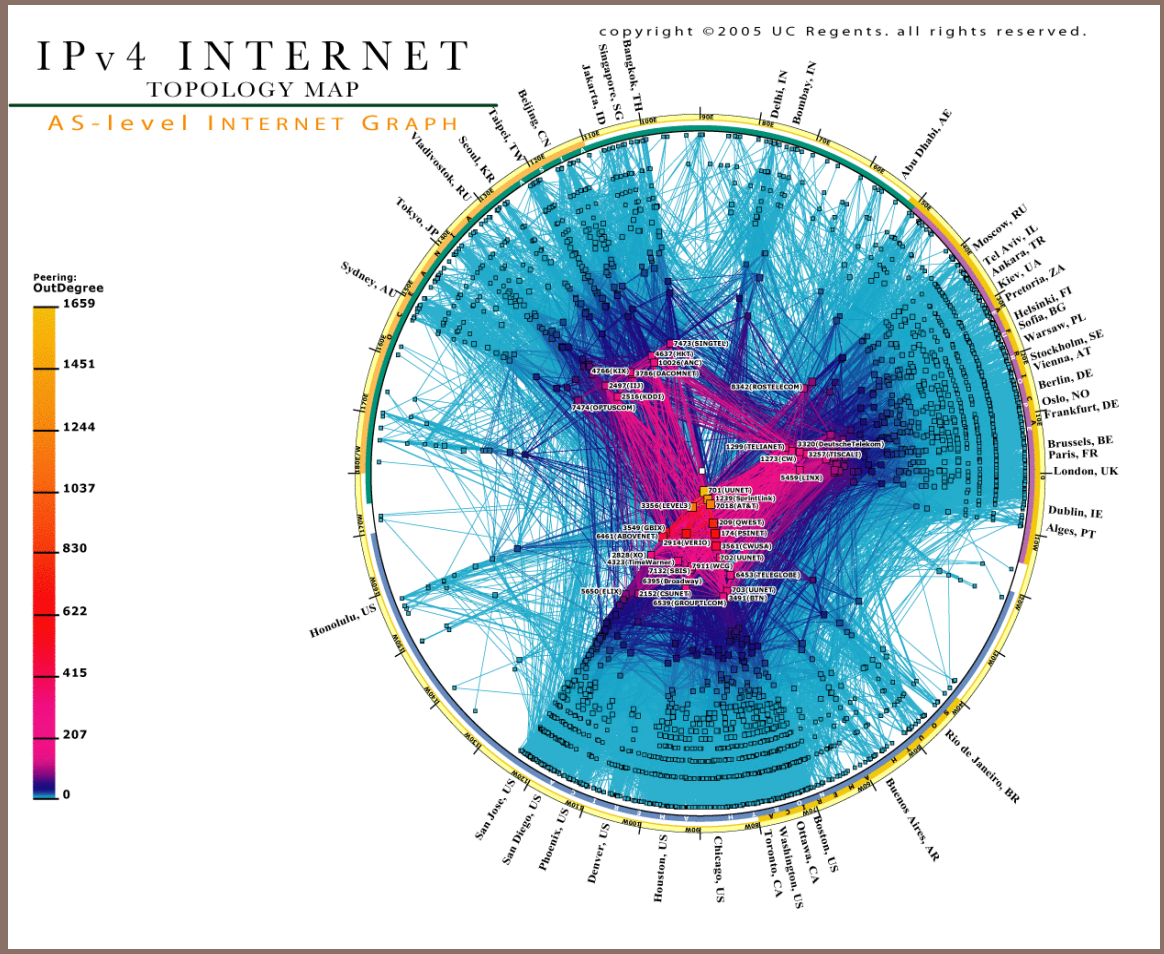


GRAPHS

Lecture 18

CS2110 – Fall 2014

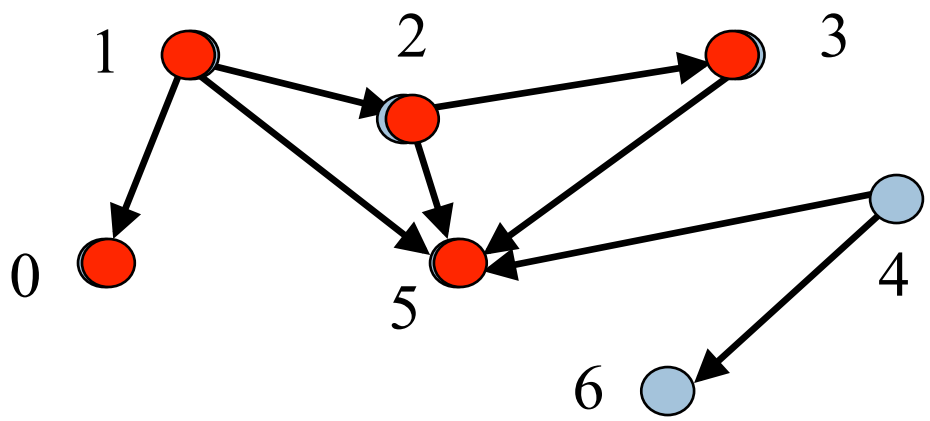
Graph Algorithms

- Search
  - depth-first search
  - breadth-first search
- Shortest paths
  - Dijkstra's algorithm
- Minimum spanning trees
  - Prim's algorithm
  - Kruskal's algorithm

Depth-First Search

Given a graph and one of it's nodes u (say node 1 below).

We want to "visit" each node reachable from u ONCE (nodes 1, 0, 2, 3, 5).



There are many paths to some nodes.

