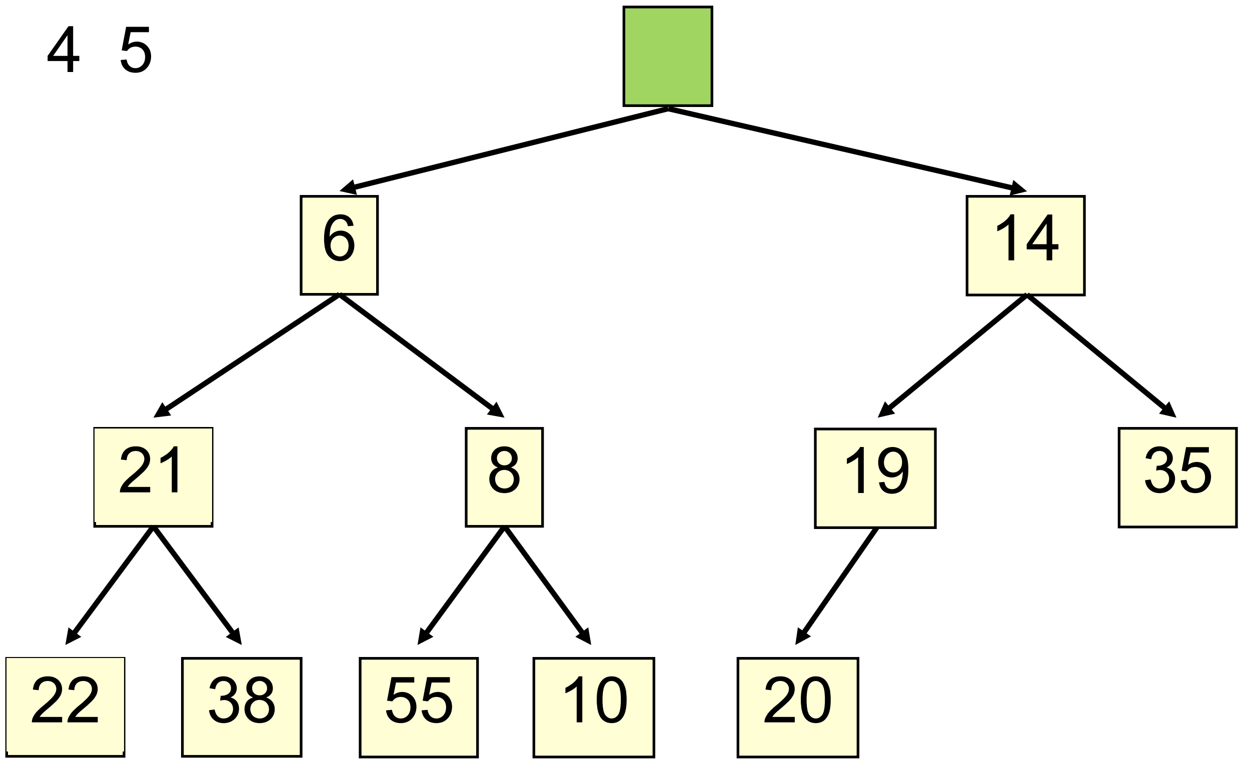


`extract()`



