

Note: Long-haul freight trucks typically serve locations at least 50 miles apart, excluding trucks that are used in movements by multiple modes and mail.

## SPANNING TREES, INTRO. TO THREADS

Lecture 23
CS2110 - Fall 2013

## A lecture with two distinct parts

$\square$ Part I: Finishing our discussion of graphs
$\square$ Today: Spanning trees
$\square$ Definitions, algorithms (Prim's, Kruskal's)
$\square$ Travelling salesman problem
$\square$ Part II: Introduction to the idea of threads
$\square$ Why do we need them?
$\square$ What is a thread?

## Undirected Trees

- An undirected graph is a tree if there is exactly one simple path between any pair of vertices



## Facts About Trees

- $|\mathrm{E}|=|\mathrm{V}|-1$
- connected
- no cycles

In fact, any two of these properties imply the third, and
 imply that the graph is a tree

## Spanning Trees

A spanning tree of a connected undirected graph ( $\mathrm{V}, \mathrm{E}$ ) is a subgraph ( $\mathrm{V}, \mathrm{E}^{\prime}$ ) that is a tree


## Spanning Trees

A spanning tree of a connected undirected graph ( $\mathrm{V}, \mathrm{E}$ ) is a subgraph ( $\mathrm{V}, \mathrm{E}^{\prime}$ ) that is a tree

- Same set of vertices V
- $\mathrm{E}^{\prime} \subseteq \mathrm{E}$
- $\left(\mathrm{V}, \mathrm{E}^{\prime}\right)$ is a tree



## Finding a Spanning Tree

A subtractive method

- Start with the whole graph - it is connected
- If there is a cycle, pick an edge on the cycle, throw it out - the graph is still connected (why?)
- Repeat until no more cycles



## Finding a Spanning Tree

A subtractive method

- Start with the whole graph - it is connected
- If there is a cycle, pick an edge on the cycle, throw it out - the graph is still connected (why?)
- Repeat until no more cycles



## Finding a Spanning Tree

A subtractive method

- Start with the whole graph - it is connected
- If there is a cycle, pick an edge on the cycle, throw it out - the graph is still connected (why?)
- Repeat until no more cycles



## Finding a Spanning Tree

An additive method

- Start with no edges - there are no cycles
- If more than one connected component, insert an edge between. them - still no cycles (why?)
- Repeat until only one



## Finding a Spanning Tree

An additive method

- Start with no edges - there are no cycles
- If more than one connected component, insert an edge between. them - still no cycles (why?)
- Repeat until only one
 component


## Finding a Spanning Tree

An additive method

- Start with no edges - there are no cycles
- If more than one connected component, insert an edge between. them - still no cycles (why?)
- Repeat until only one
 component


## Finding a Spanning Tree

An additive method

- Start with no edges - there are no cycles
- If more than one connected component, insert an edge between. them - still no cycles (why?)
- Repeat until only one
 component


## Finding a Spanning Tree

An additive method

- Start with no edges - there are no cycles
- If more than one connected component, insert an edge between. them - still no cycles (why?)
- Repeat until only one
 component


## Finding a Spanning Tree

An additive method

- Start with no edges - there are no cycles
- If more than one connected component, insert an edge between. them - still no cycles (why?)
- Repeat until only one
 component


## Minimum Spanning Trees

- Suppose edges are weighted, and we want a spanning tree of minimum cost (sum of edge weights)
- Some graphs have exactly one minimum spanning tree. Others have multiple trees with the same cost, any of which is a minimum spanning tree


## Minimum Spanning Trees

- Suppose edges are weighted, and we want a spanning tree of minimum cost (sum of edge weights)
- Useful in network routing \& other applications
- For example, to
 stream a video


