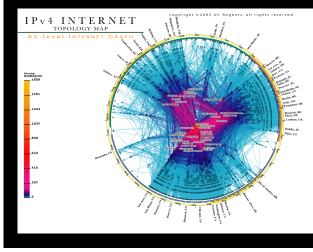
CS/ENGRD 2110
Object-Oriented Programming and Data Structures
Fall 2014

Doug James

Lecture 17: Graphs



Readings

- Chapter 28 Graphs
- Chapter 29 Graph Implementations

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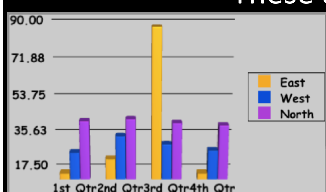
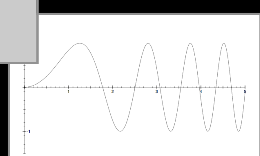
These are not Graphs





...not the kind we mean, anyway

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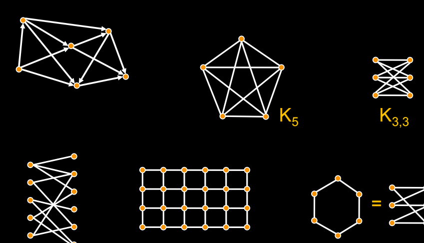
These are Graphs



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Applications of Graphs

- Communication networks; social networks
- Routing and shortest path problems
- Commodity distribution (network flow)
- Traffic control
- Resource allocation
- Numerical linear algebra (sparse matrices)
- Geometric modeling (meshes, topology, …)
- Image processing (e.g., graph cuts)
- Computer animation (e.g., motion graphs)
- Systems biology
- …

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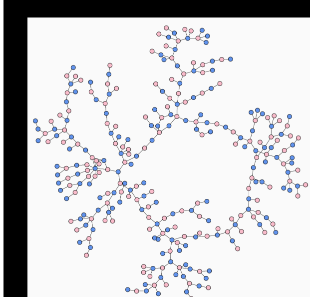
Applications of Graphs



High School Dating (Bearman, Moody, and Stovel, 2004) (Image by Mark Newman)

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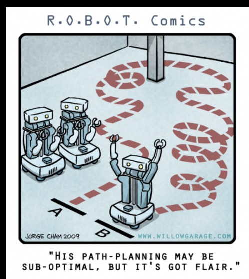
Applications of Graphs



http://www.willowgarage.com/blog/2009/09/04/robot-comics-path-planning

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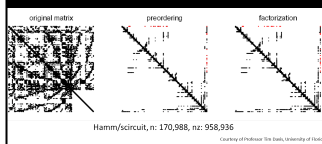

http://semiengineering.com/wp-content/uploads/2014/03/Fig13_Sparse_Matrix_reordering.png

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http://www.turbosquid.com/3d-models/3d-2015-bmw-i8-model/780918

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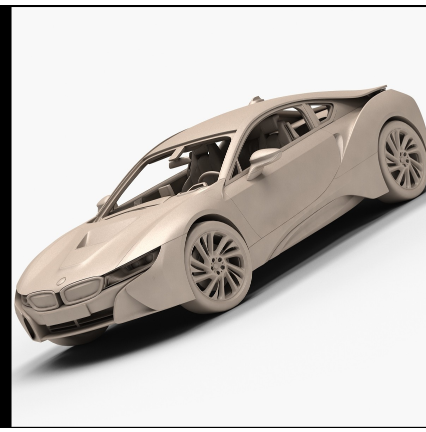

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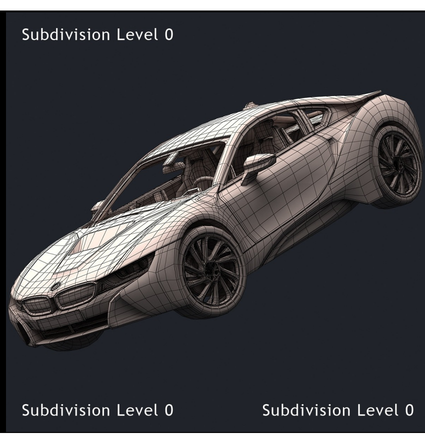