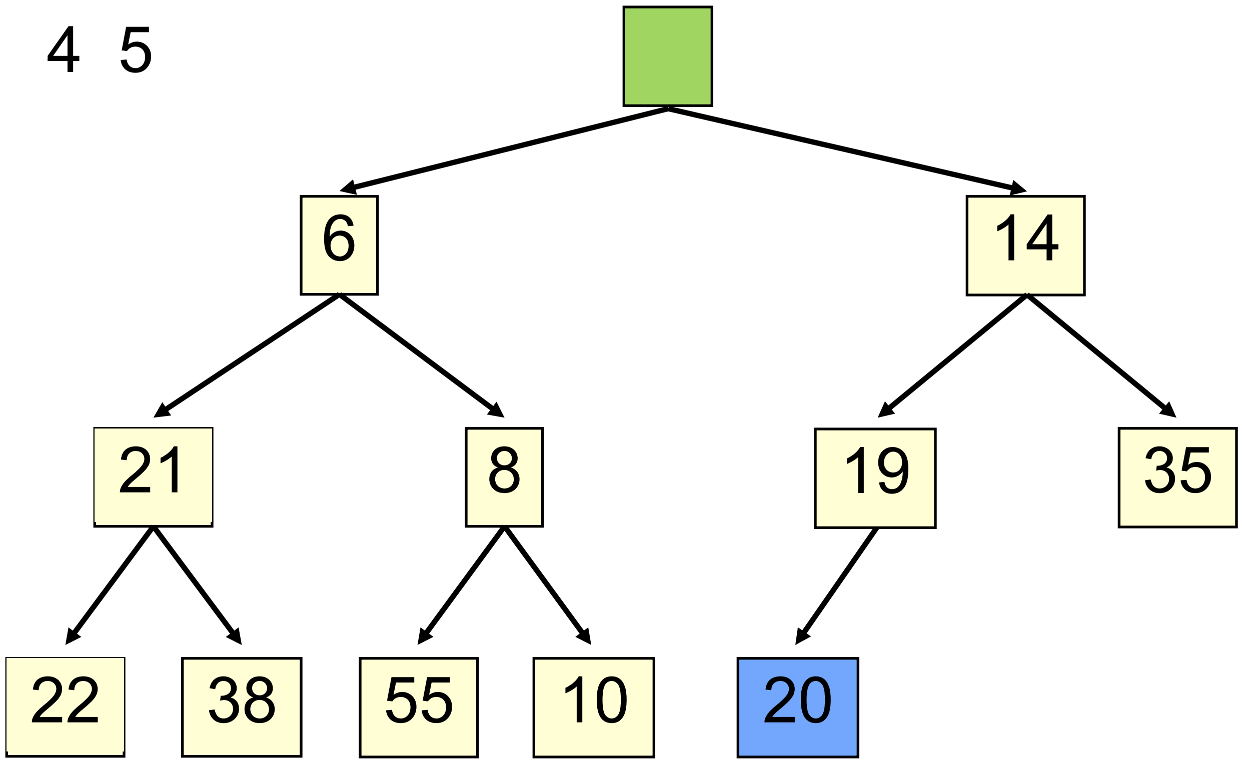
extract()

4 5



extract()

