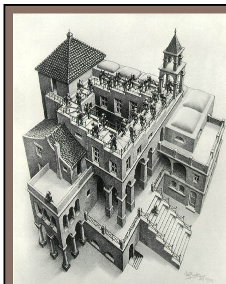

1

PRIORITY QUEUES AND HEAPS

Lecture 19
CS2110 Spring 2014

Readings and Homework

2

Read Chapter 26 to learn about heaps

Salespeople often make matrices that show all the great features of their product that the competitor's product lacks. Try this for a heap versus a BST. First, try and sell someone on a BST: List some desirable properties of a BST that a heap lacks. Now be the heap salesperson: List some good things about heaps that a BST lacks. Can you think of situations where you would favor one over the other?

With ZipUltra heaps, you've got it made in the shade my friend!

The Bag Interface

3

A Bag:

```
interface Bag<E> {
   void insert(E obj);
   E extract(); //extract some element
   boolean isEmpty();
}
```

Like a Set except that a value can be in it more than once. Example: a bag of coins

Refinements of Bag: Stack, Queue, PriorityQueue

Stacks and Queues as Lists

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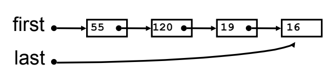
- Stack (LIFO) implemented as list
  - **`insert(),extract()`** from front of list
- Queue (FIFO) implemented as list
  - **`insert()`** on back of list, **`extract()`** from front of list
- All **`Bag`** operations are O(1)



Priority Queue

5

- A **`Bag`** in which data items are **`Comparable`**
- *lesser* elements (as determined by **`compareTo()`**) have *higher* priority
- **`extract()`** returns the element with the highest priority = least in the **`compareTo()`** ordering
- break ties arbitrarily

Priority Queue Examples

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Scheduling jobs to run on a computer
default priority = arrival time
priority can be changed by operator

Scheduling events to be processed by an event handler
priority = time of occurrence