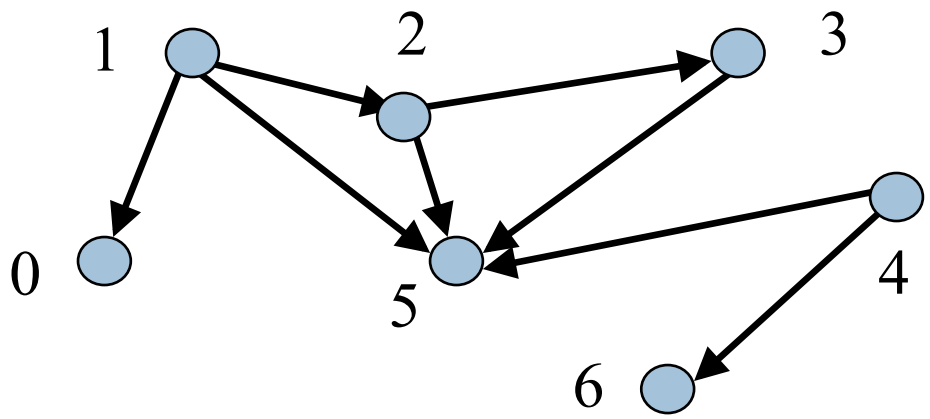
How to visit nodes only once, efficiently, and not do extra work?

Depth-First Search

boolean[] visited;

node u is visited means: visited[u] is true
To visit u means to: set visited[u] to true

Node v is REACHABLE from node u if there is a path (u, …, v) in which all nodes of the path are unvisited.



Suppose all nodes are unvisited.

Nodes REACHABLE from node 1:
1, 0, 2, 3, 5

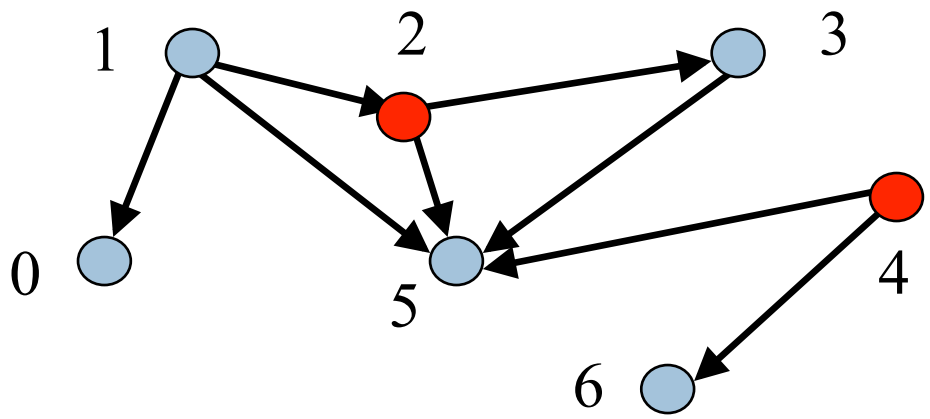
Nodes REACHABLE from 4: 4, 5, 6.