4 5



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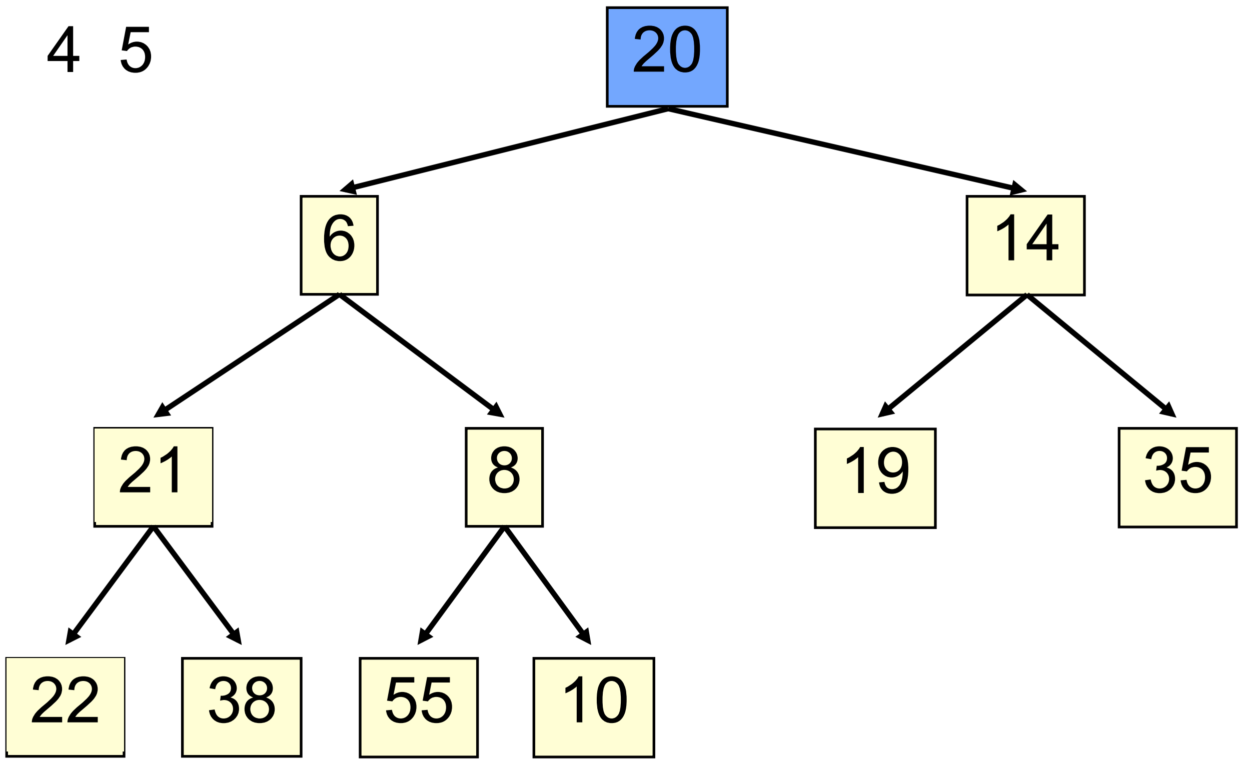
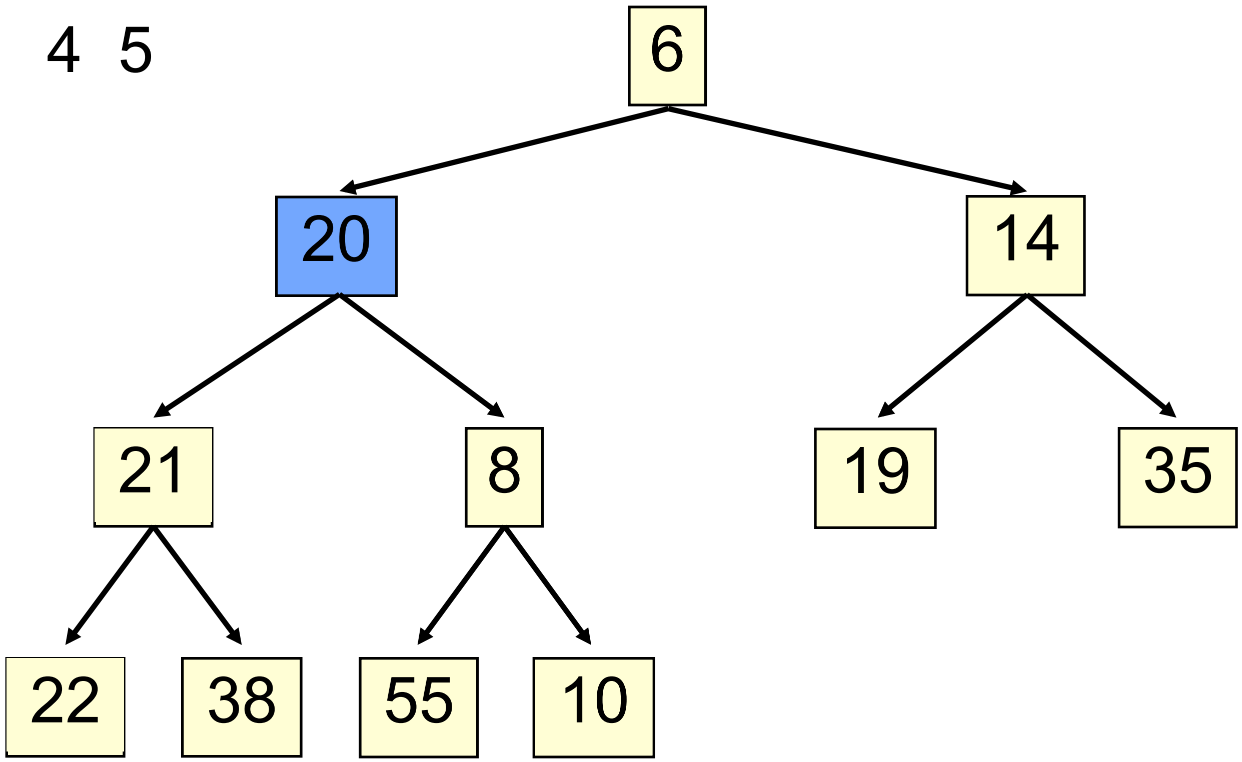