Airline check-in
first class, business class, coach
FIFO within each class

java.util.PriorityQueue<E>

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```
boolean add(E e) {...} //insert an element (insert)
void clear() {...} //remove all elements
E peek() {...} //return min element without removing
               //(null if empty)
E poll() {...} //remove min element (extract)
               //(null if empty)
int size() {...}
```

Priority Queues as Lists

8

- Maintain as unordered list
  - `insert()` put new element at front – O(1)
  - `extract()` must search the list – O(n)
- Maintain as ordered list
  - `insert()` must search the list – O(n)
  - `extract()` get element at front – O(1)
- In either case, $O(n^2)$ to process n elements

Can we do better?

Important Special Case

9

- Fixed number of priority levels 0,...,p – 1
- FIFO within each level
- Example: airline check-in

- `insert()` – insert in appropriate queue – O(1)
- `extract()` – must find a nonempty queue – O(p)

Heaps

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- A *heap* is a concrete data structure that can be used to implement priority queues
- Gives better complexity than either ordered or unordered list implementation:
  - `insert()`: O(log n)
  - `extract()`: O(log n)
- O(n log n) to process n elements
- Do not confuse with *heap memory*, where the Java virtual machine allocates space for objects – different usage of the word *heap*

Heaps

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- Binary tree with data at each node
- Satisfies the *Heap Order Invariant*:

The least (highest priority) element of any subtree is found at the root of that subtree

- Size of the heap is “fixed” at *n*. (But can usually double n if heap fills up)

Heaps

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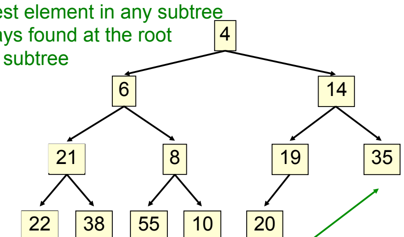
Smallest element in any subtree is always found at the root of that subtree



Note: 19, 20 < 35: Smaller elements can be deeper in the tree!

Examples of Heaps

13

- Ages of people in family tree
  - parent is always older than children, but you can have an uncle who is younger than you
- Salaries of employees of a company
  - bosses generally make more than subordinates, but a VP in one subdivision may make less than a Project Supervisor in a different subdivision

Balanced Heaps

14

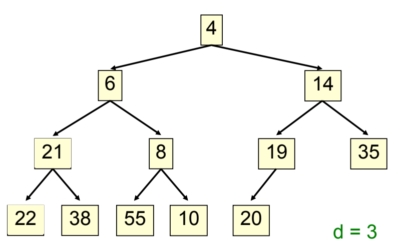
These add two restrictions:

1. Any node of depth < d – 1 has exactly 2 children, where d is the height of the tree
   – implies that any two maximal paths (path from a root to a leaf) are of length d or d – 1, and the tree has at least 2^d nodes

- All maximal paths of length d are to the left of those of length d – 1

Example of a Balanced Heap

15



d = 3

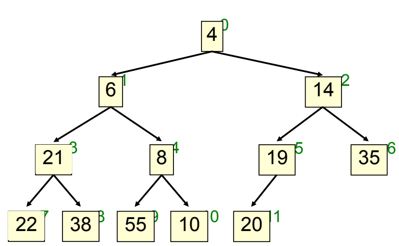
Store in an ArrayList or Vector

16

- Elements of the heap are stored in the array in order, going across each level from left to right, top to bottom
- The children of the node at array index n are at indices 2n + 1 and 2n + 2
- The parent of node n is node (n – 1)/2

Store in an ArrayList or Vector

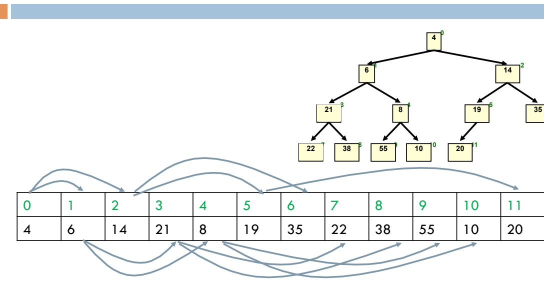
17



children of node n are found at 2n + 1 and 2n + 2

Store in an ArrayList or Vector

18



children of node n are found at 2n + 1 and 2n + 2

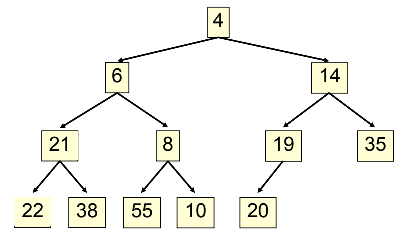
insert()

19

- Put the new element at the end of the array
- If this violates heap order because it is smaller than its parent, swap it with its parent
- Continue swapping it up until it finds its rightful place
- The heap invariant is maintained!

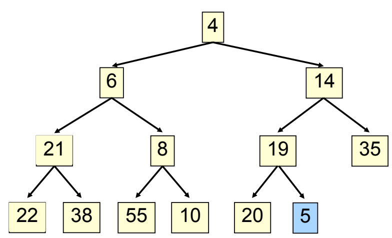
insert()

20



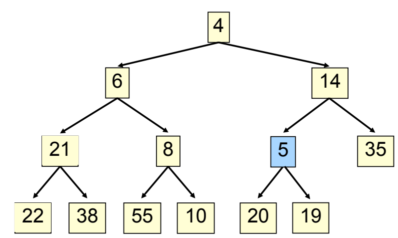
insert()

21



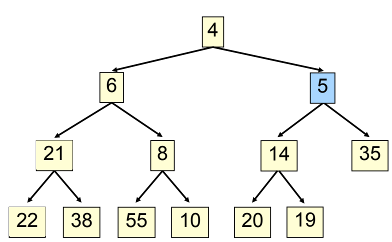
insert()

22



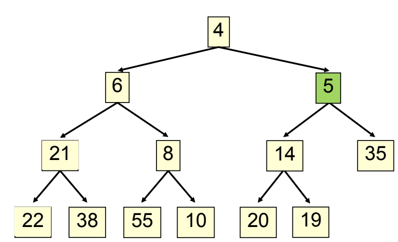
insert()

23



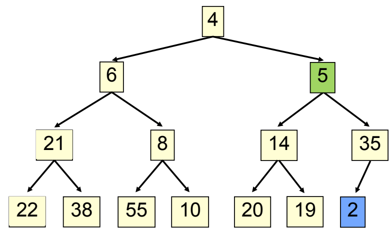
insert()

24



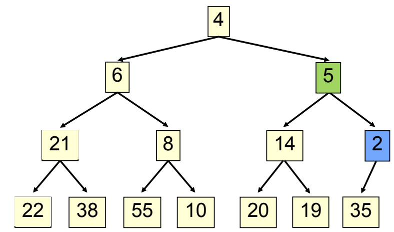
insert()

25



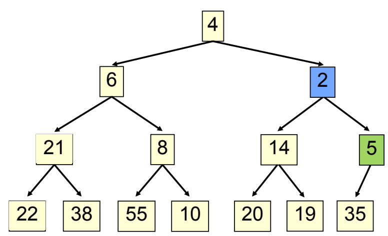
insert()

26



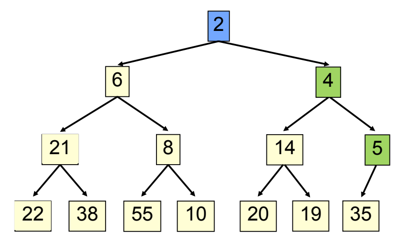
insert()

27



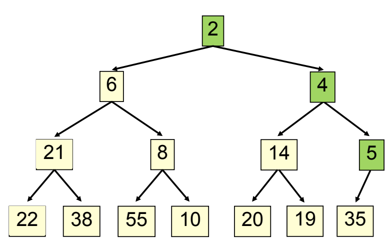
insert()

28



insert()

29



insert()

30

- Time is O(log n), since the tree is balanced
  - size of tree is exponential as a function of depth
  - depth of tree is logarithmic as a function of size

insert()

31