Depth-First Search

boolean[] visited;

node u is visited means: visited[u] is true
To visit u means to: set visited[u] to true

Node u is REACHABLE from node v if there is a path (u, …, v) in which all nodes of the path are unvisited.



Red nodes: visited.
Blue nodes: unvisited

Nodes REACHABLE from 1: 1, 0, 5

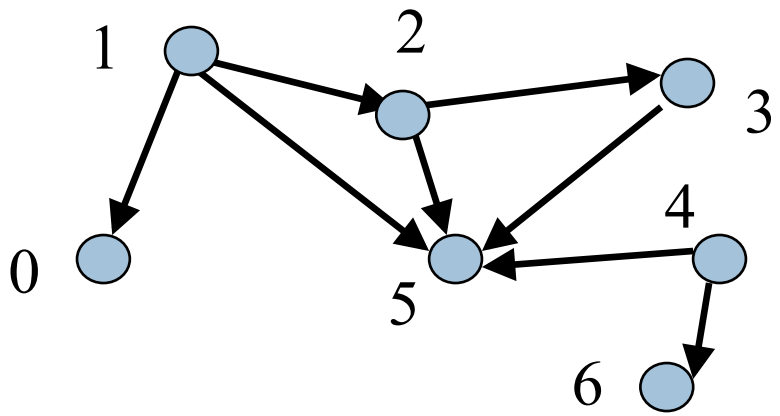
Nodes REACHABLE from 4: none

Depth-First Search

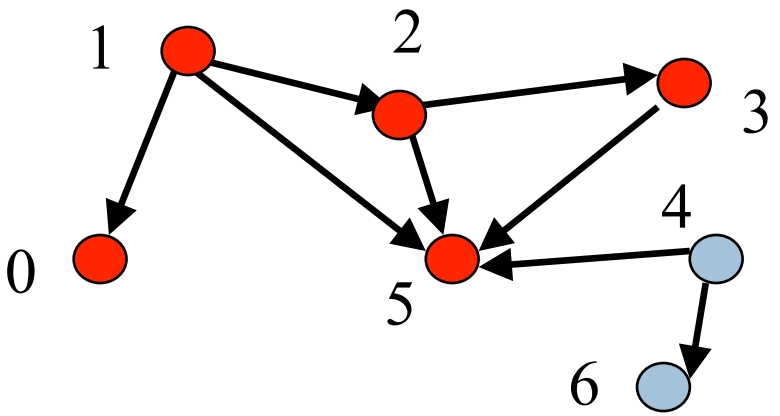

```
/** Node u is unvisited. Visit all nodes
    that are REACHABLE from u. */
public static void dfs(int u) {
}
```

Let u be 1
The nodes that are REACHABLE from node 1 are 1, 0, 2, 3, 5

Start



End

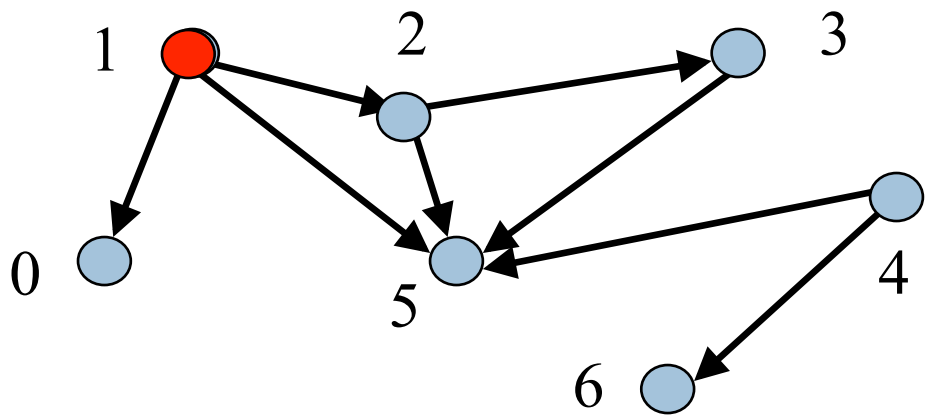


Depth-First Search


```
/** Node u is unvisited. Visit all nodes
    that are REACHABLE from u. */
public static void dfs(int u) {
      visited[u]= true;


}
```