`extract()`

4 5



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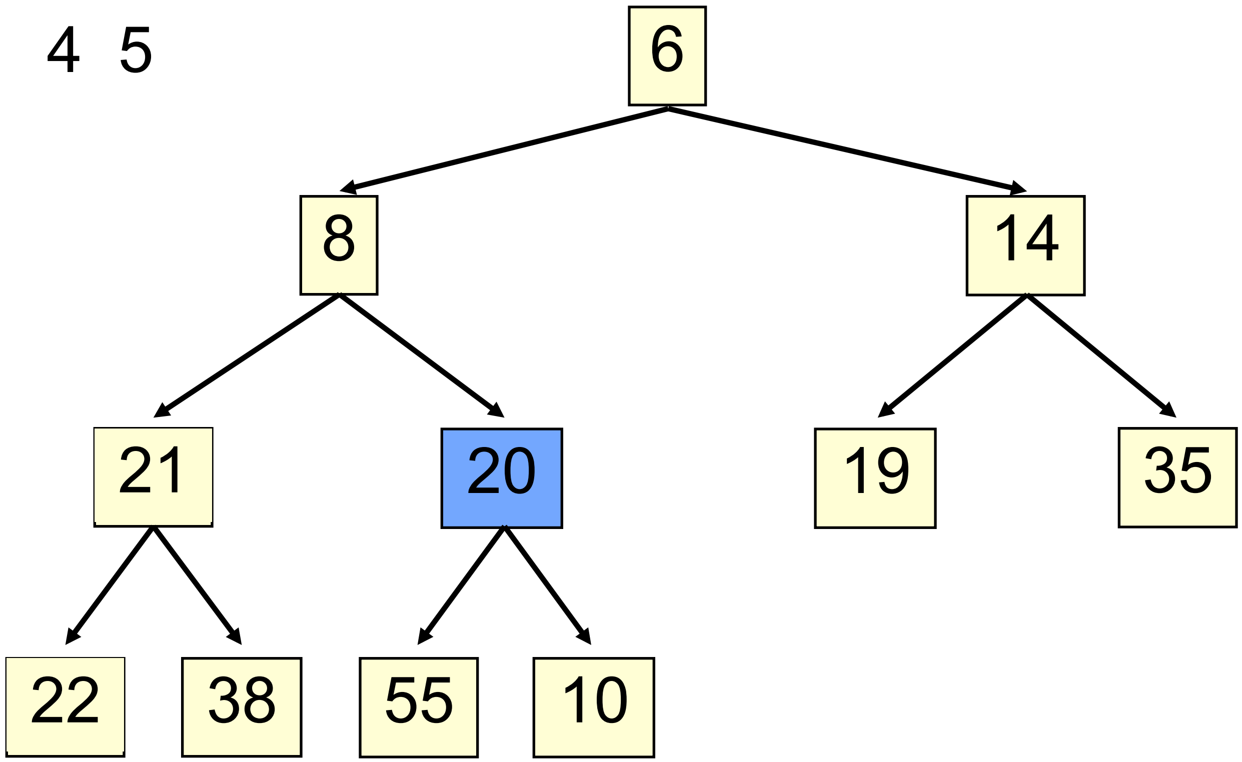