```
/** An instance of a priority queue */
class PriorityQueue<E> extends java.util.Vector<E> {

  /** Insert e into the priority queue */
  public void insert(E e) {
    super.add(e); //add to end of array
    bubbleUp(size() - 1); // given on next slide
  }
```

insert()

32

```
class PriorityQueue<E> extends java.util.Vector<E> {

 /** Bubble element k up the tree */
 private void bubbleUp (int k) {
   int p= (k-1)/2;   // p is the parent of k
   // inv: Every element satisfies the heap property except
   //        element k might be smaller than its parent
   while (k > 0   &&   get(k).compareTo(get(p)) < 0) {
        swap elements k and p;
        k= p;
        p= (k-1)/2;
   }
}
```

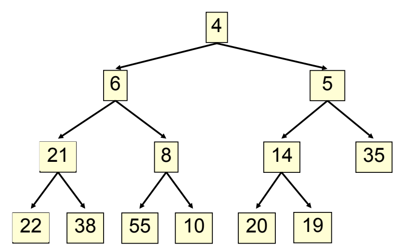
extract()

33

- Remove the least element – it is at the root
- This leaves a hole at the root – fill it in with the last element of the array
- If this violates heap order because the root element is too big, swap it down with the smaller of its children
- Continue swapping it down until it finds its rightful place
- The heap invariant is maintained!

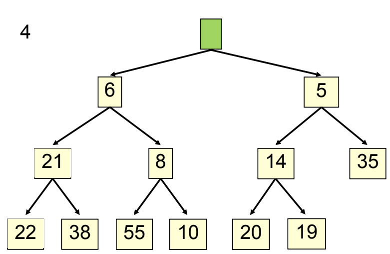
extract()

34



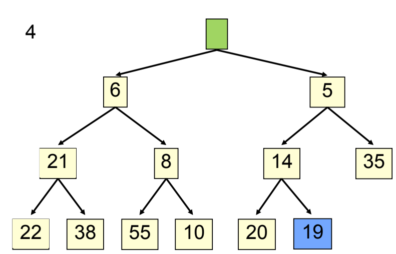
extract()

35



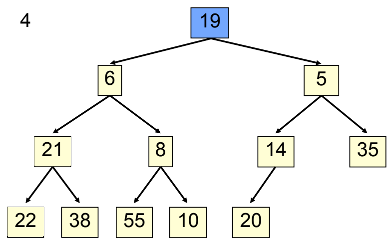
extract()

36



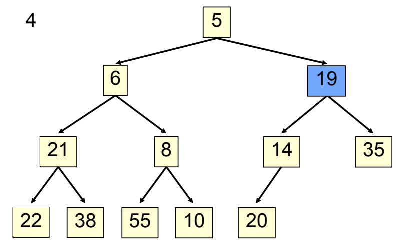
extract()

37



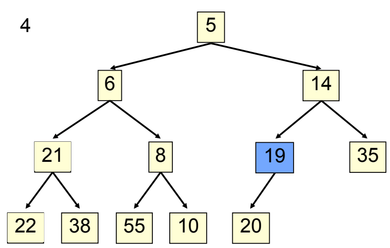
extract()

38



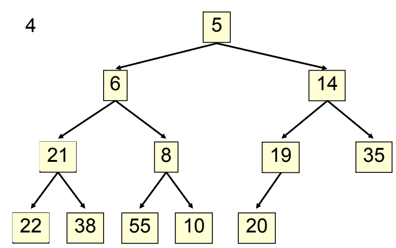
extract()

39



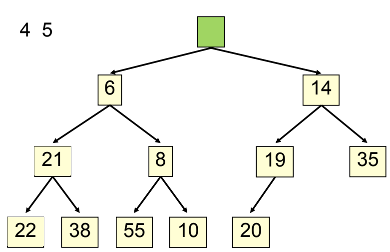
extract()

40



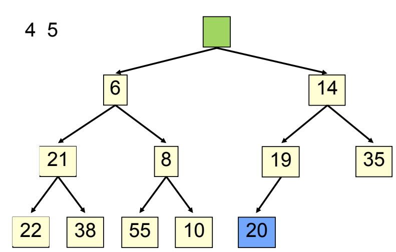
extract()

41



extract()

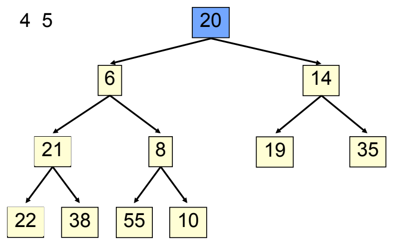
42



extract()

43

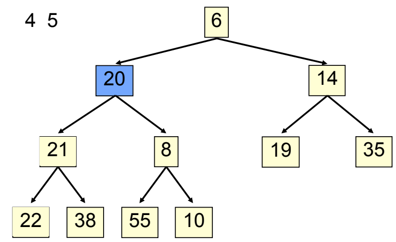
4 5



extract()

44

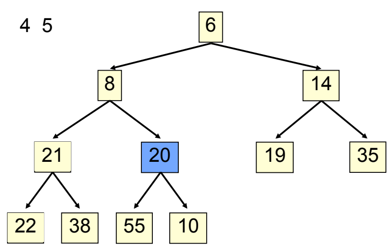
4 5



extract()

45

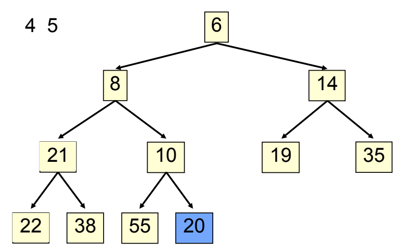
4 5



extract()

46

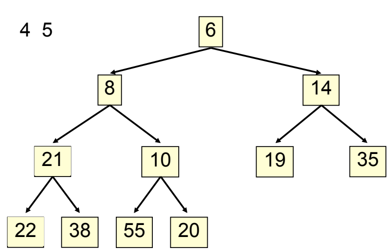
4 5



extract()

47

4 5



extract()

48

Time is O(log n), since the tree is balanced

extract()

49