## 3 Greedy Algorithms

A. Find a max weight edge - if it is on a cycle, throw it out, otherwise keep it


## 3 Greedy Algorithms

A. Find a max weight edge - if it is on a cycle, throw it out, otherwise keep it


## 3 Greedy Algorithms

A. Find a max weight edge - if it is on a cycle, throw it out, otherwise keep it


## 3 Greedy Algorithms

A. Find a max weight edge - if it is on a cycle, throw it out, otherwise keep it


## 3 Greedy Algorithms

A. Find a max weight edge - if it is on a cycle, throw it out, otherwise keep it


## 3 Greedy Algorithms

A. Find a max weight edge - if it is on a cycle, throw it out, otherwise keep it


## 3 Greedy Algorithms

A. Find a max weight edge - if it is on a cycle, throw it out, otherwise keep it


## 3 Greedy Algorithms

A. Find a max weight edge - if it is on a cycle, throw it out, otherwise keep it


## 3 Greedy Algorithms

B. Find a min weight edge - if it forms a cycle with edges already taken, throw it out, otherwise keep it

Kruskal's algorithm


## 3 Greedy Algorithms

B. Find a min weight edge - if it forms a cycle with edges already taken, throw it out, otherwise keep it

Kruskal's algorithm


## 3 Greedy Algorithms

B. Find a min weight edge - if it forms a cycle with edges already taken, throw it out, otherwise keep it

Kruskal's algorithm


## 3 Greedy Algorithms

B. Find a min weight edge - if it forms a cycle with edges already taken, throw it out, otherwise keep it

Kruskal's algorithm


## 3 Greedy Algorithms

B. Find a min weight edge - if it forms a cycle with edges already taken, throw it out, otherwise keep it

Kruskal's algorithm


## 3 Greedy Algorithms

B. Find a min weight edge - if it forms a cycle with edges already taken, throw it out, otherwise keep it

Kruskal's algorithm


## 3 Greedy Algorithms

B. Find a min weight edge - if it forms a cycle with edges already taken, throw it out, otherwise keep it

Kruskal's algorithm


## 3 Greedy Algorithms

C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle

Prim's algorithm (reminiscent of Dijkstra's algorithm)


## 3 Greedy Algorithms

C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle

Prim's algorithm (reminiscent of Dijkstra's algorithm)


## 3 Greedy Algorithms

C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle

Prim's algorithm (reminiscent of
Dijkstra's algorithm)


## 3 Greedy Algorithms

C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle

Prim's algorithm (reminiscent of
Dijkstra's algorithm)


## 3 Greedy Algorithms

C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle

Prim's algorithm (reminiscent of
Dijkstra's algorithm)


## 3 Greedy Algorithms

C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle

Prim's algorithm (reminiscent of
Dijkstra's algorithm)


## 3 Greedy Algorithms

C. Start with any vertex, add min weight edge extending that connected component that does not form a cycle

Prim's algorithm (reminiscent of
Dijkstra's algorithm)


## 3 Greedy Algorithms

- When edge weights are all distinct, or if there is exactly one minimum spanning tree, the 3 algorithms all find the identical tree



## Prim's Algorithm

```
prim(s) {
    D[s] = O; mark s; //start vertex
    while (some vertices are unmarked) {
        v = unmarked vertex with smallest D;
        mark v;
        for (each w adj to v) {
            D[w] = min(D[w], c(v,w));
        }
    }
```

- $O\left(n^{2}\right)$ for adj matrix
- While-loop is executed $n$ times
- For-loop takes O(n) time
$\square \mathrm{O}(\mathrm{m}+\mathrm{n} \log \mathrm{n})$ for adj list
- Use a PQ
$\square$ Regular PQ produces time $O(n+m \log m)$
- Can improve to $O(m+n \log n)$ using a fancier heap


## Greedy Algorithms

$\square$ These are examples of Greedy Algorithms
$\square$ The Greedy Strategy is an algorithm design technique

> - Like Divide \& Conquer
$\square$ Greedy algorithms are used to solve optimization problems

- The goal is to find the best solution
$\square$ Works when the problem has the greedy-choice property
- A global optimum can be reached by making locally optimum choices
- Example: the Change Making Problem: Given an amount of money, find the smallest number of coins to make that amount
- Solution: Use a Greedy Algorithm
- Give as many large coins as you can
- This greedy strategy produces the optimum number of coins for the US coin system
- Different money system $\Rightarrow$ greedy strategy may fail
- Example: old UK system


## Similar Code Structures

while (some vertices are unmarked) \{
$\mathrm{v}=$ best of unmarked vertices;
mark v;
for (each w adj to v) update w;
\}