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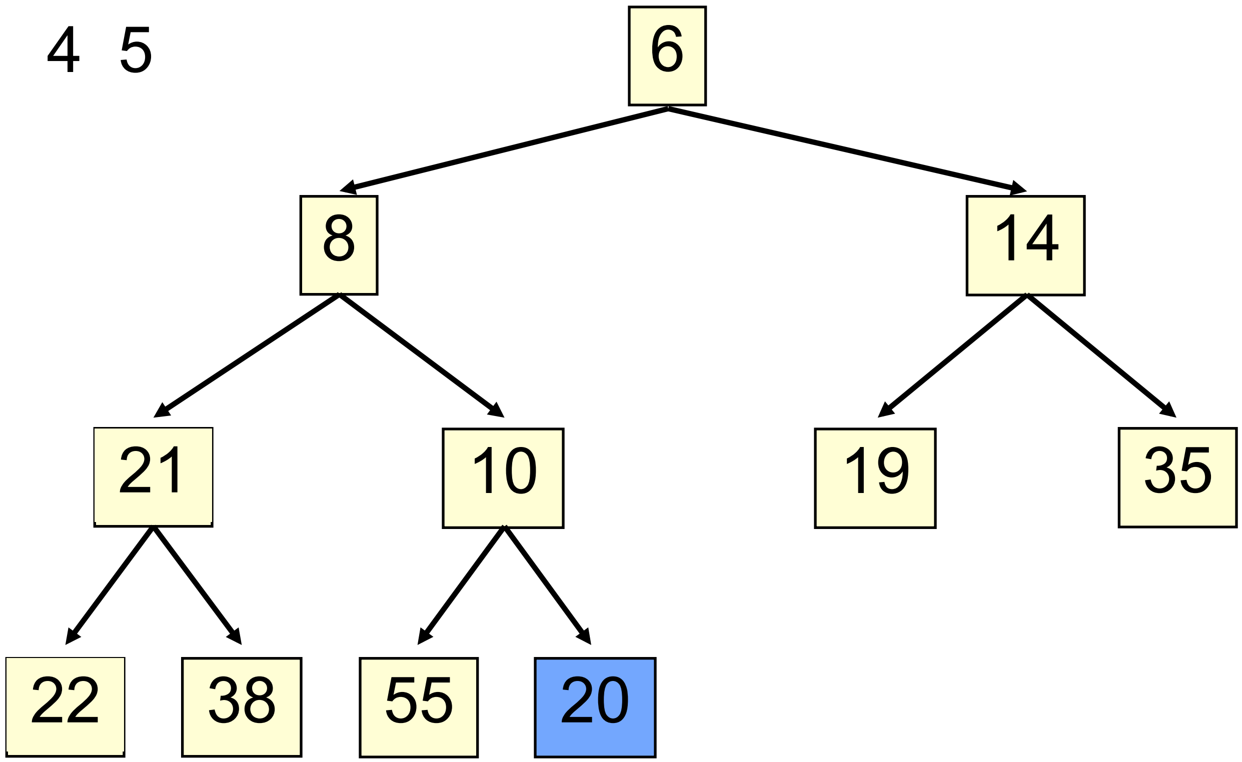
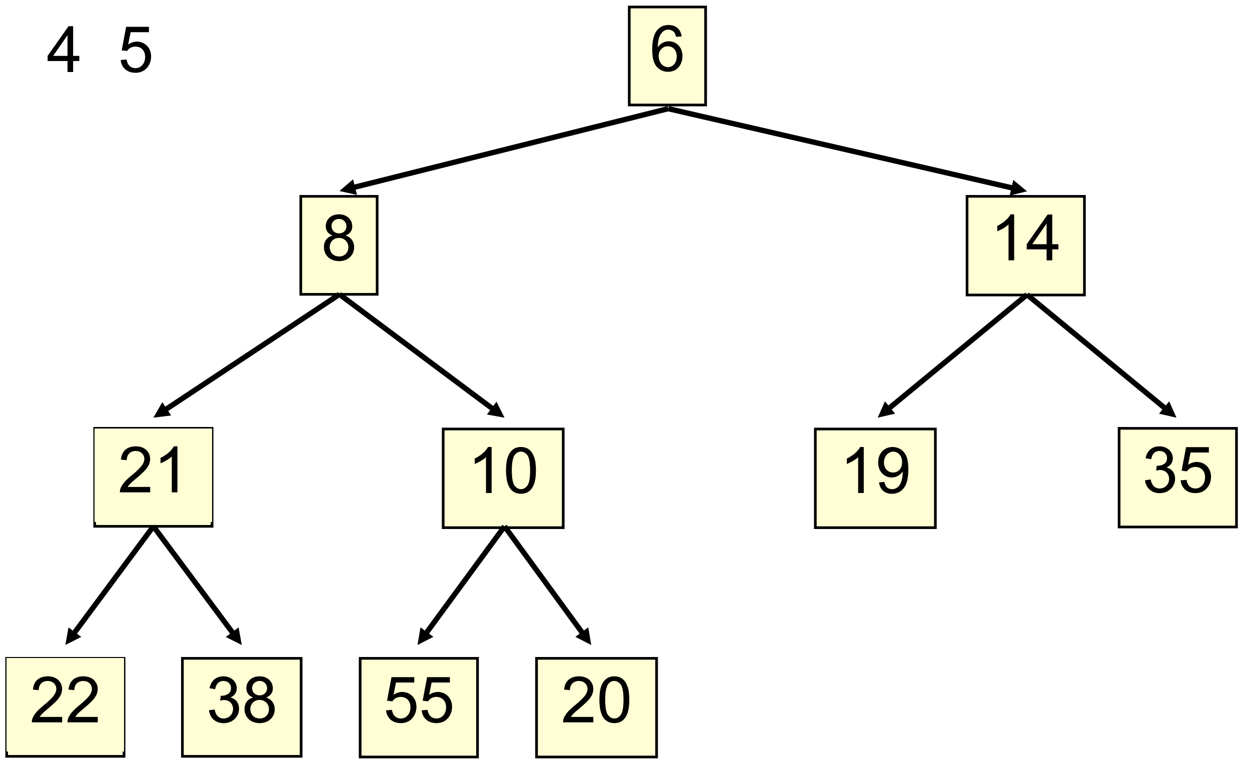