Let u be 1
The nodes that are REACHABLE from node 1 are
1, 0, 2, 3, 5

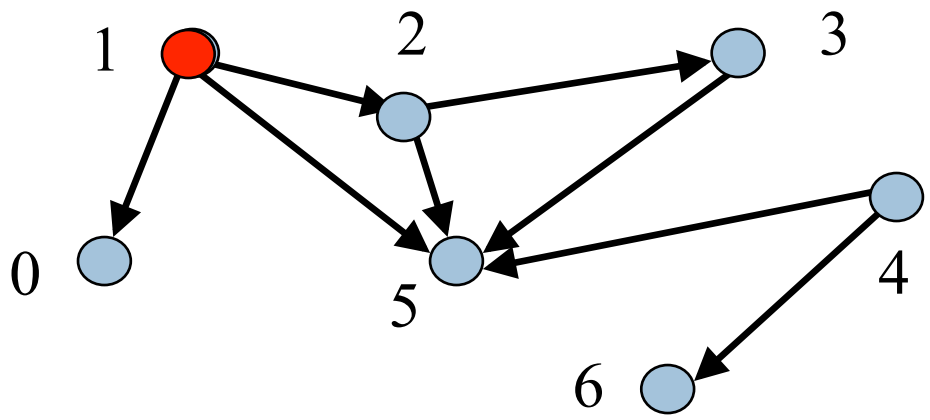


Depth-First Search


```
/** Node u is unvisited. Visit all nodes
    that are REACHABLE from u. */
public static void dfs(int u) {

    visited[u]= true;

    for each edge (u, v) leaving u:
        if v is unvisited then dfs(v);
}
```



Let u be 1
The nodes to be visited are
0, 2, 3, 5

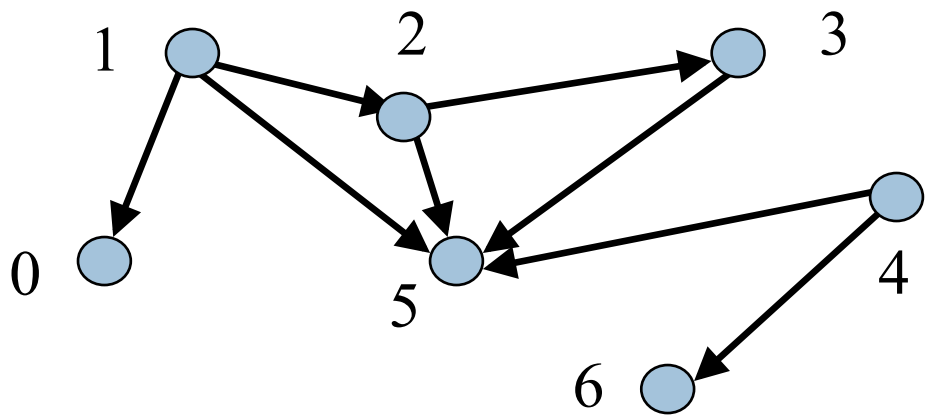
Have to do dfs on all unvisited neighbors of u

Depth-First Search


```
/** Node u is unvisited. Visit all nodes
    that are REACHABLE from u. */
public static void dfs(int u) {

    visited[u]= true;

    for each edge (u, v) leaving u:
        if v is unvisited then dfs(v);
}
```



Let u be 1
Nodes to be visited are: 0, 2, 3, 5

Suppose the loop visits neighbors in numerical order. Then dfs(1) visits the nodes in this order: 1, 0, 2, 3, 5
Depth First!

Depth-First Search


```
/** Node u is unvisited. Visit all nodes
    that are REACHABLE from u. */
public static void dfs(int u) {

     visited[u]= true;

     for each edge (u, v) leaving u:
        if v is unvisited then dfs(v);
}
```

Notes:

Suppose n nodes are REACHABLE along e edges (in total). What is Worst-case execution?

Worst-case space?

Depth-First Search


```
/** Node u is unvisited. Visit all nodes
    that are REACHABLE from u. */
public static void dfs(int u) {
     visited[u]= true;
     for each edge (u, v) leaving u:
        if v is unvisited then dfs(v);
}
```

Example: Use different way (other than array visited) to know whether a node has been visited

Example: We really haven't said what data structures are used to implement the graph.