- Breadth-first-search (bfs)
-best: next in queue
-update: $\mathrm{D}[\mathrm{w}]=\mathrm{D}[\mathrm{v}]+1$
- Dijkstra's algorithm
-best: next in priority queue
-update: $D[w]=\min (D[w], D[v]+c(v, w))$
- Prim's algorithm
-best: next in priority queue
-update: $\mathrm{D}[\mathrm{w}]=\min (\mathrm{D}[\mathrm{w}], \mathrm{c}(\mathrm{v}, \mathrm{w}))$
here $c(v, w)$ is the $v \rightarrow w$ edge weight


## Traveling Salesman Problem

$\square$ Given a list of cities and the distances between each pair, what is the shortest route that visits each city exactly once and returns to the origin city?
$\square$ Basically what we want the butterfly to do in A6! But we don't mind if the butterfly revisits a city (Tile), or doesn't use the very shortest possible path.
$\square$ The true TSP is very hard (NP complete)... for this we want the perfect answer in all cases, and can't revisit.
$\square$ Most TSP algorithms start with a spanning tree, then "evolve" it into a TSP solution. Wikipedia has a lot of information about packages you can download...


## THREADS: WHO NEEDS ‘EM?

Introduction to the concept...

## The Multicore Trend

$\square$ Moore's Law: Computer speeds and memory densities nearly double each year
$\square$ But we no longer are getting this speed purely by running a faster CPU clock
$\square$ CPU = "central processor unit"
$\square$ CPU clock roughly determines instructions / second for the computer


## Issue: A fast computer runs hot

$\square$ Power dissipation rises as the square of the CPU clock rate
$\square$ Chips were heading towards melting down!
$\square$ Multicore: with four
CPUs (cores) on one chip, even if we run each at half speed we get more overall performance!


## How a computer works

$\square$ Your program translates to machine instructions
$\square$ CPU has a pointer into the code: Program Counter
$\square$ To execute an instruction, it fetches what the PC points to, decodes it, fetches the arguments, and performs the required action (such as add two numbers, then store at some location)
$\square$ We call this a "thread of execution" or a "context of execution"
$\square$ One CPU == 1 thread, right? Well, not really....

## Each program has its own thread!

$\square$ Earliest days: shared one CPU among many programs by just having it run a few instructions each, "round robin"
$\square$ Program A gets to run 10,000 instructions
$\square$ Then pause A, "context switch" to B, run 10,000 of B

- Then pause $B$, context switch to $C$, run 10,000 for $C .$. .
$\square$ This makes one CPU seem like N (slower) CPUs
$\square$ With the new trend toward multicore we can have a lot of threads all concurrently active


## Keeping those

## cores busy

- The operating system provides support for multiple "processes"
- In reality there there may be fewer processors than processes
- Processes are an illusion - at the hardware level, lots of multitasking
- memory subsystem
- video controller
- buses
- instruction prefetching
- Virtualization can even let one machine create the illusion of many machines (they share disks, etc)



## How is a Thread defined?

$\square$ A separate "execution" that runs within a single program and can perform a computational task independently and concurrently with other threads
$\square$ Many applications do their work in just a single thread: the one that called main() at startup
$\square$ But there may still be extra threads...
$\square$... Garbage collection runs in a "background" thread
$\square$ GUls have a separate thread that listens for events and "dispatches" upcalls
$\square$ Today: learn to create new threads of our own

## What is a Thread in Java?

$\square$ A thread is a kind of object that "independently computes"
$\square$ Has an associated stack and local variables (context)
$\square$ Needs to be created, like any object
$\square$ Then "started". This causes some method (like main()) to be invoked. It runs side by side with other thread in the same program and they see the same global data
$\square$ The actual execution could occur on distinct CPU cores, but Java could also simulate multiple cores. You can't really tell which approach Java is using

## Concurrency

$\square$ Concurrency refers to a single program in which several threads are running simultaneously
$\square$ Special problems arise: These threads literally access the same shared memory regions at the same time!
$\square$ They are at risk of interfering with each other, e.g. if one thread is modifying a complex structure like a heap while another is trying to read it
$\square \ln \operatorname{cs} 2110$ we focus on simple ways to use this model without bugs introduced by interference