extract()

4 5



`extract()`

Time is O(log n), since the tree is balanced

extract()


```
/** Remove and return the smallest element
    return null if list is empty) */
public E extract() {
   if (size() == 0) return null;
   E temp=  get(0);           // smallest value is at root
   set(0, get(size() – 1));   // move last element to the root
   setSize(size() - 1);       // reduce size by 1
   bubbleDown(0);
   return temp;
}
```

```
/** Bubble the root down to its heap position.
  Pre: tree is a heap except: root may be >than a child */
private void bubbleDown() {
    int k= 0;
    // Set c to smaller of k's children
    int c= 2*k + 2;                         // k's right child
    if (c > size()-1 || get(c-1).compareTo(get(c)) < 0) c= c-1;
    // inv tree is a heap except: element k may be > than a child.
    //      Also. k's smallest child is element c
    while (c < size()  &&  get(k).compareTo(get(c) > 0) {
        Swap elements at k and c;
        k= c;
        c= 2*k + 2;                 // k's right child
        if (c > size()-1 || get(c-1).compareTo(get(c)) < 0) c= c-1;
    }
}
```

HeapSort

Given a `Comparable[]` array of length n,

- Put all n elements into a heap – O(n log n)
- Repeatedly get the min – O(n log n)

```
public static void heapSort(Comparable[] b) {
   PriorityQueue<Comparable> pq=
      new PriorityQueue<Comparable>(b);
   for (int i = 0; i < b.length; i++) {
      b[i] = pq.extract();
   }
}
```

One can do the two stages in the array itself, in place, so algorithm takes O(1) space.

PQ Application: Simulation

- Example: Probabilistic model of bank-customer arrival times and transaction times, how many tellers are needed?
  - Assume we have a way to generate random inter-arrival times
  - Assume we have a way to generate transaction times
  - Can simulate the bank to get some idea of how long customers must wait

Time-Driven Simulation

- Check at each *tick* to see if any event occurs

Event-Driven Simulation

- Advance clock to next event, skipping intervening *ticks*
- This uses a PQ!