That's all there is to the basic dfs. You may have to change it to fit a particular situation.

If you don't have this spec and you do something different, it's probably wrong.

Depth-First Search in an OO fashion


```
public class Node {
    boolean visited;
    List<Node> neighbors;

    /** This node is unvisited. Visit all nodes
        REACHABLE from this node */
    public void dfs() {
        visited= true;
        for (Node n: neighbors) {
            if (!n.visited) n.dfs()
        }
    }
}
```

Each node of the graph is an Object of class Node

No need for a parameter. The object is the node

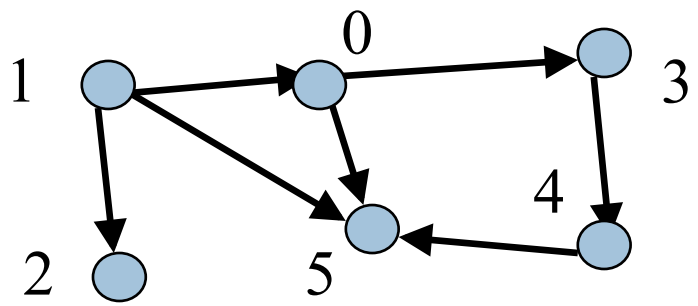
Depth-First Search written iteratively


```
/** Node u is unvisited. Visit all node REACHABLE from u. */
public static void dfs(int u) {
     Stack s= (u);          // Not Java
     // inv: all nodes that have to be visited are
     //       REACHABLE from some node in s
    while (   s is not empty    ) {
          u=  s.pop();    // Remove top stack node, put in u
          if (u has not been visited) {
               visit u;
               for each edge (u, v) leaving u:
                   s.push(v);
          }
     }
}
```

Depth-First Search written iteratively


```
/** u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
       u=  s.pop();
       if (u has not been visited) {
          visit u;
          for each edge (u, v):
              s.push(v);
       }
    }
}
```

Call dfs(1)



1

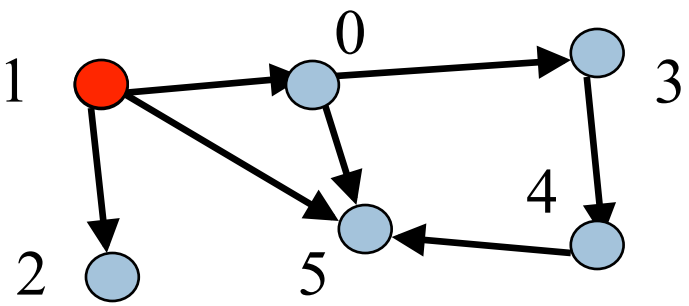
stack s

Depth-First Search written iteratively


```
/** u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
       u=  s.pop();
       if (u has not been visited) {
          visit u;
          for each edge (u, v):
              s.push(v);
       }
    }
}
```

Call dfs(1)

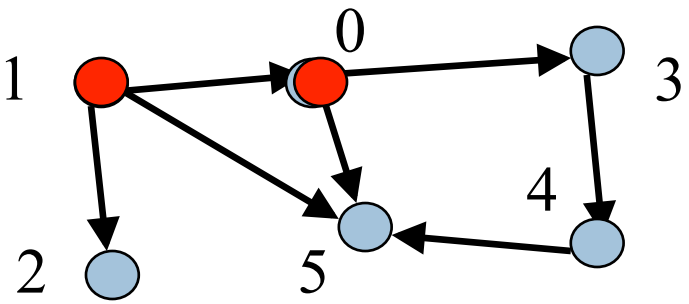
Iteration 0



0
2
5
1
stack s

Depth-First Search written iteratively


```
/** u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Stack s= (u);
    while (s is not empty) {
       u=  s.pop();
       if (u has not been visited) {
          visit u;
          for each edge (u, v):
              s.push(v);
       }
    }
}
```



Call dfs(1)

Iteration 1

Yes, 5 is put on stack twice, once for each edge to it. It will be visited only once.