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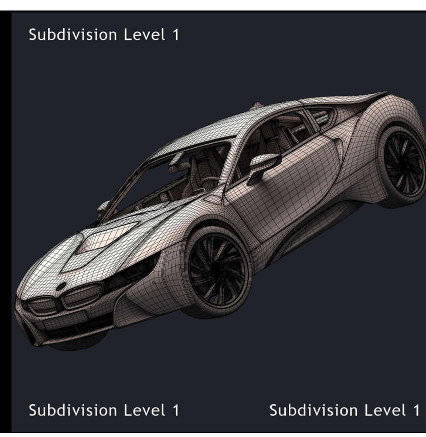

http://www.turbosquid.com/3d-models/3d-2015-bmw-i8-model/780918

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Graph Definitions

- A directed graph (or digraph) is a pair (V, E) where
  - V is a set
  - E is a set of ordered pairs (u,v) where $u,v \in V$
    - Usually require $u \neq v$ (i.e., no self-loops)
- An element of V is called a vertex or node
- An element of E is called an edge or arc
- |V| = size of V, often denoted n
- |E| = size of E, often denoted m

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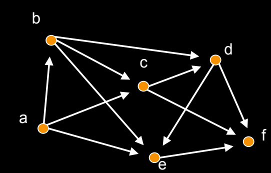
Example Directed Graph (Digraph)



V = {a,b,c,d,e,f}
E = {(a,b), (a,c), (a,e), (b,c), (b,d), (b,e), (c,d), (c,f), (d,e), (d,f), (e,f)}

|V| = 6, |E| = 11

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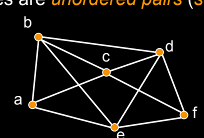
Example Undirected Graph

An *undirected graph* is just like a directed graph, except the edges are *unordered pairs* (*sets*) {u,v}

Example:



V = {a,b,c,d,e,f}
E = {{a,b}, {a,c}, {a,e}, {b,c}, {b,d}, {b,e}, {c,d}, {c,f}, {d,e}, {d,f}, {e,f}}

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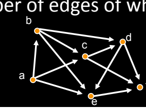
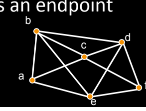
Some Graph Terminology

- Vertices u and v are called the source and sink of the directed edge (u,v), respectively
- Vertices u and v are called the endpoints of (u,v)
- Two vertices are adjacent if they are connected by an edge
- The outdegree of a vertex u in a directed graph is the number of edges for which u is the source
- The indegree of a vertex v in a directed graph is the number of edges for which v is the sink
- The degree of a vertex u in an undirected graph is the number of edges of which u is an endpoint





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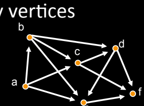
More Graph Terminology

- A path is a sequence $v_0,v_1,v_2,\ldots,v_p$ of vertices such that $(v_i,v_{i+1}) \in E$, $0 \leq i \leq p-1$
- The length of a path is its number of edges
  - In this example, the length is 5
- A path is simple if it does not repeat any vertices
- A cycle is a path $v_0,v_1,v_2,\ldots,v_p$ such that $v_0 = v_p$
- A cycle is simple if it does not repeat any vertices except the first and last
- A graph is acyclic if it has no cycles
- A directed acyclic graph is called a dag



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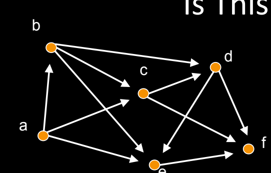
Is This a Dag?



- Intuition:
  - If it's a dag, there must be a vertex with indegree zero
- This idea leads to an algorithm
  - A digraph is a dag if and only if we can iteratively delete indegree-0 vertices until the graph disappears

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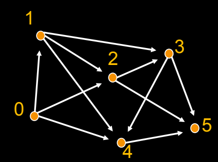
Topological Sort

- We just computed a topological sort of the dag
  - This is a numbering of the vertices such that all edges go from lower- to higher-numbered vertices



- Useful in job scheduling with precedence constraints

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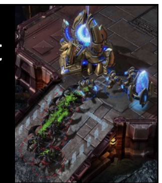
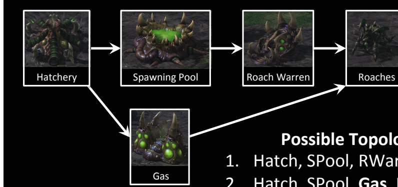
Example of Topological Sort



- Starcraft II build order: Roach Rush



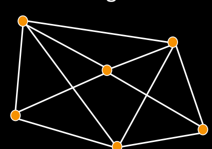
Possible Topological Sorts

1. Hatch, SPool, RWarren, **Gas**, Roaches
2. Hatch, SPool, **Gas**, RWarren, Roaches
3. Hatch, **Gas**, SPool, RWarren, Roaches

Timing is everything though ;)

Graph Coloring

- A coloring of an undirected graph is an assignment of a color to each node such that no two adjacent vertices get the same color



- How many colors are needed to color this graph?