```
/** Remove and return the smallest element
    return null if list is empty) */
public E extract() {
    if (size() == 0) return null;
    E temp=  get(0);          // smallest value is at root
    set(0, get(size() – 1));  // move last element to the root
    setSize(size() - 1);      // reduce size by 1
    bubbleDown(0);
    return temp;
}
```

50

```
/** Bubble the root down to its heap position.
  Pre: tree is a heap except: root may be >than a child */
private void bubbleDown() {
    int k= 0;
    // Set c to smaller of k's children
    int c= 2*k + 2;                      // k's right child
    if (c > size()-1 || get(c-1).compareTo(get(c)) < 0) c= c-1;
    // inv tree is a heap except: element k may be > than a child.
    //       Also. k's smallest child is element c
    while (c < size()  &&  get(k).compareTo(get(c) > 0) {
        Swap elements at k and c;
        k= c;
        c= 2*k + 2;                  // k's right child
        if (c > size()-1 || get(c-1).compareTo(get(c)) < 0) c= c-1;
    }
}
```

HeapSort

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Given a `Comparable[]` array of length n,

- Put all n elements into a heap – O(n log n)
- Repeatedly get the min – O(n log n)

```
public static void heapSort(Comparable[] b) {
  PriorityQueue<Comparable> pq=
     new PriorityQueue<Comparable>(b);
   for (int i = 0; i < b.length; i++) {
      b[i] = pq.extract();
   }
}
```

One can do the two stages in the array itself, in place, so algorithm takes O(1) space.

PQ Application: Simulation

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- Example: Probabilistic model of bank-customer arrival times and transaction times, how many tellers are needed?
  - Assume we have a way to generate random inter-arrival times
  - Assume we have a way to generate transaction times
  - Can simulate the bank to get some idea of how long customers must wait

Time-Driven Simulation

- Check at each *tick* to see if any event occurs

Event-Driven Simulation

- Advance clock to next event, skipping intervening *ticks*
- This uses a PQ!