Using a stack causes depth-first search

3
~~0~~
~~2~~
5

stack s

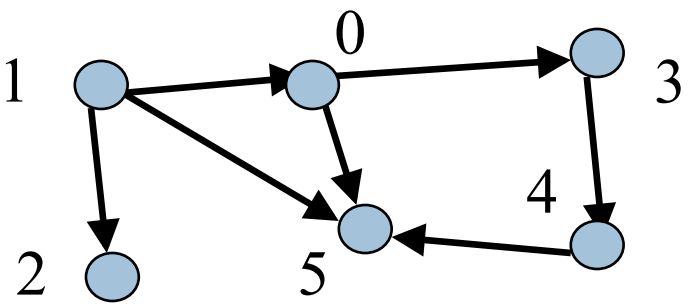
Breadth-First Search


```
/** Node u is unvisited. Visit all node REACHABLE from u. */
public static void dfs(int u) {
     Queue q= (u);          // Not Java
     // inv: all nodes that have to be visited are
     //       REACHABLE from some node in q
    while (   q is not empty   ) {
          u=  q.removeFirst();    // Remove first node, put in u
          if (u has not been visited) {
               visit u;
               for each edge (u, v) leaving u:
                   q.append(v);
          }
     }
}
```

Breadth-First Search


```
/** u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Queue q= (u);
    while (q is not empty) {
        u=  q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v):
                append(v);
        }
    }
}
```

Call dfs(1)



1

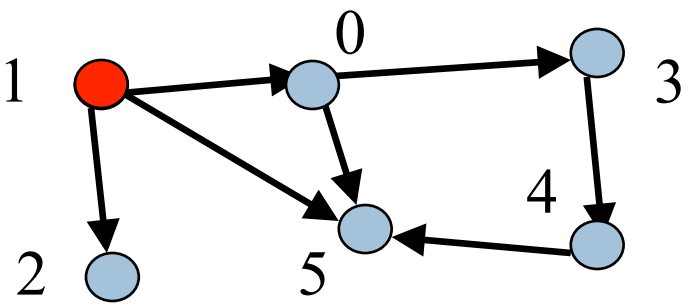
Queue q

Breadth-First Search


```
/** u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Queue q= (u);
    while (q is not empty) {
        u=  q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v):
                append(v);
        }
    }
}
```

Call dfs(1)

Iteration 0



0 1 2 5

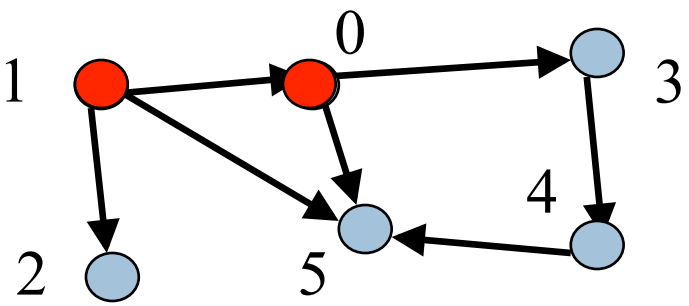
Queue q

Breadth-First Search


```
/** u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Queue q= (u);
    while (q is not empty) {
        u=  q.popFirst();
        if (u has not been visited) {
            visit u;
            for each edge (u, v):
                append(v);
        }
    }
}
```

Call dfs(1)

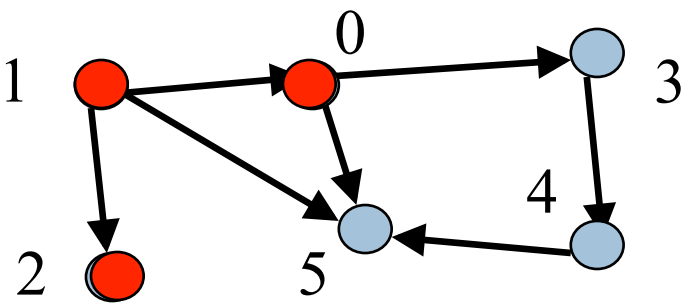
Iteration 1



Queue q

Breadth-First Search


```
/** u is unvisited. Visit all nodes REACHABLE from u. */
public static void dfs(int u) {
    Queue q= (u);
    while (q is not empty) {
       u=  q.popFirst();
       if (u has not been visited) {
          visit u;
          for each edge (u, v):
              append(v);
       }
    }
}
```



Call dfs(1)

Iteration 2

Breadth first:
(1) Node u
(2) All nodes 1 edge from u
(3) All nodes 2 edges from u
(4) All nodes 3 edges from u
…

2 5 3 5
Queue q