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Graph Coloring

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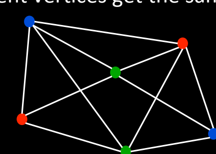

- How many colors are needed to color this graph?

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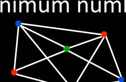
An Application of Coloring

- Vertices are jobs
- Edge (u,v) is present if jobs u and v each require access to the same shared resource, and thus cannot execute simultaneously
- Colors are time slots to schedule the jobs
- Minimum number of colors needed to color the graph = minimum number of time slots required



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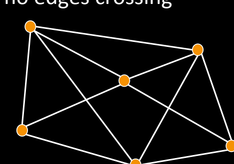
Planarity

- A graph is planar if it can be embedded in the plane with no edges crossing



- Is this graph planar?

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Planarity

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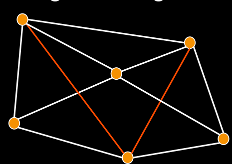

- Is this graph planar?
  - Yes

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Planarity

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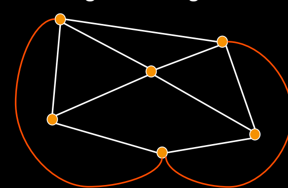

- Is this graph planar?
  - Yes

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Detecting Planarity

- Kuratowski's Theorem

K_5 $K_{3,3}$

- A graph is planar if and only if it does not contain a copy of K_5 or $K_{3,3}$ (possibly with other nodes along the edges shown)

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Four-Color Theorem:
Every planar graph is 4-colorable.
(Appel & Haken, 1976)

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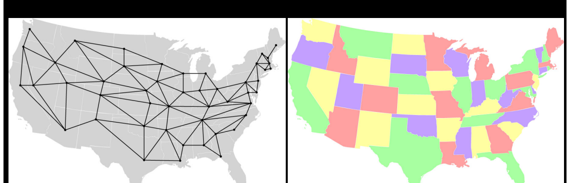
Another 4-colored planar graph



http://www.cs.cmu.edu/~bryant/boolean/maps.html

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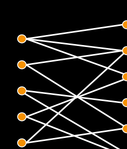
Bipartite Graphs

- A directed or undirected graph is bipartite if the vertices can be partitioned into two sets such that all edges go between the two sets
- The following are equivalent
  - G is bipartite
  - G is 2-colorable
  - G has no cycles of odd length



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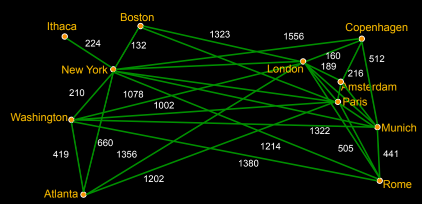
Traveling Salesperson



- Find a path of minimum distance that visits every city

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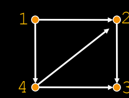
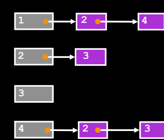
Representations of Graphs



Adjacency List



Adjacency Matrix

	1	2	3	4
1	0	1	0	1
2	0	0	1	0
3	0	0	0	0
4	0	1	1	0

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Adjacency Matrix or Adjacency List?

- Definitions
  - n = number of vertices
  - m = number of edges
  - d(u) = degree of u = number of edges leaving u
- Adjacency Matrix
  - Uses space $O(n^2)$
  - Can iterate over all edges in time $O(n^2)$
  - Can answer "Is there an edge from u to v?" in O(1) time
  - Better for dense graphs (lots of edges)
- Adjacency List
  - Uses space O(m+n)
  - Can iterate over all edges in time O(m+n)
  - Can answer "Is there an edge from u to v?" in O(d(u)) time
  - Better for sparse graphs (fewer edges)

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Graph Algorithms

- Search
  - depth-first search
  - breadth-first search
- Shortest paths
  - Dijkstra's algorithm
- Minimum spanning trees
  - Prim's algorithm
  - Kruskal's algorithm

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Readings

- Chapter 28 Graphs
- Chapter 29 Graph Implementations

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